



تحلیل سازه ۱

دکتر حر خسروی

ویدئو کامل جلسات تحلیل سازه مطابق با این جزوه، در کانال یوتیوب به
آدرس زیر قابل دسترسی می باشد.

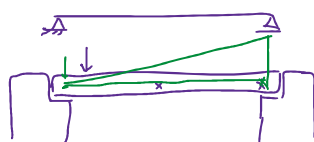
@daneshesaze

مهر ۱۴۰۳

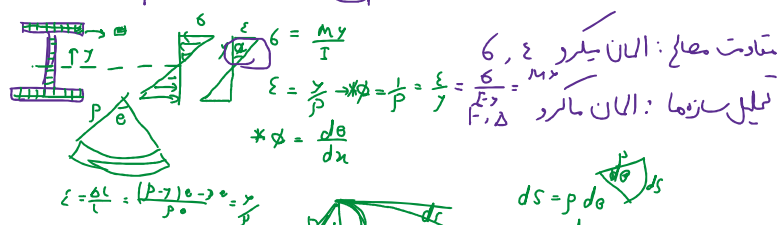
تحلیل سازه ها I

سرفصل درسی

- ① پایداری و معین سازه ها
- ② تحلیل سازه ها معین استاتیکی
- ③ محاسب تغییر شکل ها سازه بر روش ها سطح، بارالایک و تیر مزدوج
- ④ محاسب تغییر شکل ها بر روش ها انرژی شامل کار مجازی و بار واحد، قضایای دانتلیانو و ماکسول
- ⑤ تحلیل سازه ها معین بر روش نیروی (نرو) : روش سگاری تغییر شکل ها
- ⑥ خط تأثیر سازه ها معین



موضوع درسی تحلیل سازه ها:



$$\epsilon = \frac{\Delta L}{L} = \frac{p \cdot \Delta x}{L} = \frac{p}{E} \cdot \frac{\Delta x}{L}$$

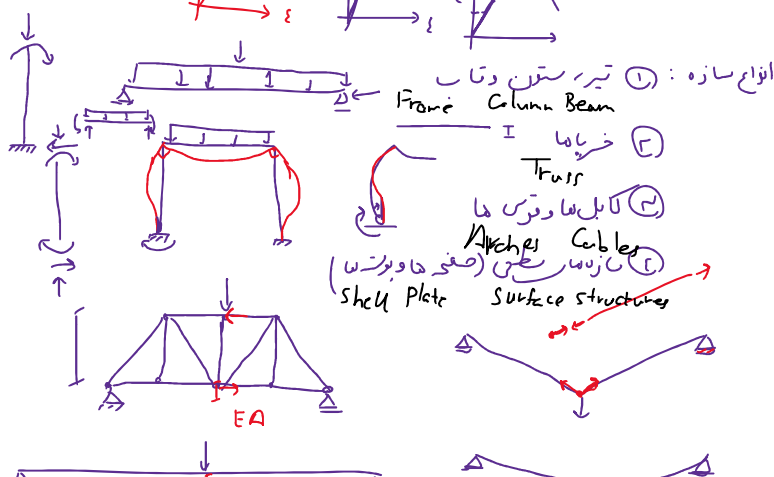
$$\phi = \frac{d\epsilon}{dx} = \frac{p}{E} \cdot \frac{1}{L}$$

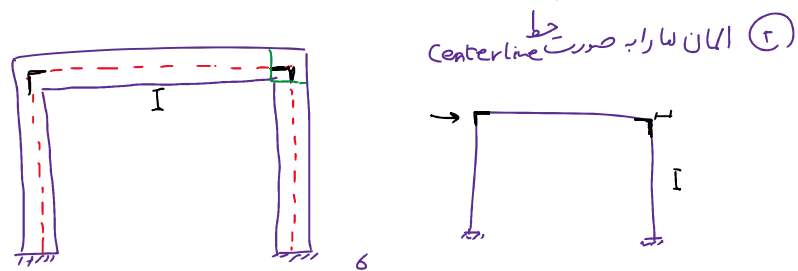
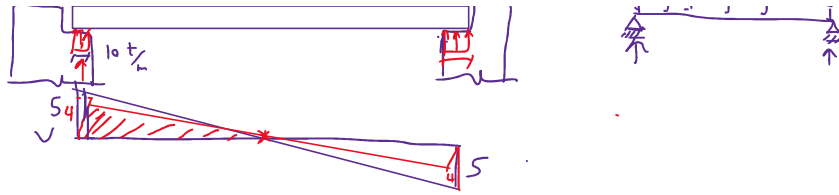
$$\theta = \frac{d\phi}{dx} = \frac{p}{E} \cdot \frac{1}{L^2}$$

$$\delta = \frac{p}{E} \cdot \frac{1}{L^3}$$

$$\phi = \frac{d\epsilon}{dx} = \frac{p}{E} \cdot \frac{1}{L} = \frac{M}{EI}$$

$$\theta = \frac{d\phi}{dx} = \frac{p}{E} \cdot \frac{1}{L^2} = \frac{M}{EI}$$





(۲) المان ها را به صورت خط centerline

(۳) رفتار خط مصالح $\epsilon = E \epsilon$ $\frac{1}{\rho} = \frac{M}{EI}$

(۴) صرف تغییر شکل ها (موضع تغییر شکل ها را بچند)

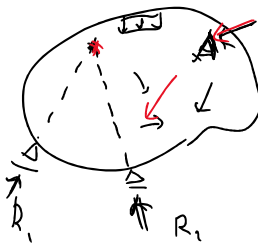


video2

مض اول:

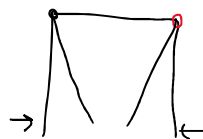
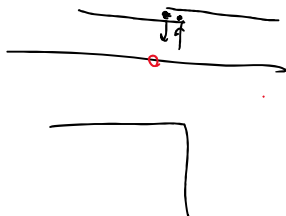
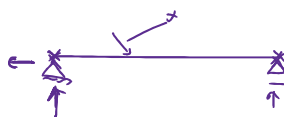
باید ارس و معین سازه ما

$\sum F_x = 0$ $\sum F_y = 0$ $\sum M_A = 0$
 $\sum F_x = m a_x$ $\sum F_y = m a_y$ $\sum M = I \alpha$

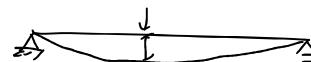


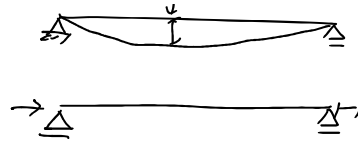
$\sum M_A = 0$ $\sum M_B = 0$ $\sum F_y = 0$
 به شرطی که محور عمود بر استوار AB عمود باشد.

$\sum M_A = 0$ $\sum M_B = 0$ $\sum M_C = 0$
 به شرطی که A، B، C در یک استوار باشند.



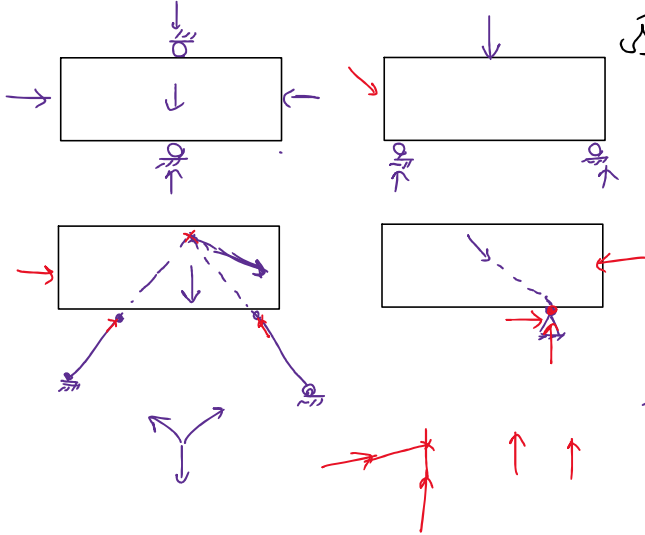
حرکت Rigid body motion
تغییر شکل deformation





بایداری و معین یک جسم صلب

① لازم است که مجموع ناپایداری رتبه‌های
میلنگه‌ها صفر باشد

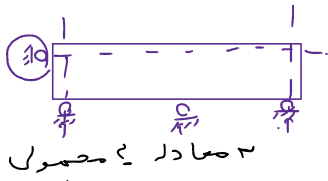


تعادل ناپایدار

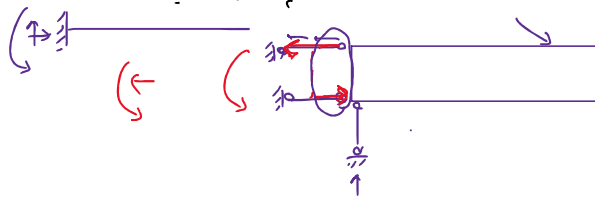


② مجموع میلنگه‌ها صفر

بایداری و معین

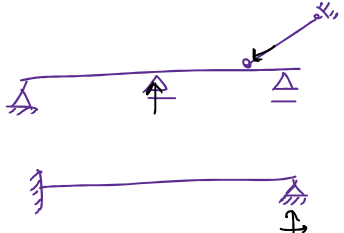


۳ معادله در ۳ مجهول



③ این را به مجموع میلنگه‌ها صفر

$$5 - 3 = 2$$

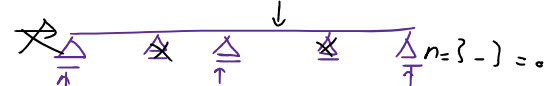


بایداری نامعین

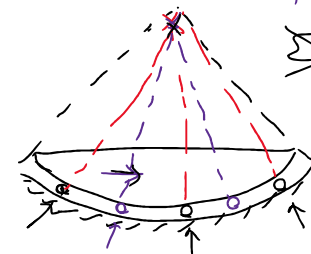
$$2$$

④ به مجموع میلنگه‌ها صفر لازم است ولی کافی نیست

ناپایداری هندسی خارجی

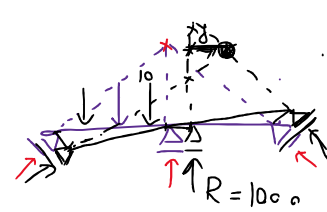
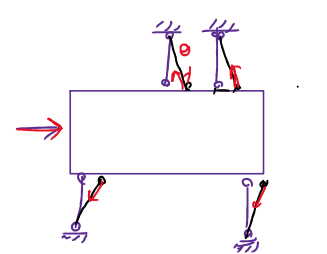


$$n = 3 - 3 = 0$$



ناپایداری هندسی

$$n = 3 - 3 = 0$$

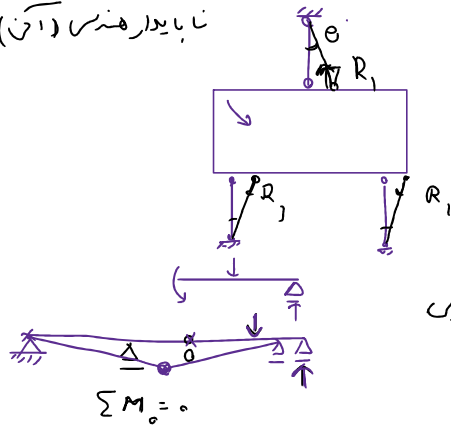


ناپایداری هندسی (آهن)



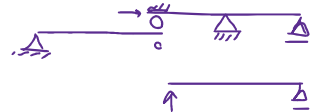
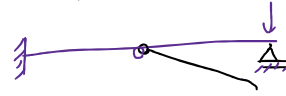
سایداره نه ی

نایداره نه ی (آژن)



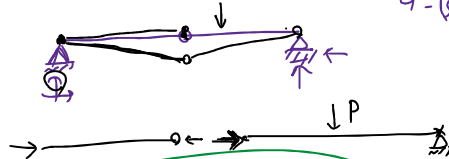
نایدارات تکی

سایدارس ر صعیب سازه



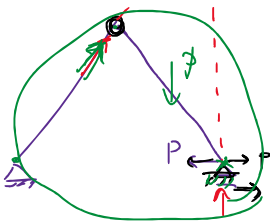
$$\sum M_o = 0$$

$$4 - (3 + 1) = 0$$

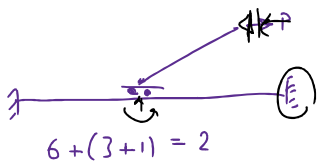
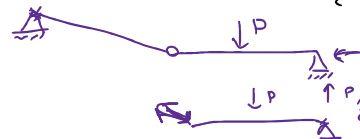
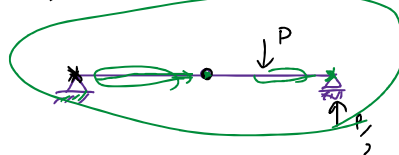
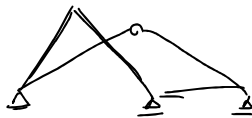


$$\sum M_o = 0$$

$$\sum F_x = 0$$



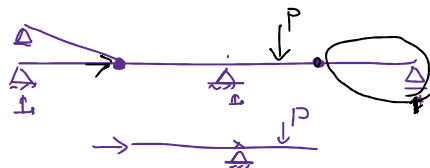
نایداره نه ی داخلی



$$6 + (3 + 1) = 2$$



$$n = 7 - (3 + 4) = 0$$



$$n = 5 - (3 + 2) = 0$$

video3

ترکیب سایدار اجسام صلب

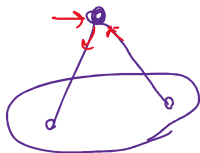
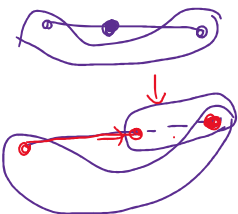
① ترکیب یک جسم صلب یکد متصل

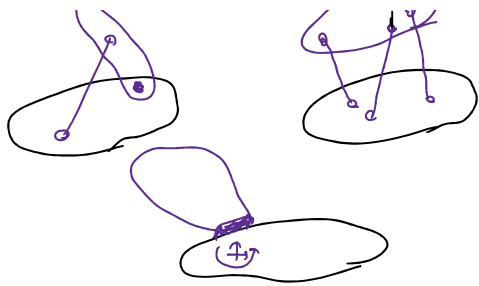
حوافل آ محیدل
(ءء ملء)

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases}$$

② ترکیب سایدار دو جسم صلب

$$\sum F_x = 0$$

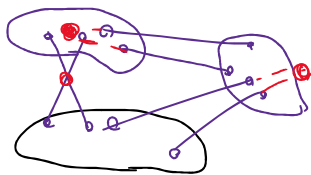




⑤ ترکیب پایدار دو جسم صلب

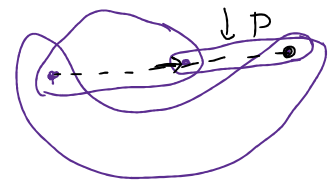
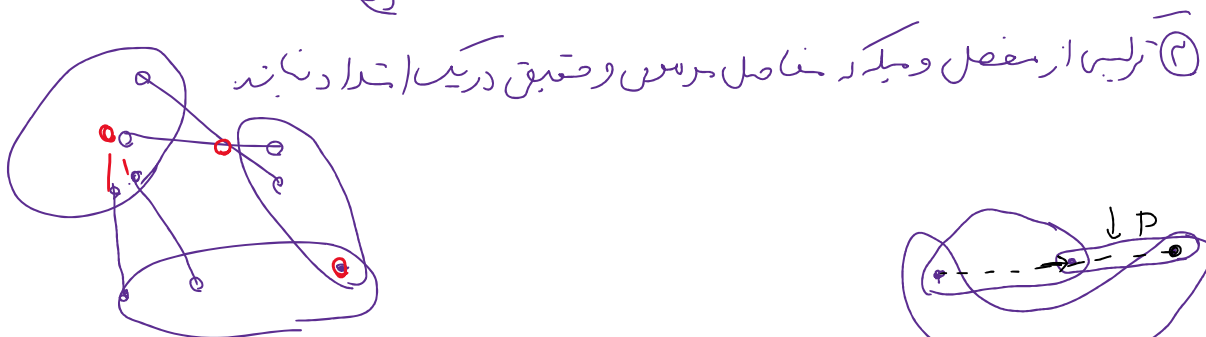
$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases} \rightarrow \text{حالات ۳ محمول}$$

⑥ ترکیب پایدار سه جسم صلب

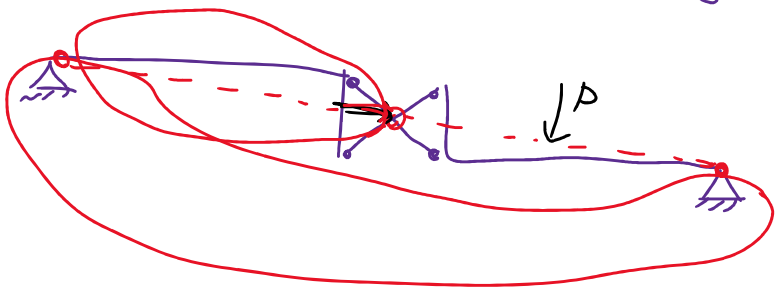
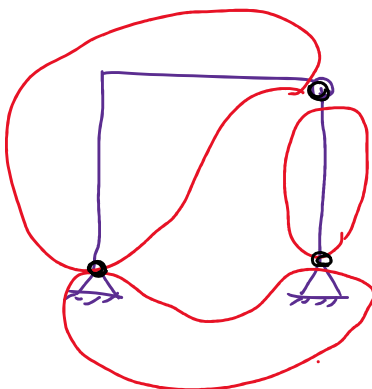


⑦ محمول ۴ معادله
⑧ مفصل در ترکیب استوار نباشد

⑨ یک میله در سه جسم صلب با یک میله وصل شود و
مناصل هر دو در ترکیب استوار نباشد

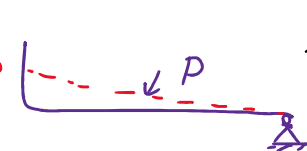


مثال:

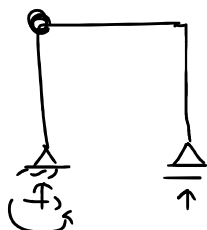


$n=0$

پایدار



ناپایدار



$n=4-3=1$

تعداد معادلات - تعداد محمولات = n

درجه نامعین سازه:

محمولات تکیه‌ها

تعداد محمولات تکیه‌ها $|n|$

تعداد محبورات تکیه‌گاه‌ها (r)

① غلغلی



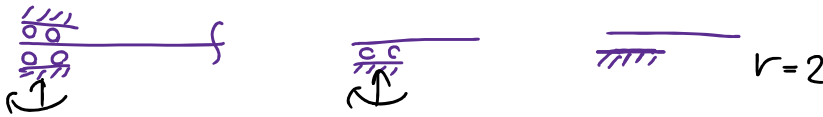
② مفصلی



③ مفصل بزرگ



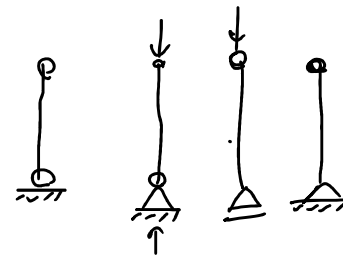
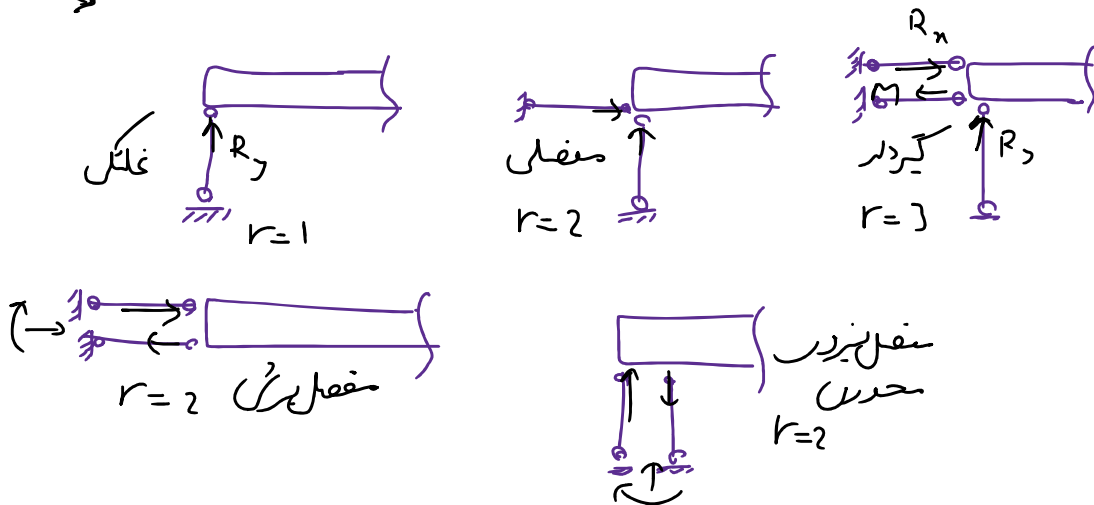
④ مفصل نیزه محدود



⑤ گیردار

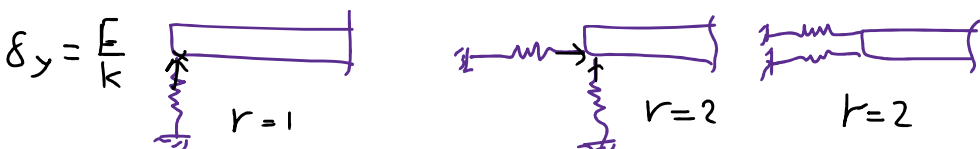


⑥ میلان



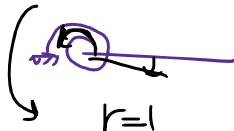
⑦ تکیه‌گاه لارجامی

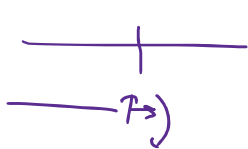
۱- فنر محدود



۲- فنر جریقی

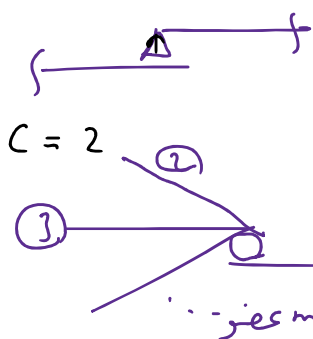
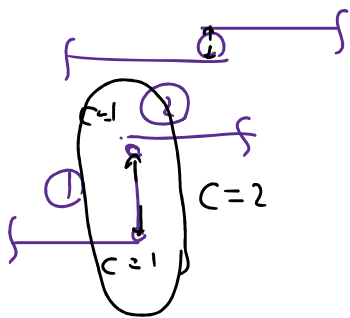
⑧ تکیه‌گاه ترکیبی





①
 $\sum F_x = 0 \quad C = 1$

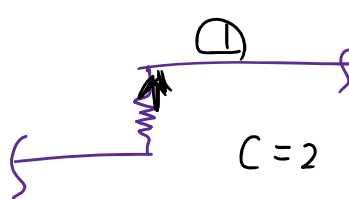
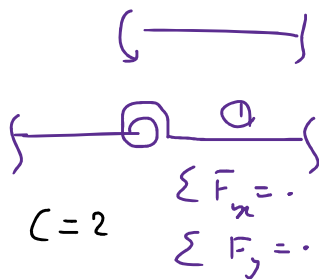
④ مفصل محدد داخلی



⑤ غلنگ داخلی

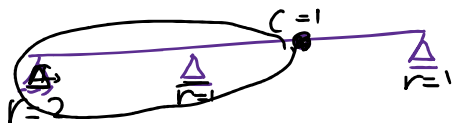
①
 $\sum F_x = 0$
 $\sum M = 0$

$C = 2(m-1)$

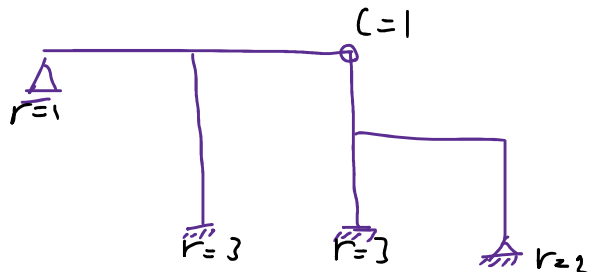


⑥ فنر داخلی

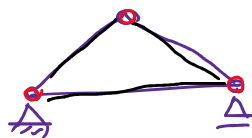
مثال:



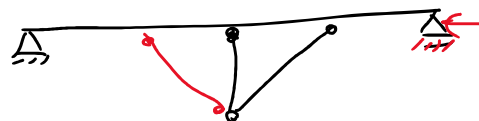
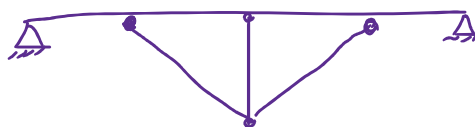
$n = 4 - (3 + 1) = 0$



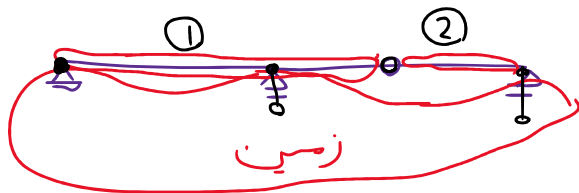
$n = 9 - (3 + 1) = 5$



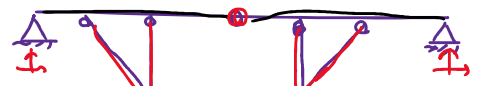
$n = 0$



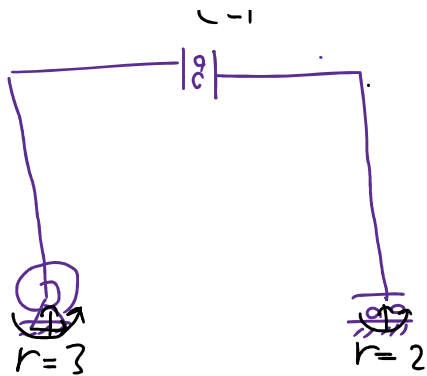
$n = 2$



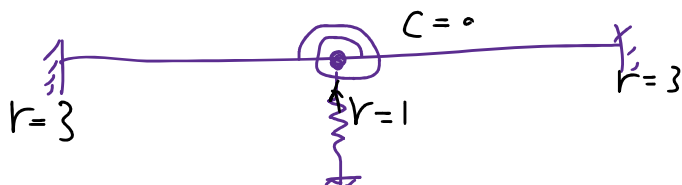
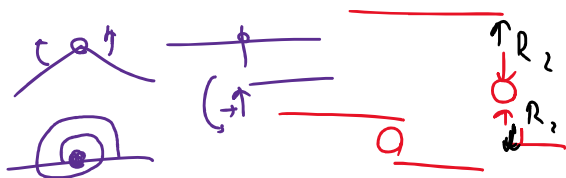
$C = 1$



$n = 1$



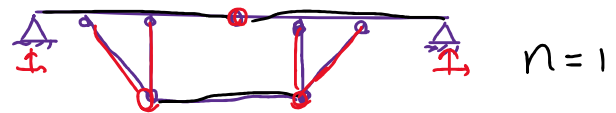
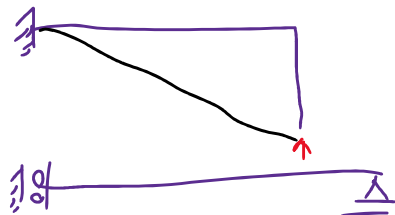
$$n = 5 - (3 + 1) = 1$$



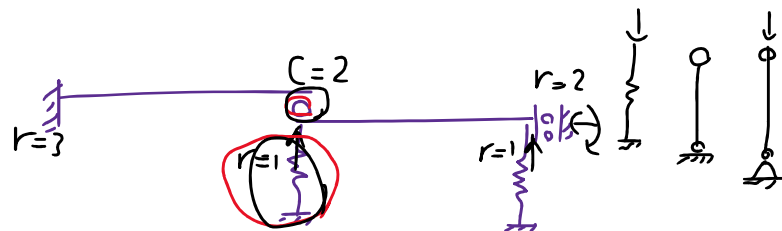
$$n = 7 - (3 + 0) = 4$$

کابل درستی $n = 2$

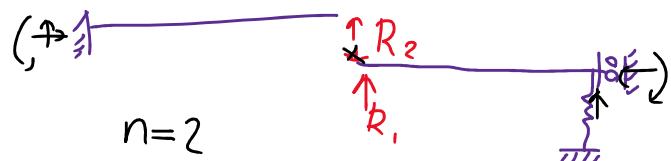
کابل درنگ $n = 6 - (3 + 2) = 1$



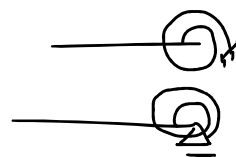
$$n = 1$$



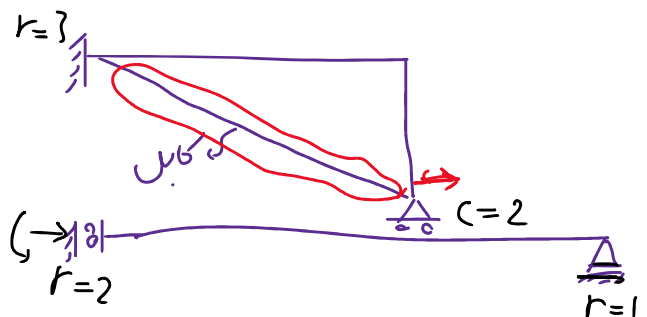
$$n = 7 - (3 + 2) = 2$$



$$n = 2$$

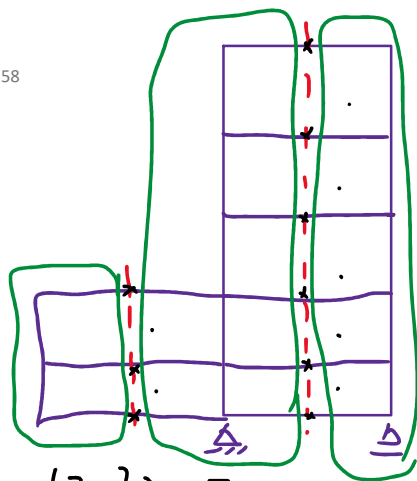


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$$n = 1$$

$$n = 2$$

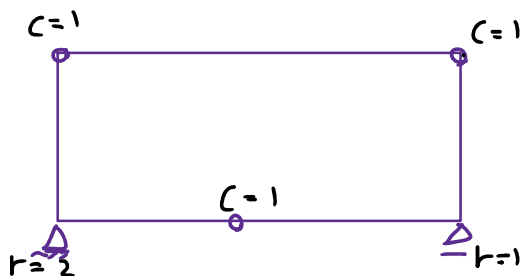


$$h = (3 + 9 \times 3) - (3 \times 3) = 7 \times 3$$

$$h = (3 + 6 \times 3) - (2 \times 3) = 5 \times 3 \quad h = (3 + 2 \times 3) - (2 \times 3) = 3$$

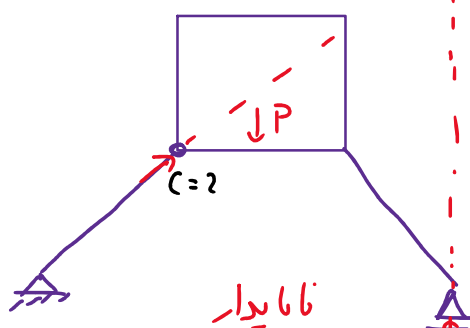
$$n = (3 + r) - (3 + c)$$

معادلات شرط
تعداد واشر
کجه گاه
تعداد حلقه بسته

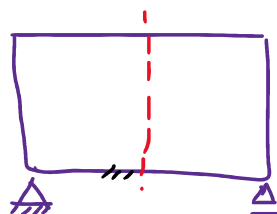
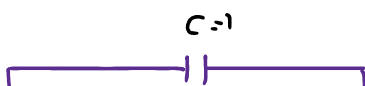


$$h = (3 + 1 \times 3) - (3 + 3) = 0$$

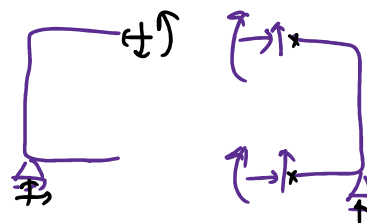
$$n = (3 \times 4 + 3) - (4 \times 3 + 3) = 0$$



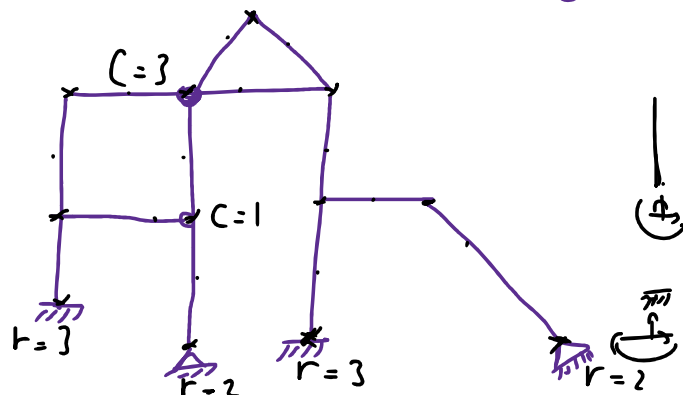
$$n = (3 + 1 \times 3) - (3 + 2) = 1$$



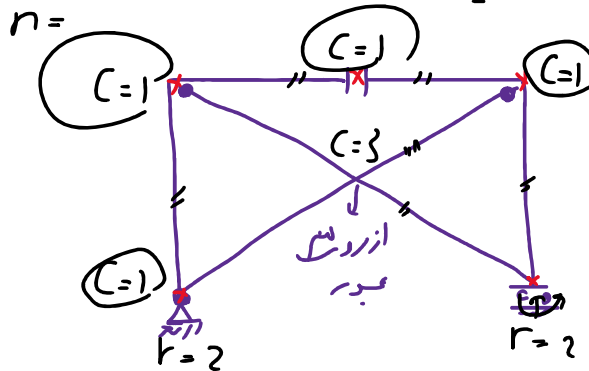
حلقه بسته



$$n = (13 \times 3 + 10) - (12 \times 3 + 4) = 9$$

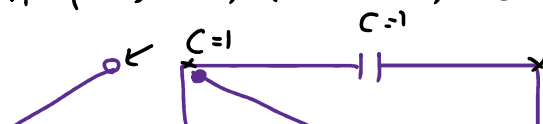


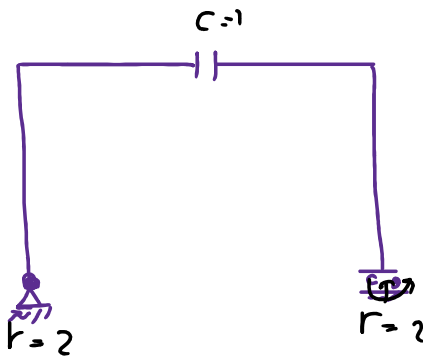
$$n = (10 + 2 \times 3) - (3 + 4) = 9$$



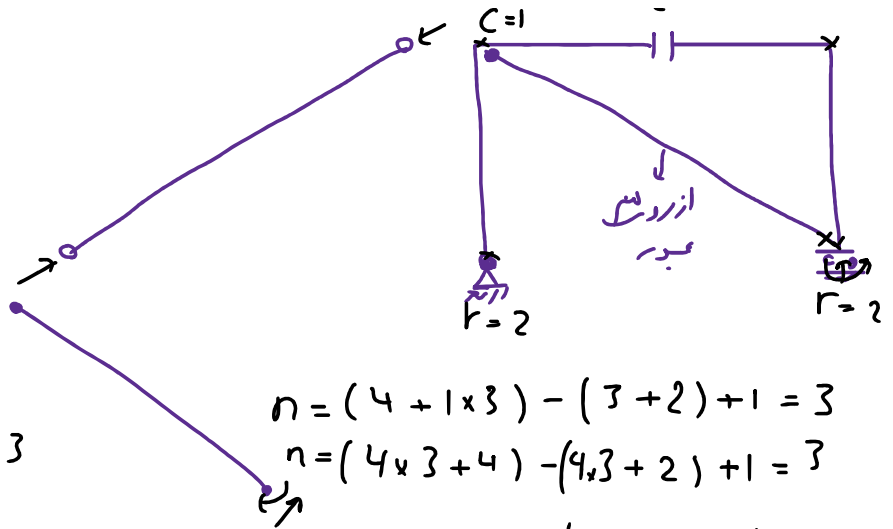
$$n = (4 + 3 \times 3) - (3 + 7) = 3$$

$$n = (6 \times 3 + 4) - (5 \times 3 + 4) = 3$$





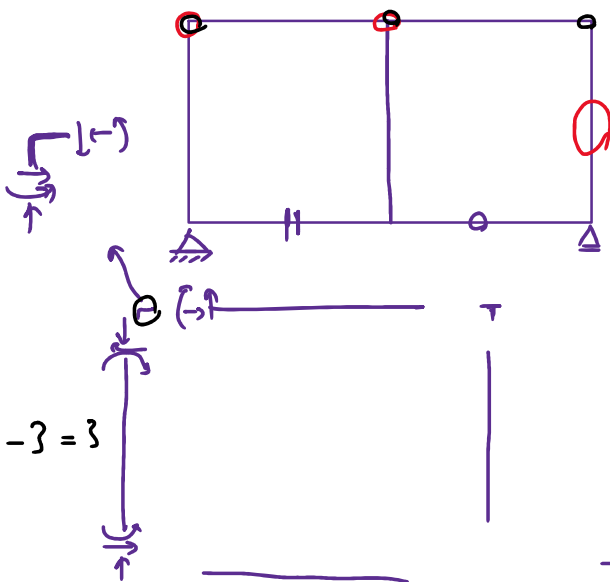
$$h = (4 + 0) - (3 + 1) + 1 + 2 = 3$$



$$n = (4 + 1 \times 3) - (3 + 2) + 1 = 3$$

$$n = (4 \times 3 + 4) - (4 \times 3 + 2) + 1 = 3$$

رابطه دوم درجه نامعین متابعا

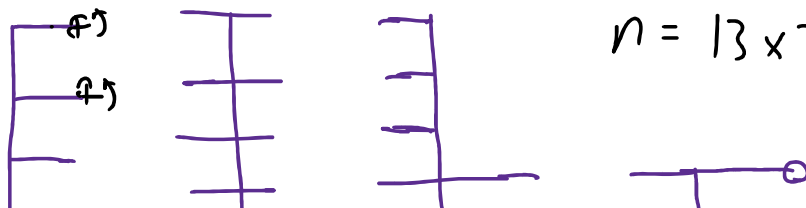
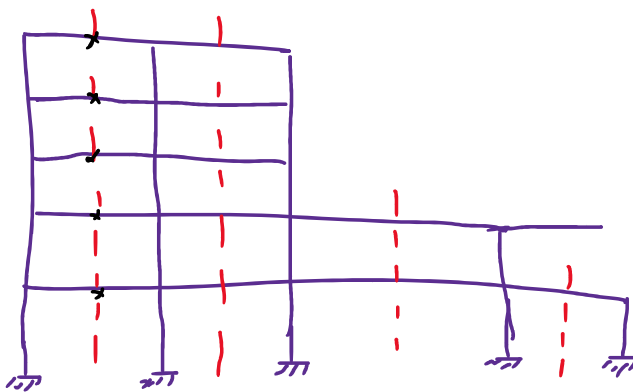


$$n = (3m + r) - (3j + c)$$

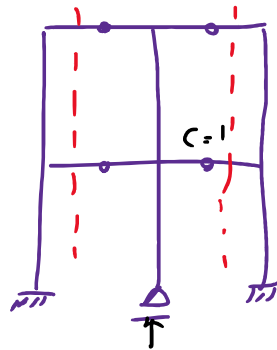
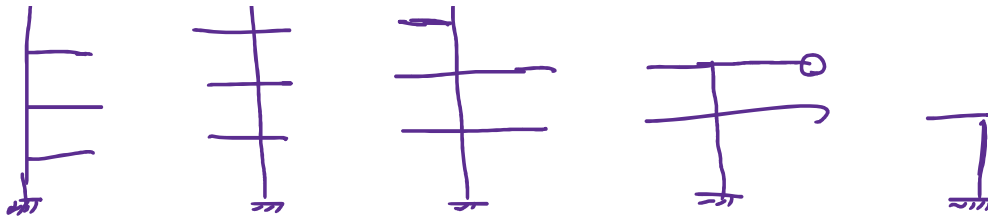
که تعداد اعضا (m) و تعداد گره ها (r) و تعداد اجزای (j) و تعداد اجزای (c)

$$6 - 3 = 3$$

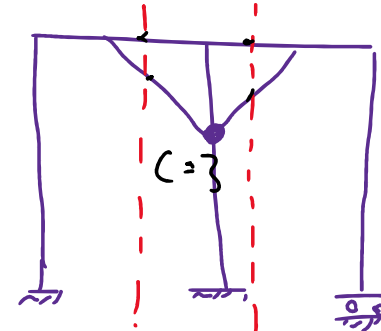
رشته درختی برر متابعا



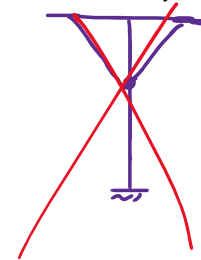
$$n = 13 \times 3 = 39$$



$$n = (4 \times 3) - (2 + 4) = 6$$



$$n = 4 \times 3 - (1 + 3) = 8$$



بایداری و معین خرابها

انواع خرابها:

① خراب ساده

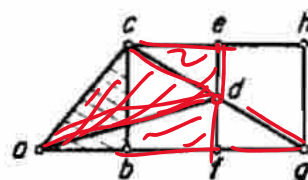
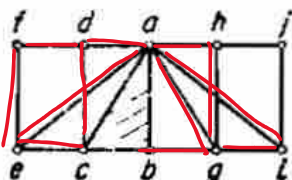
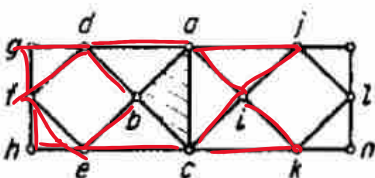
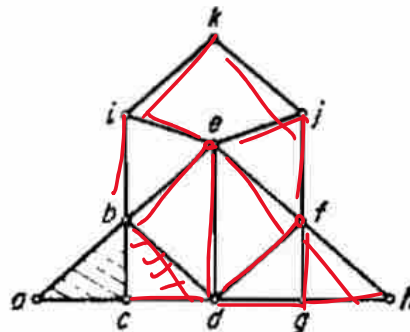
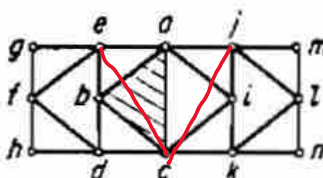
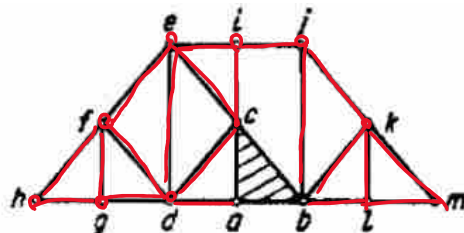
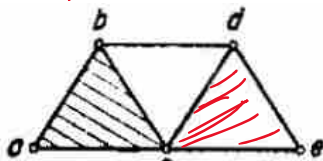
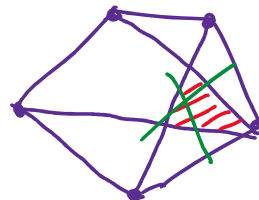
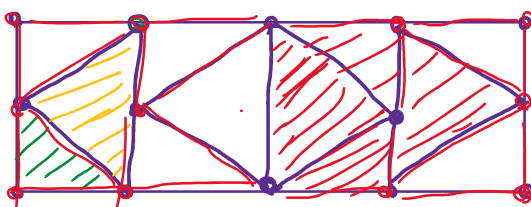
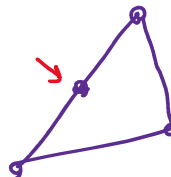
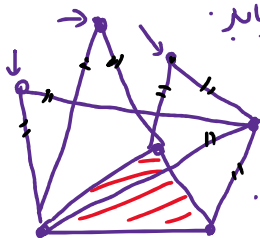
② خراب مرکب

③ خراب پیچیده (بفرخ)

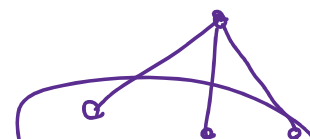
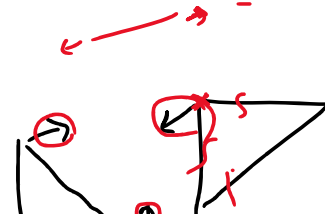
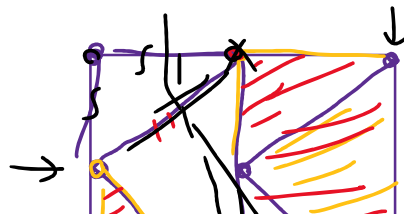
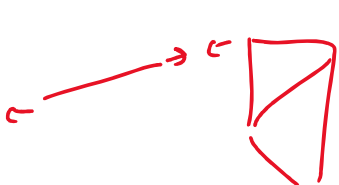
① خراب ساده

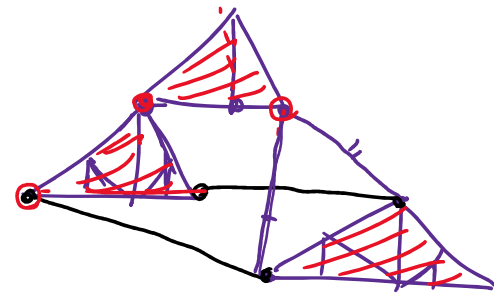
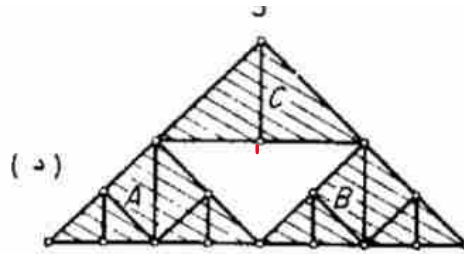
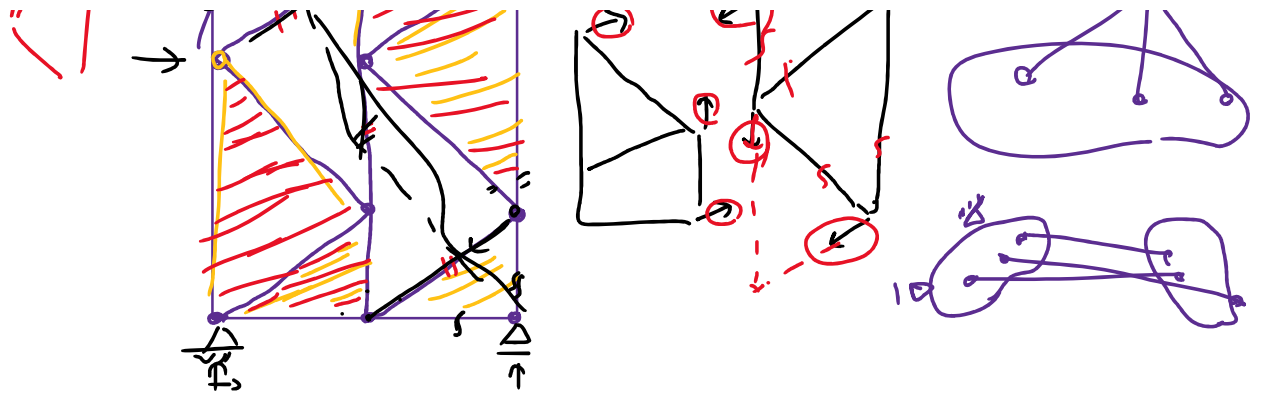
از تریب باید یک جبهه و یک منصل (یک مثلث) شروع شود

با دومین و یک منصل لستری می یابد

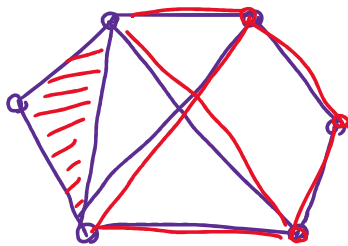


② خراب مرکب: خرابی است که از تریب باید دو یا چند خراب ساده ای در خود

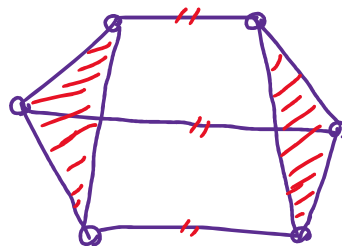




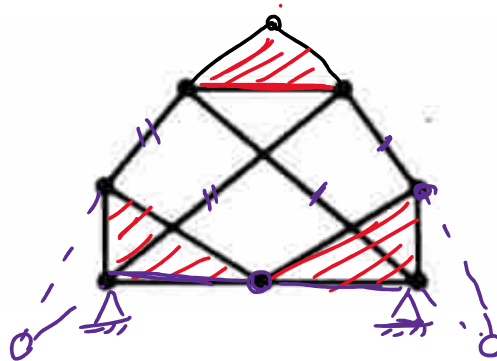
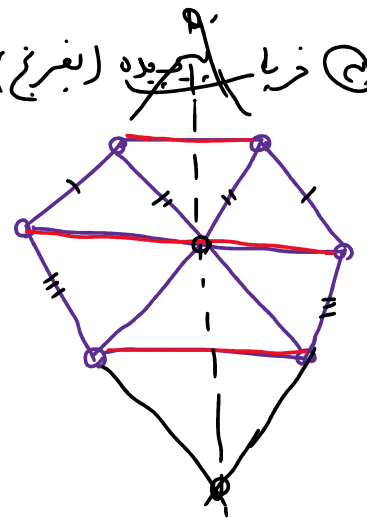
② خرابی‌های پیچیده (بفرغی) خرابی‌ها در استخفات خرابی‌ها در مرکب باشد.



ساد



مرکب



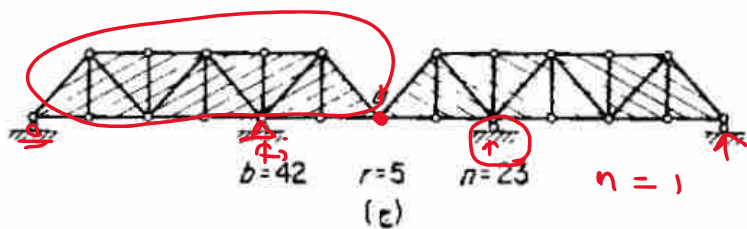
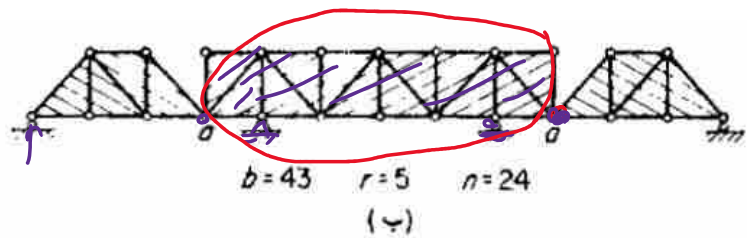
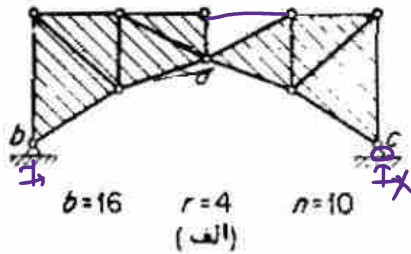
$$n = (m + r) - 2j$$

تعداد اضلاع
تعداد مفاصل
تعداد اعضا
تعداد دگر

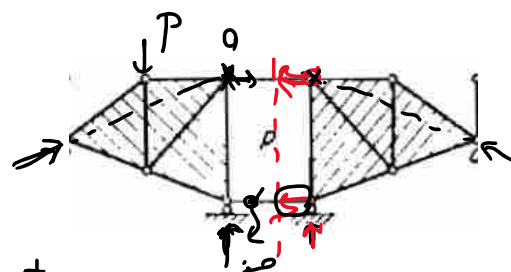
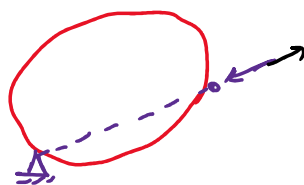
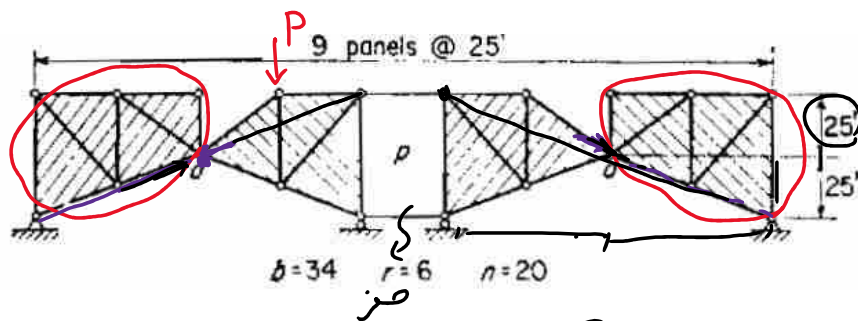
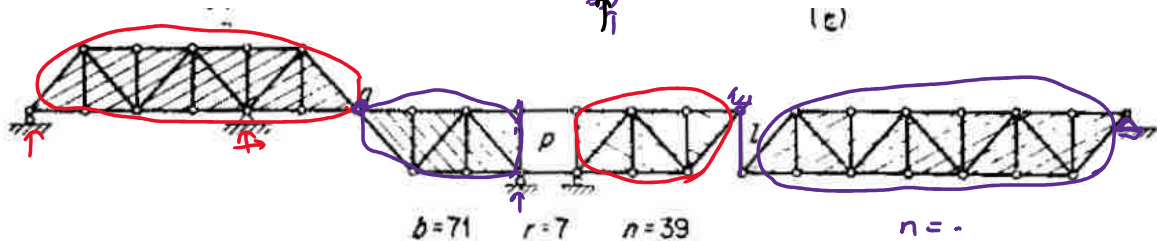
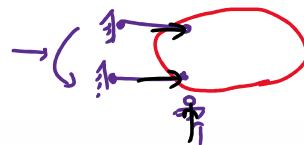
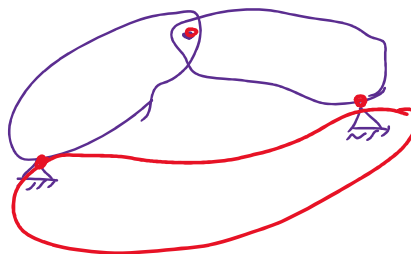
سداد منقول
تعداد محصول
تعداد ارجاع

$$16 + 4 - 10(2) = 0$$

مثال: درجه نامعین و پایداری خنجر را بررسی کنید.

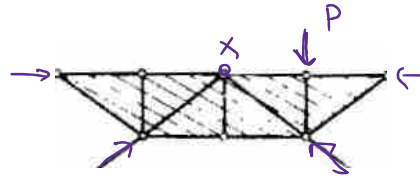
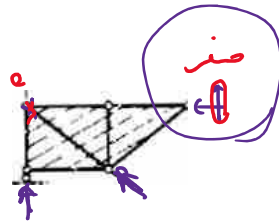
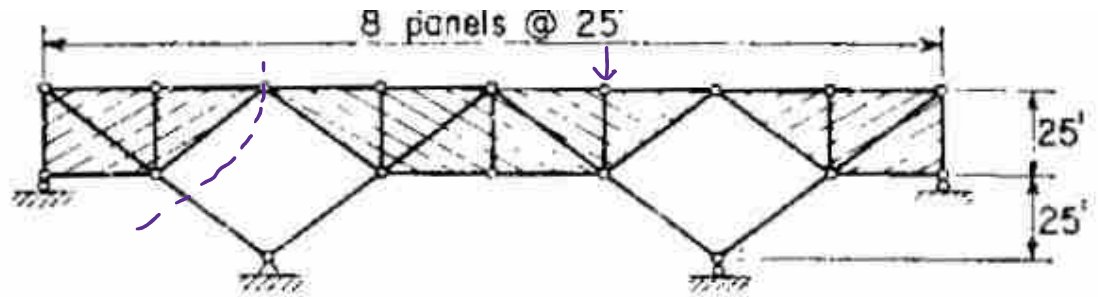


$$42 + 5 - 46 = 1$$

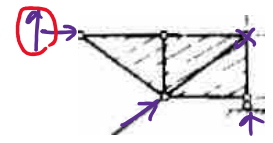




$$\sum M_o \neq 0$$



مركز



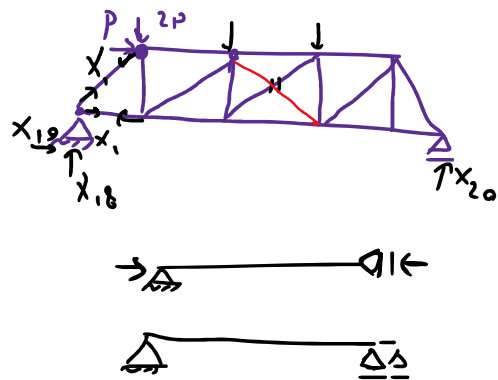
$$\sum M_x \neq 0 \rightarrow$$

$$\sum M_o = 0 \rightarrow P(2a) = 0 \rightarrow P = 0$$

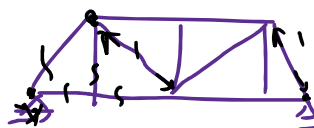
$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1,20}x_{20} = P_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2,20}x_{20} = P_2 \\ \vdots \\ a_{20,1}x_1 + \dots + a_{20,20}x_{20} = P_{20} \end{cases}$$

$$[A] \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{20} \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \\ \vdots \\ P_{20} \end{Bmatrix} \rightarrow \begin{cases} |A| \neq 0 \rightarrow \text{جواب یکتا} \\ |A| = 0 \rightarrow \text{جواب یکتا ندارد} \end{cases}$$

بررسی ناپایداری خرابی مابین روشی در تیرچین ضرایب



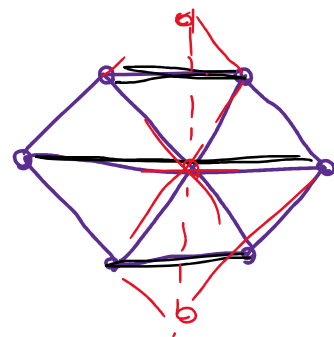
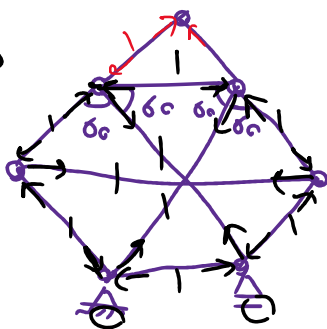
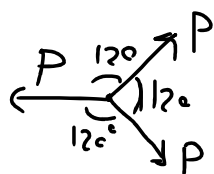
بررسی ناپایداری خرابی مابین روشی بار صفر



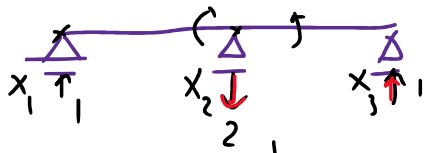
در یک سازه معین، اگر بارگذاری خارجی نداشته باشیم، نیروی درونی اعضا باید صفر باشد.

حال اگر بتوان یک دسته جواب غیر صفر پیدا کرد (بدون اینکه هیچ تناقضی در معادلات تعادل ایجاد شود) می توان نتیجه گرفت که ساختار ناپایدار است.

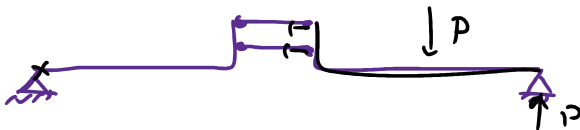
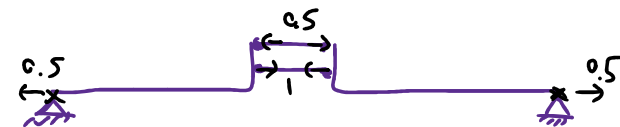
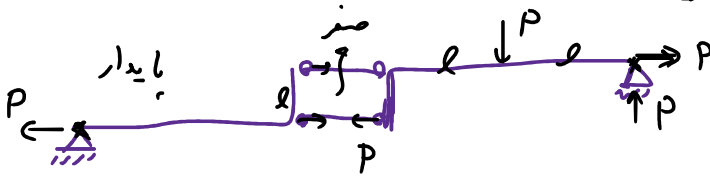
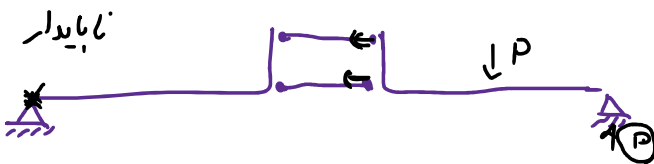
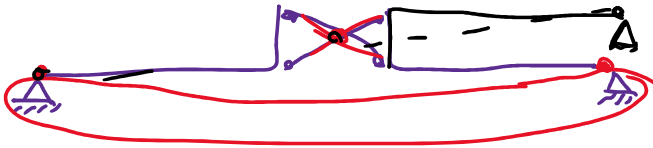
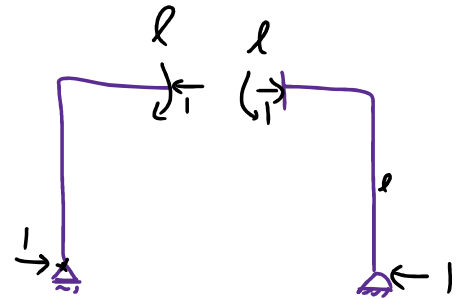
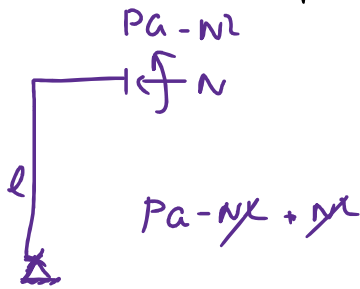
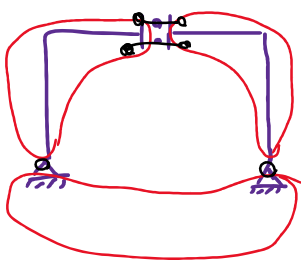
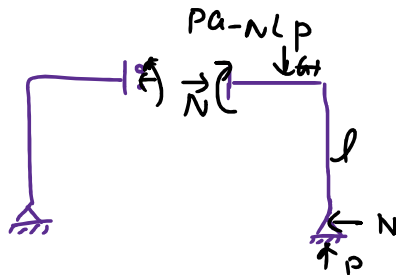
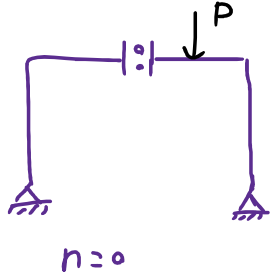
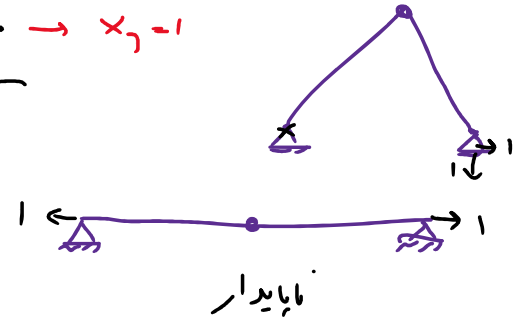
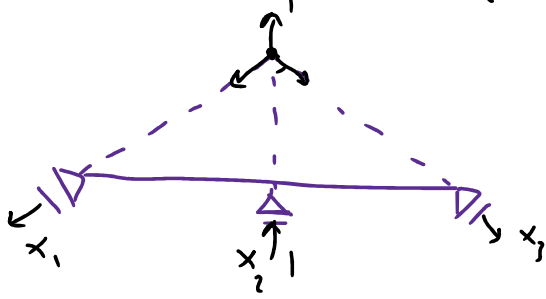
* روشی بار صفر نقطه ناپایداری سازه را اجابت می کند.



$n = 0$

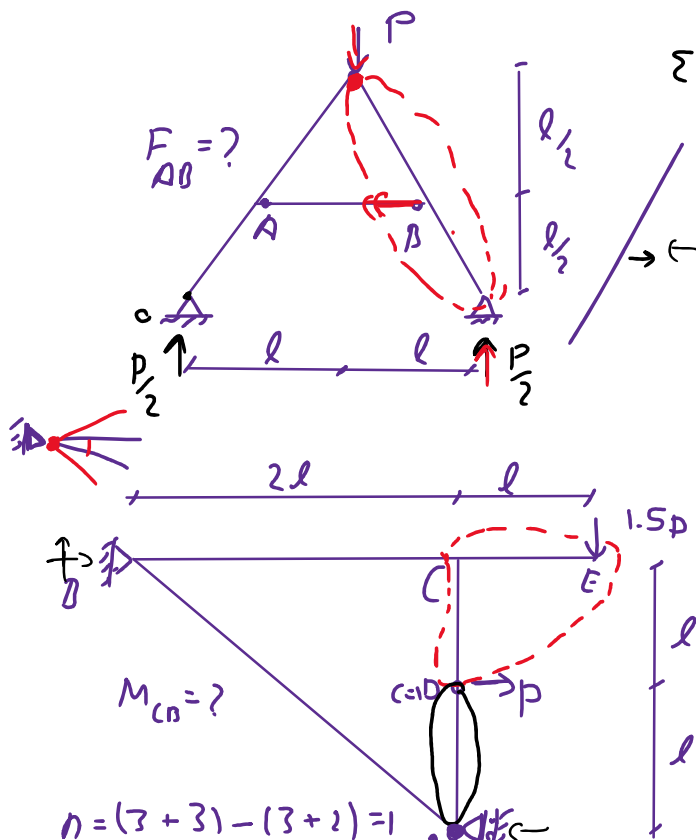
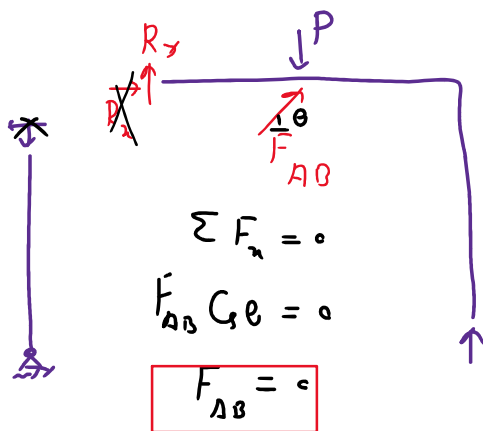
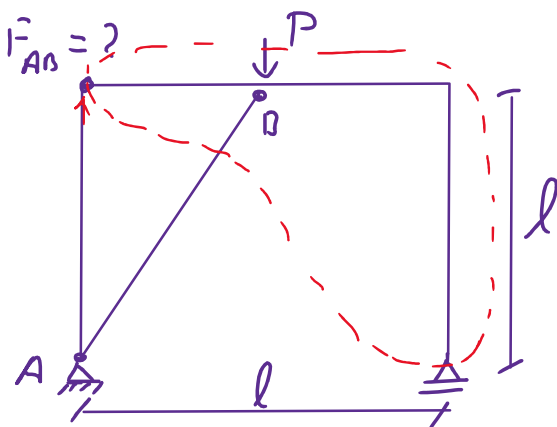
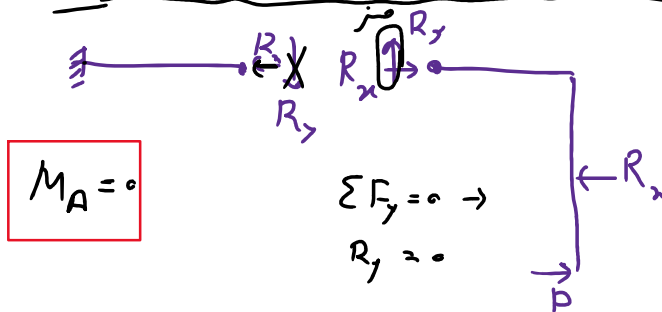
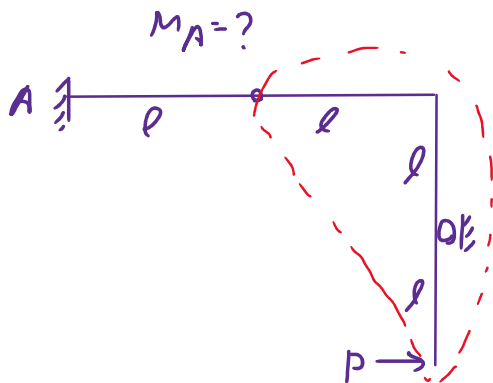


$$\begin{cases} x_1 + x_2 + x_3 = 0 & x_2 = -2 \\ x_1 - x_3 = 0 & \rightarrow x_3 = 1 \end{cases}$$

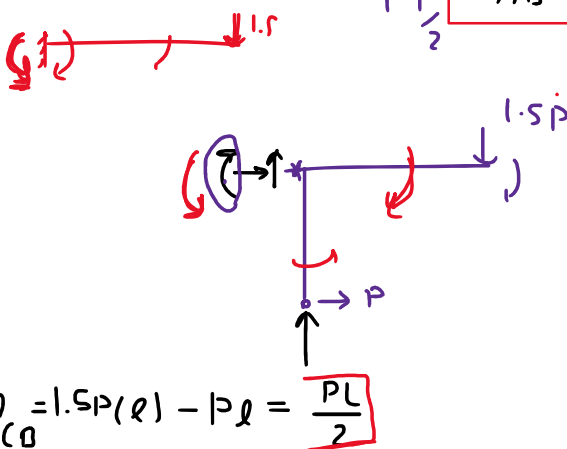
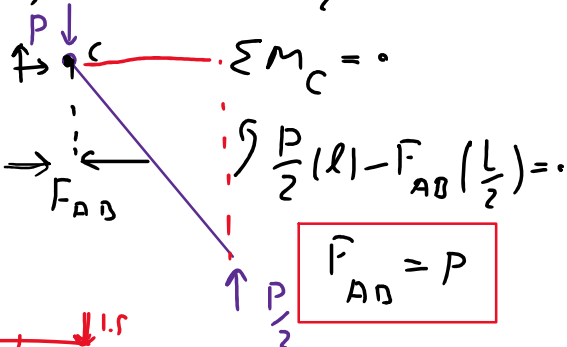


فصل دوم: تحلیل سازه هارمین

استفاده از معادلات قادل و دیگر ارام آزاد



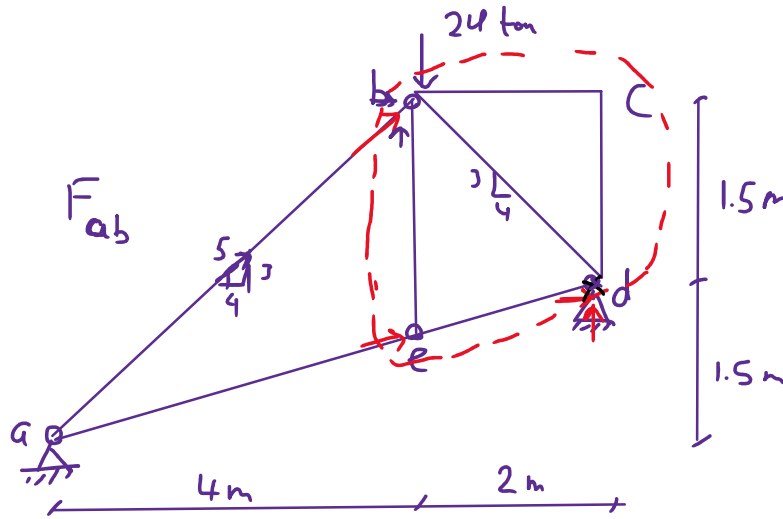
$$\sum M_y = 0 \rightarrow Pl - R_y(2l) = 0 \rightarrow R_y = \frac{P}{2}$$



$$n = (3 + 3) - (3 + 1) = 1$$


$$M_{CB} = 1.5P(l) - Pl = \frac{PL}{2}$$

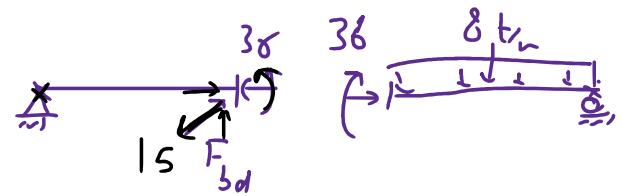
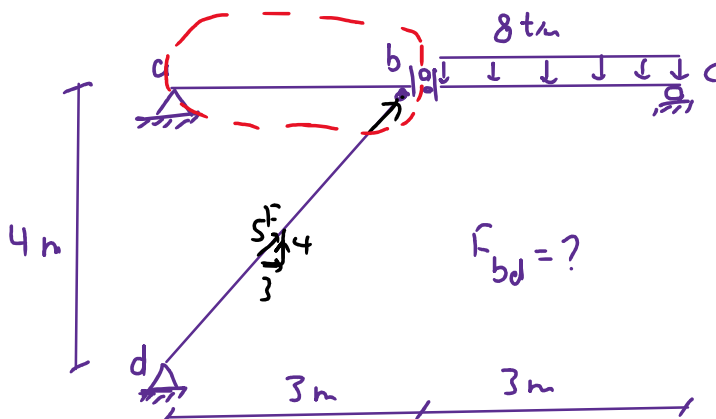
$$1.5P(l) - Pl + M_{CB} = 0 \rightarrow M_{CB} = -\frac{PL}{2}$$



$$\sum M_d = 0$$

$$24(2) - F\left(\frac{4}{5}\right)(1.5) - F\left(\frac{3}{5}\right)(2) = 0$$

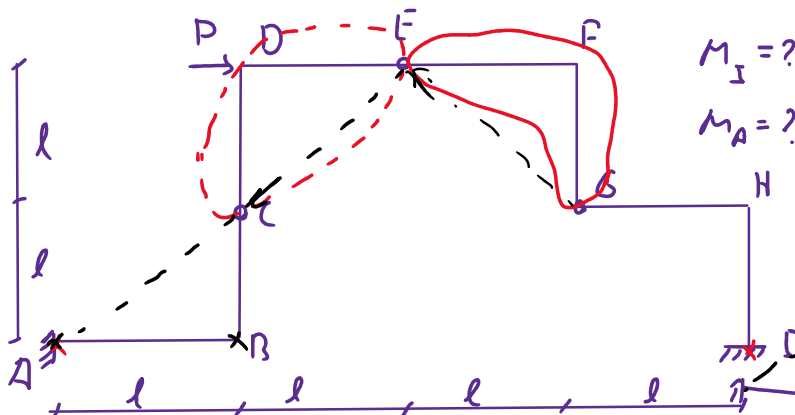
$$F_{ab} = 20 \text{ tons}$$



$$8(3)(1.5) - M = 0$$

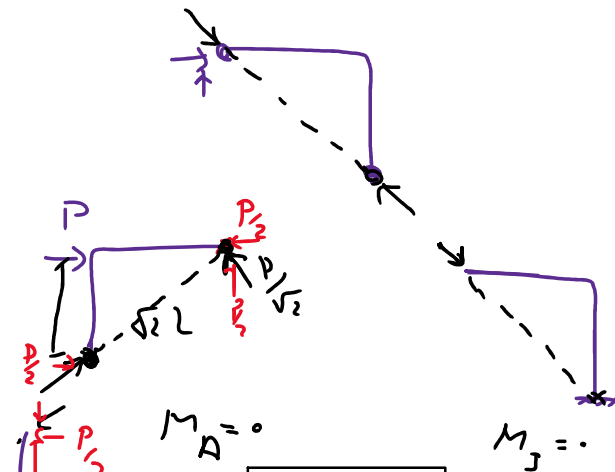
$$\frac{4}{5}F(3) + 36 = 0 \rightarrow F_{bd} = -15$$

P, V, M



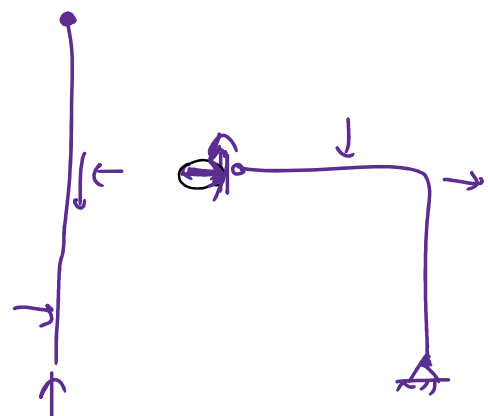
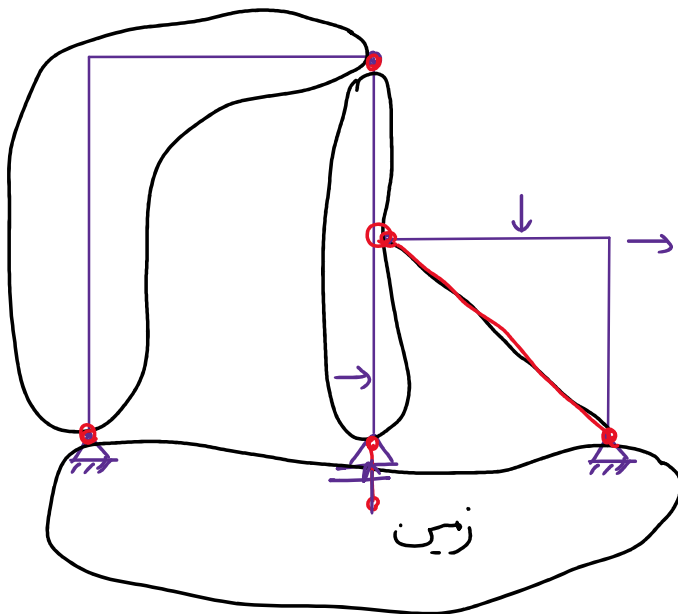
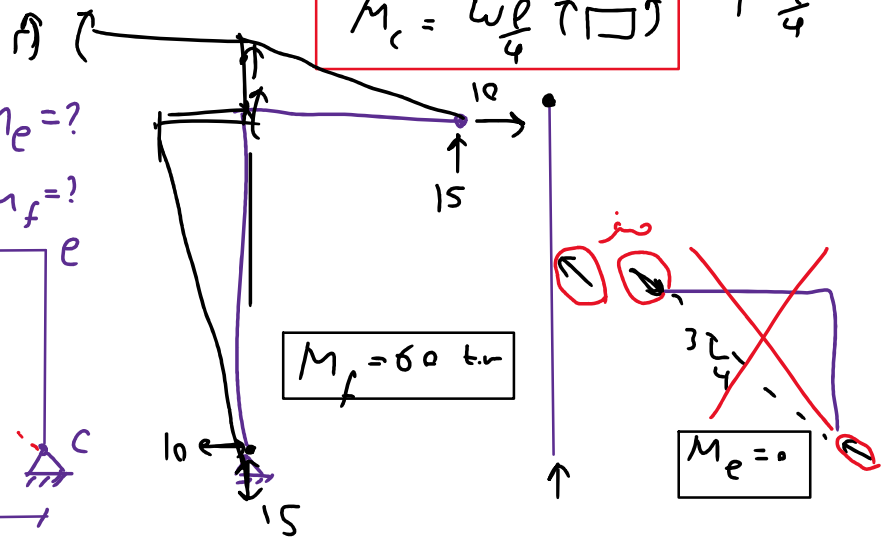
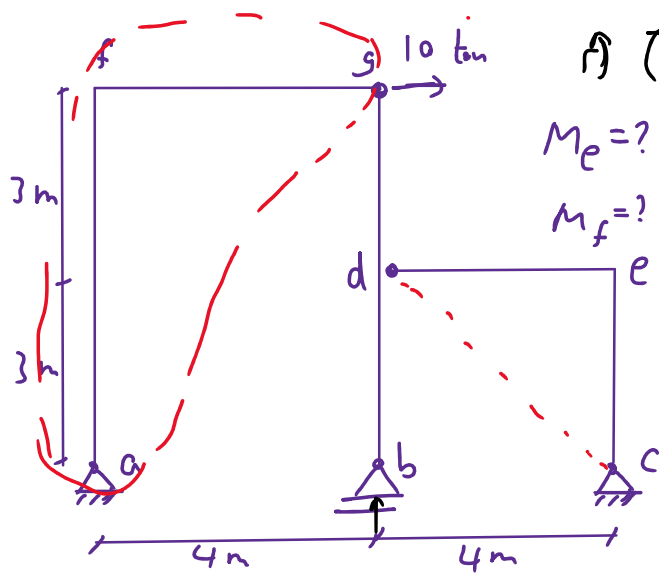
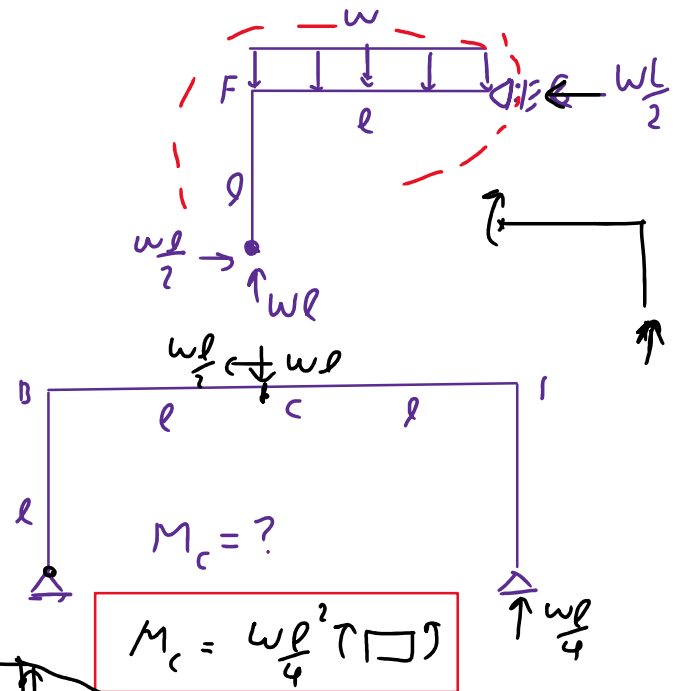
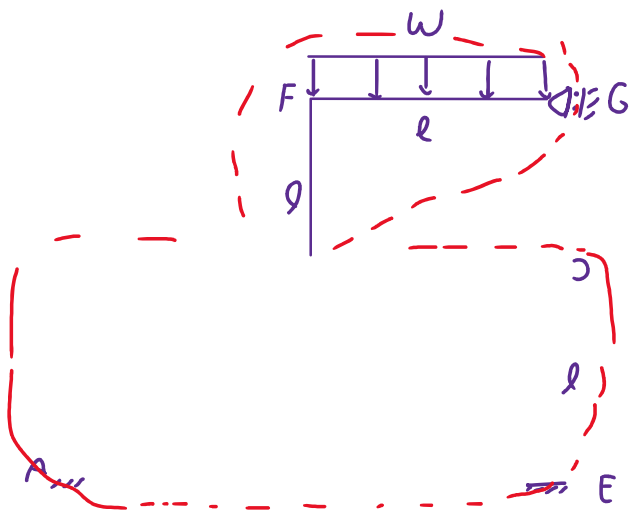
$$M_I = ?$$

$$M_A = ?$$

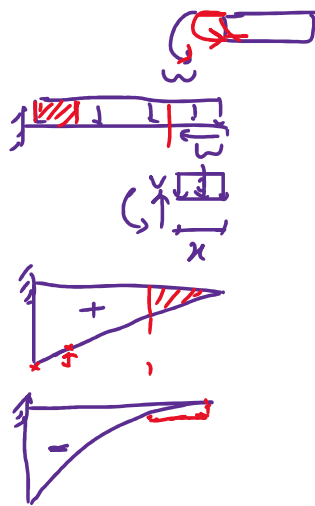
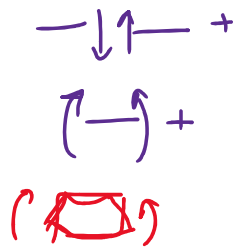


$$M_A = 0$$

$$M_B = \frac{PL}{2}$$

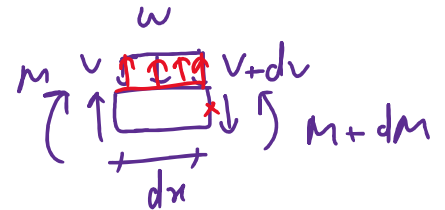


تحليل زوئیه سبب
① سبب
② قافیه
③ خرابه



$$V = wx$$

$$M = \frac{wx^2}{2}$$



$$\sum F_y = 0 \rightarrow \cancel{V} + w dx - (\cancel{V} + dV) = 0 \rightarrow \frac{dV}{dx} = +w$$

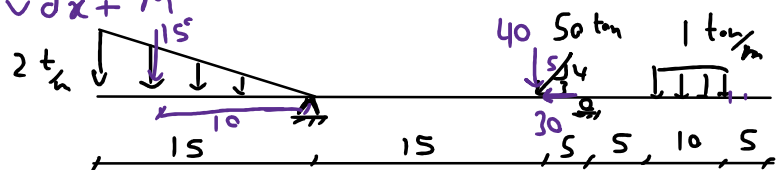
$$\sum M = 0 \rightarrow V dx + \cancel{M} + w \cancel{\frac{dx^2}{2}} - (\cancel{M} + dM) = 0 \rightarrow \frac{dM}{dx} = V$$

$$\begin{cases} \frac{dV}{dx} = w \\ \frac{dM}{dx} = V \end{cases}$$

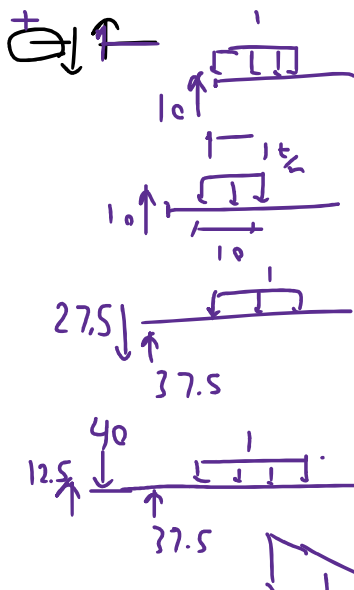
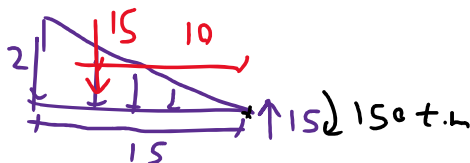
$$\rightarrow \begin{cases} V = \int w dx + V_0 \\ M = \int V dx + M_0 \end{cases}$$

$$\frac{dM}{dx} = V$$

$$40 \times 15 + 10 \times 30 - 15 \times 10 = 200$$

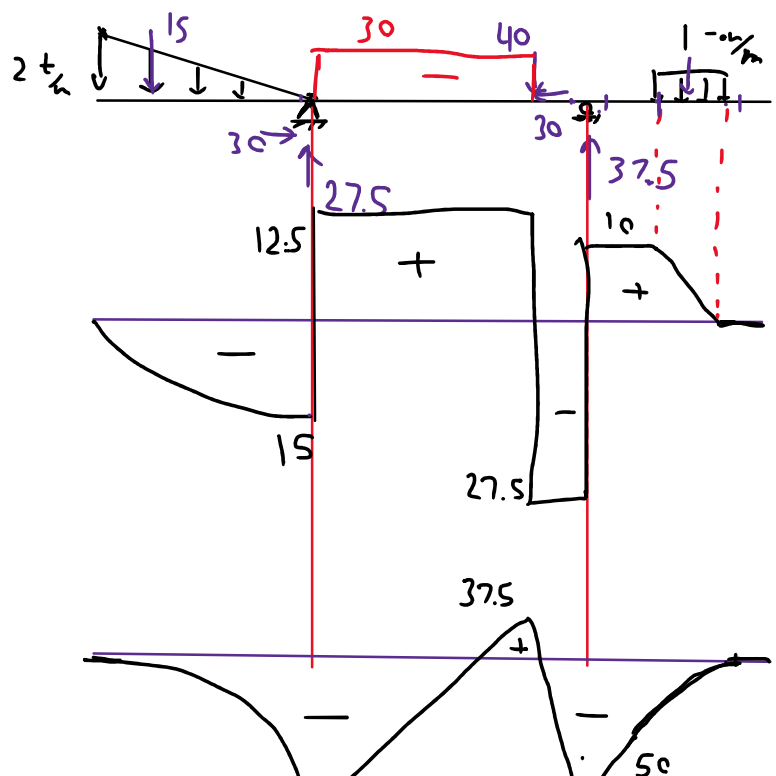


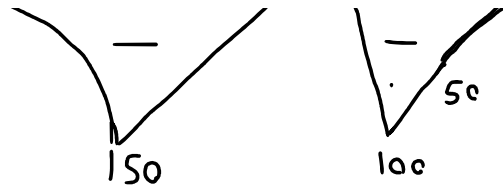
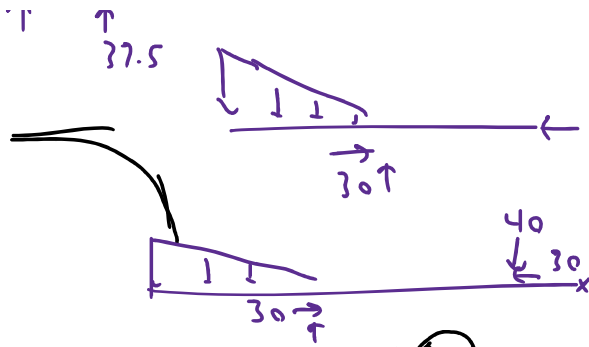
مثال:



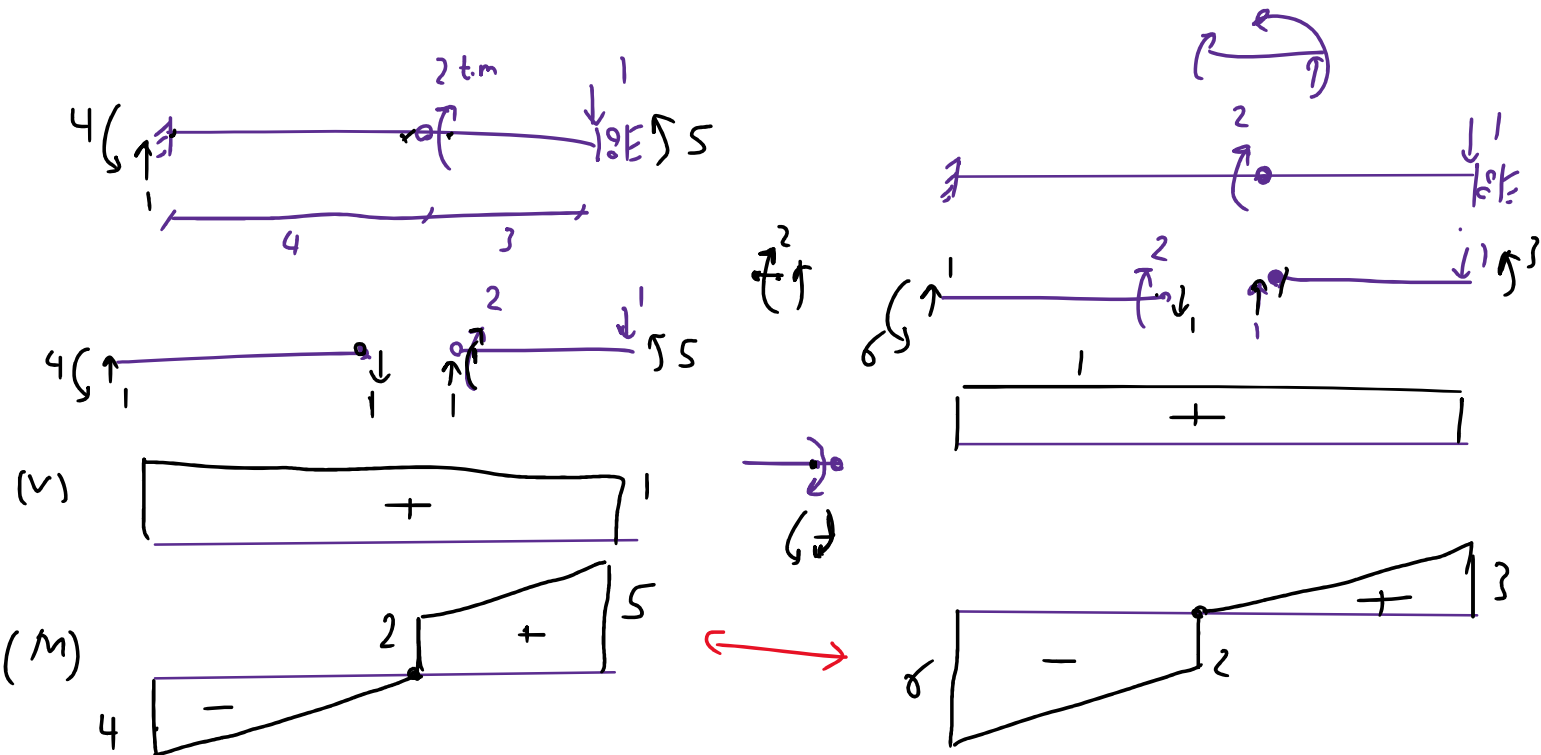
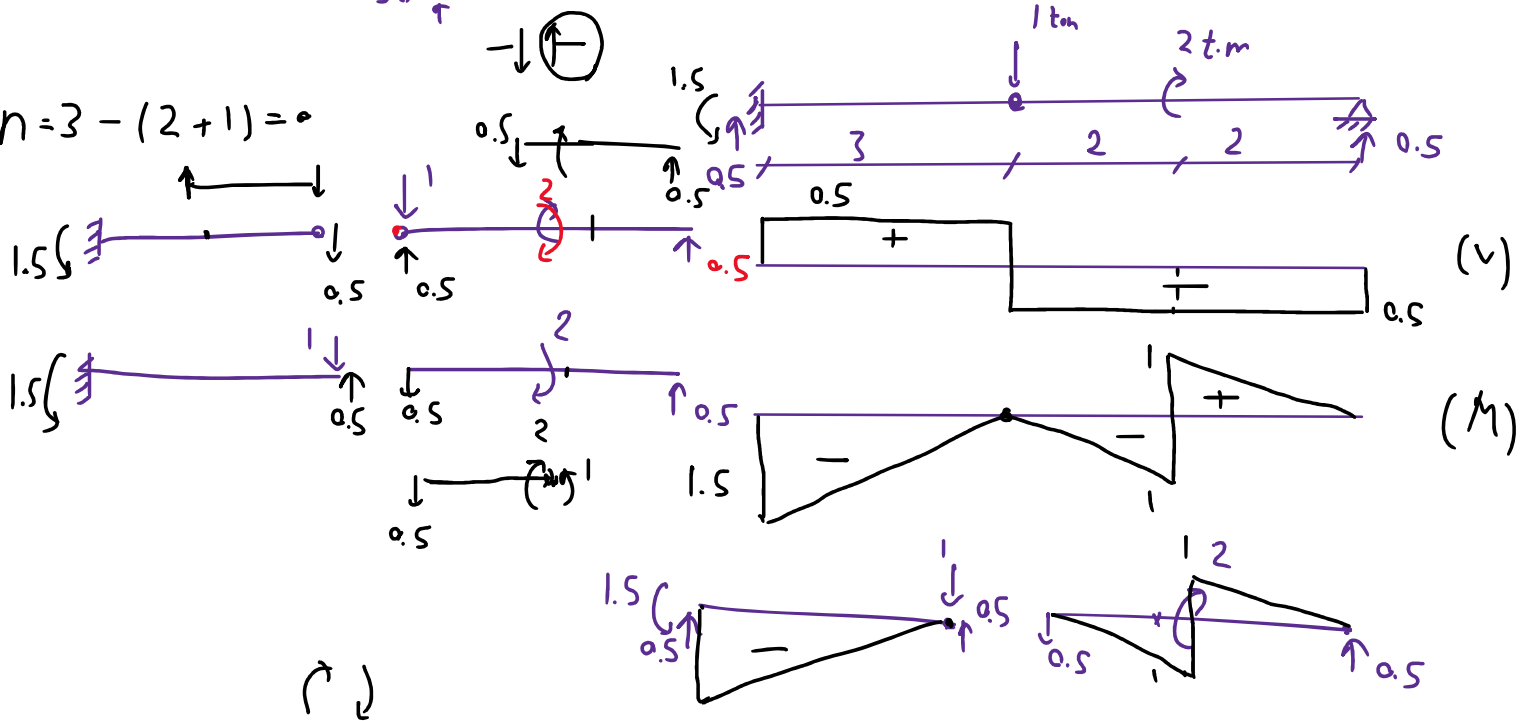
(V)

(M)

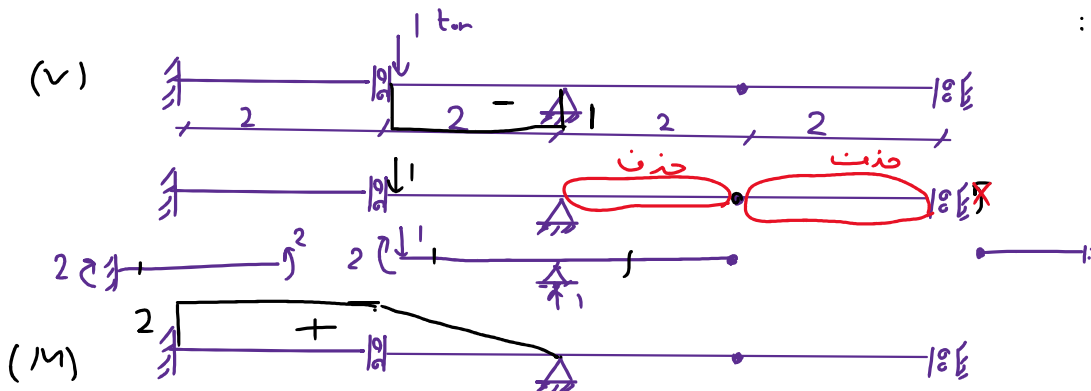




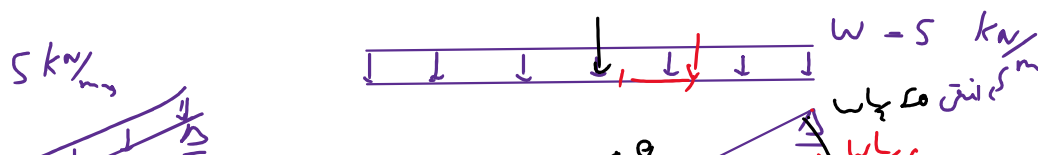
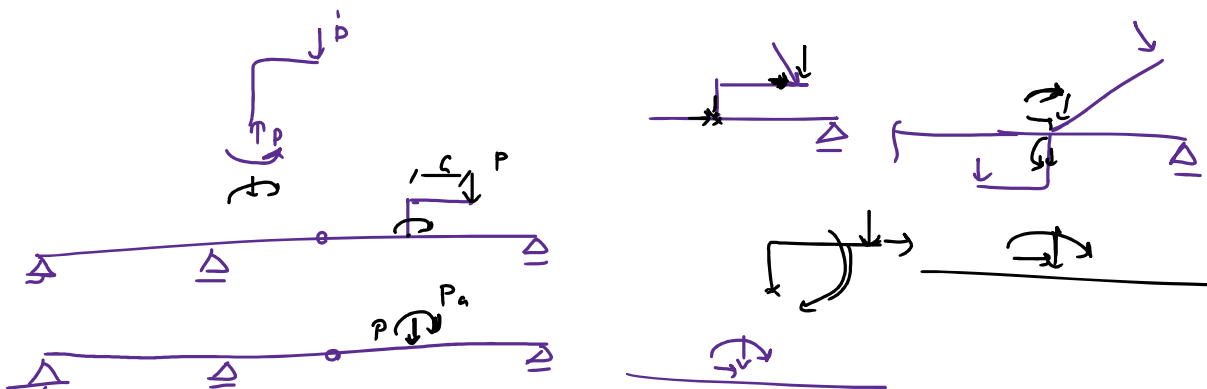
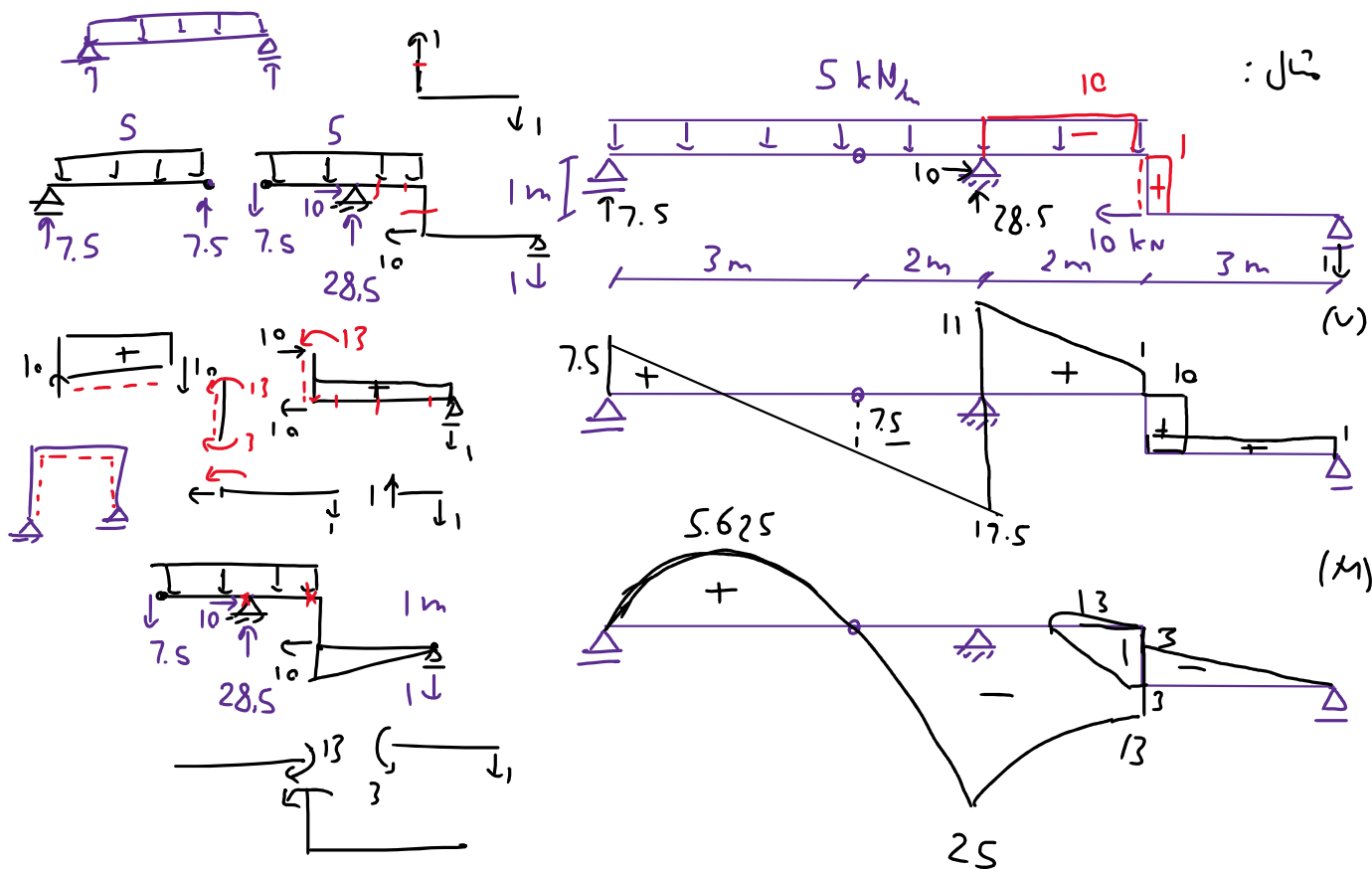
$$n = 3 - (2 + 1) = 0$$

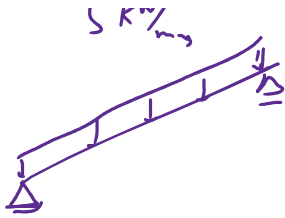


مثال:

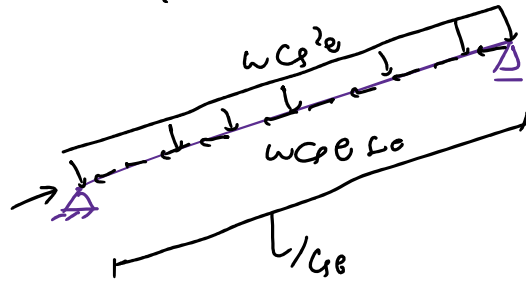
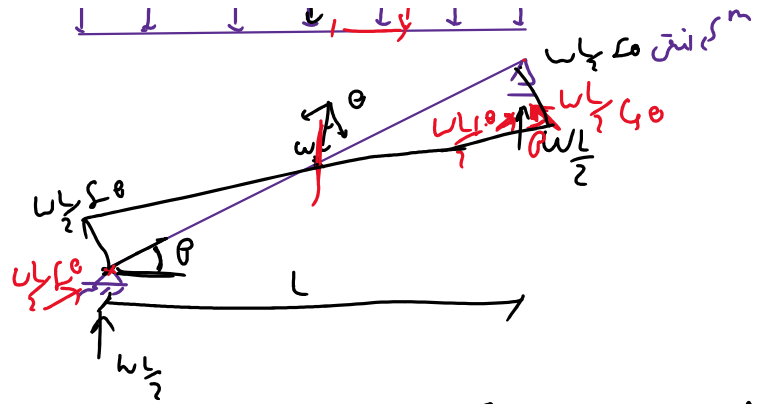


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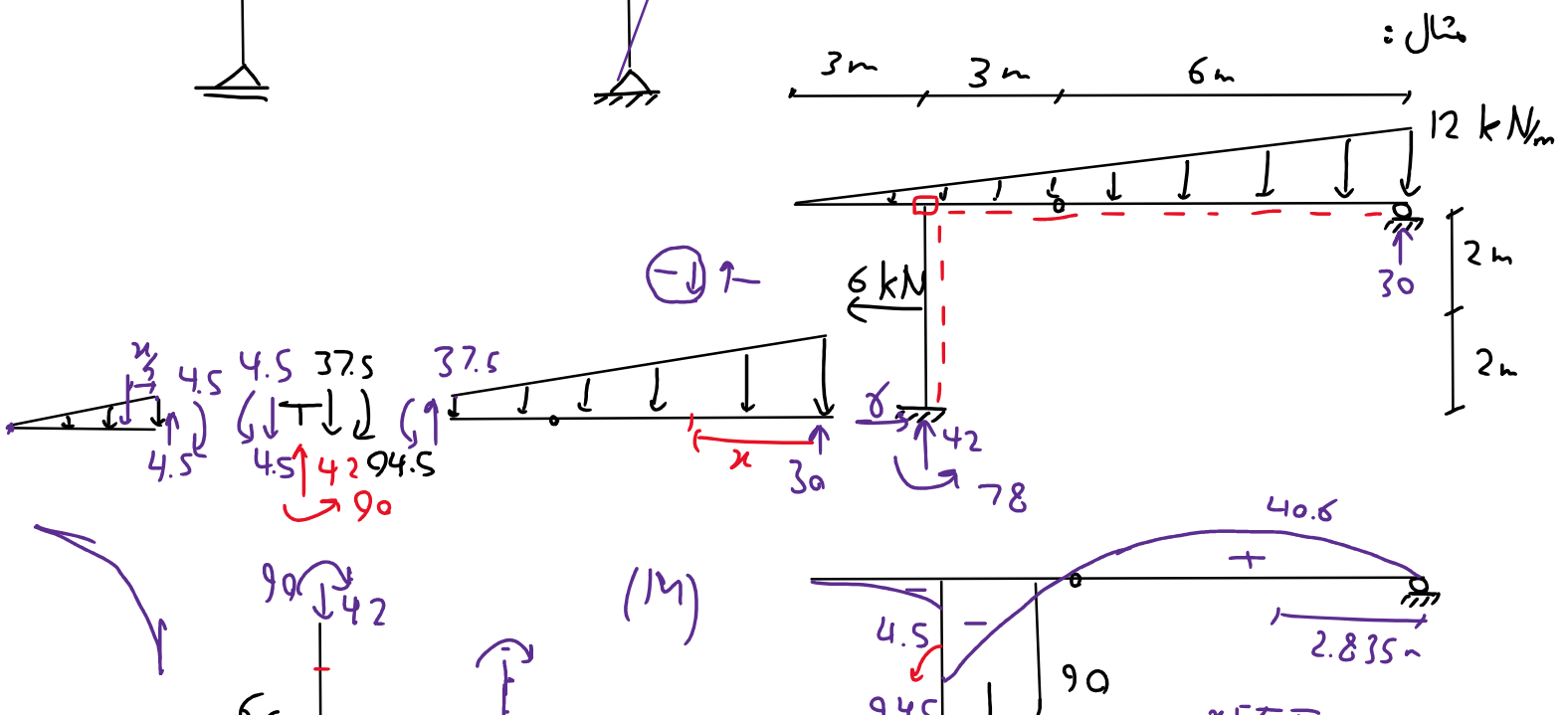
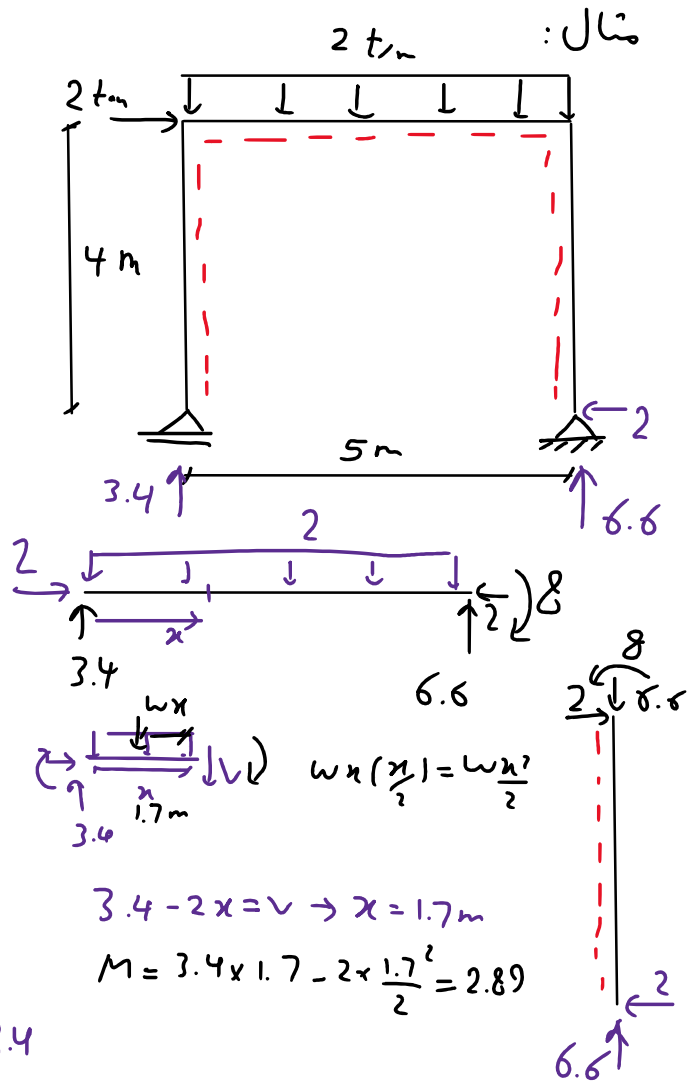
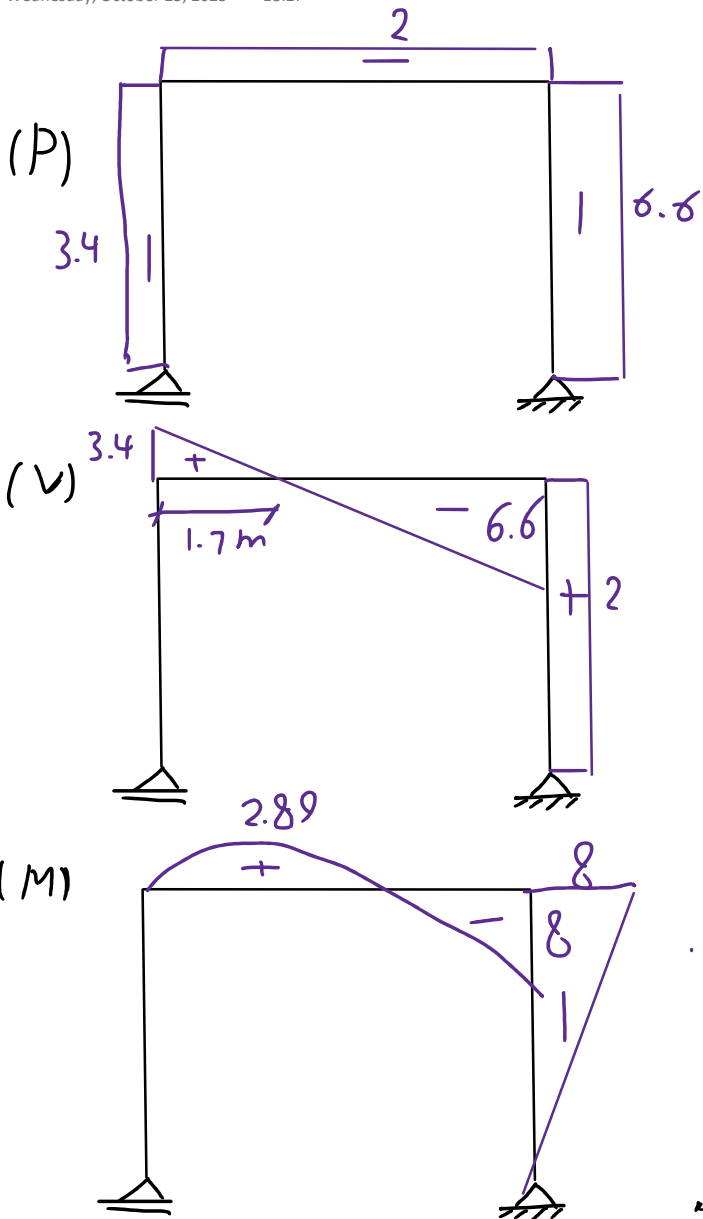


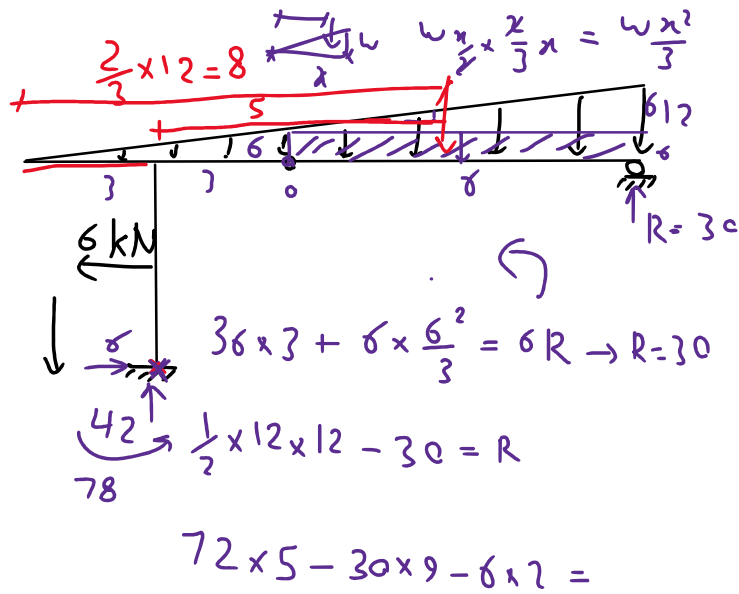
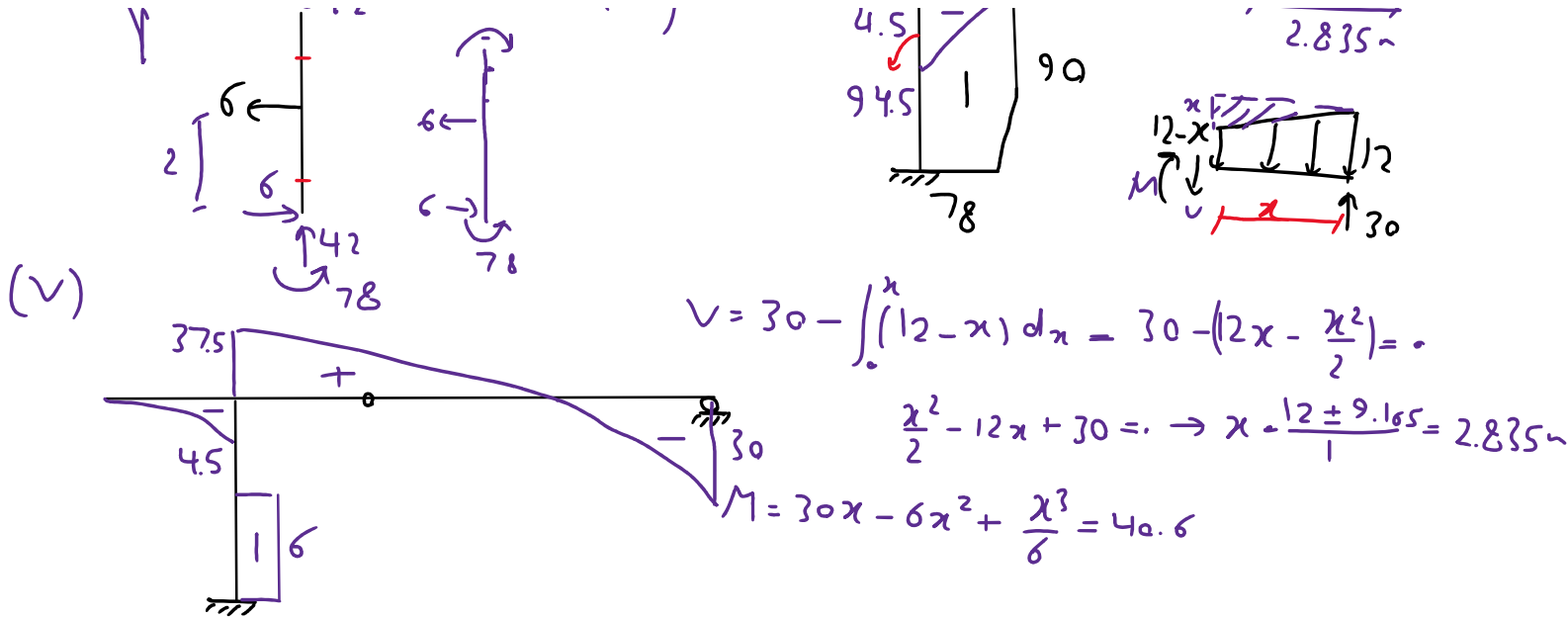


$$\frac{wk}{\frac{k}{G\theta}} = w C \theta$$

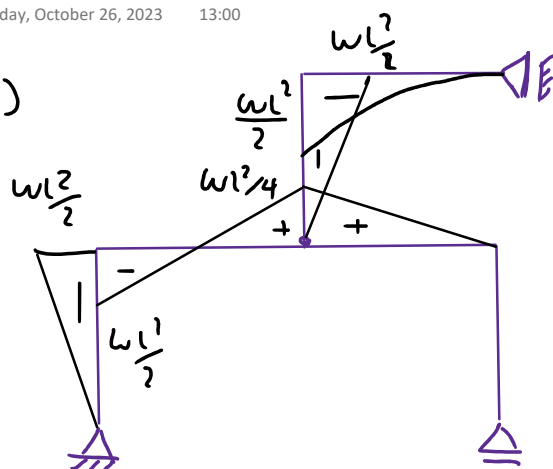


$$\frac{w C \theta \left(\frac{L}{G\theta} \right)^2}{8}$$

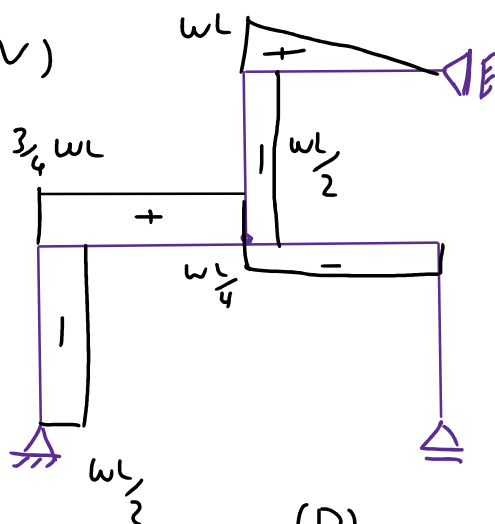




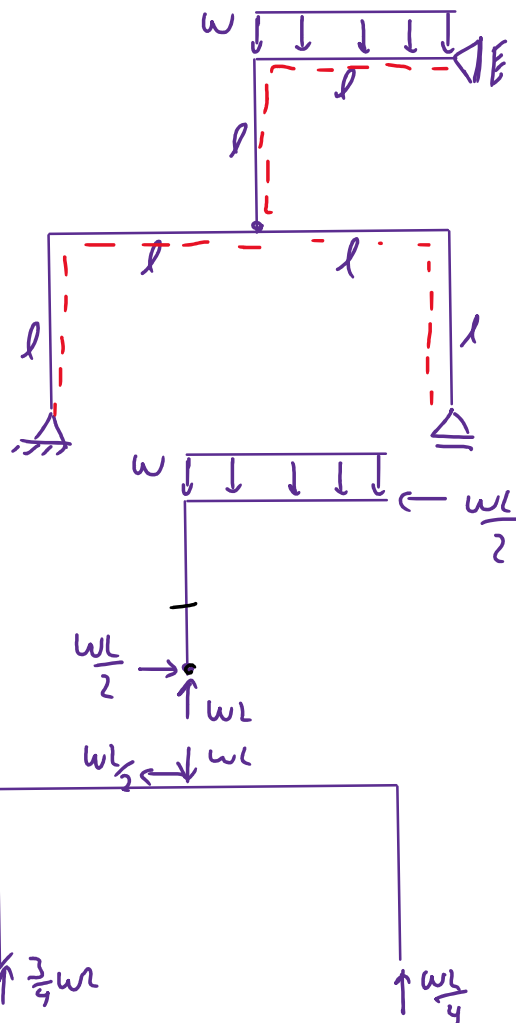
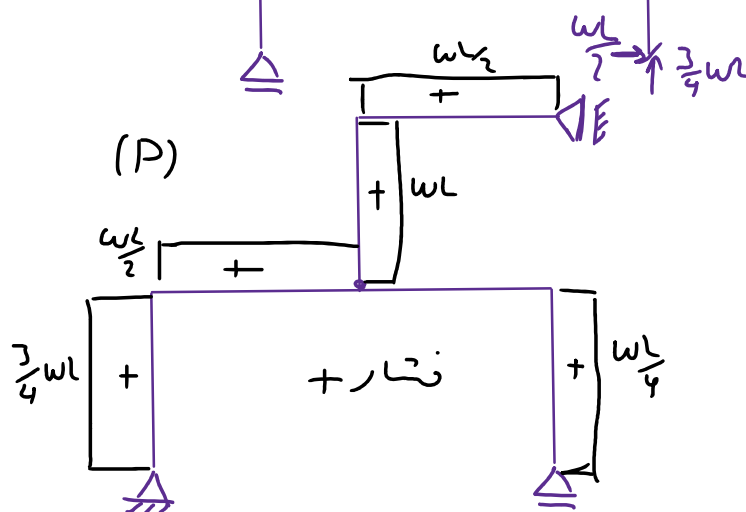
(M)



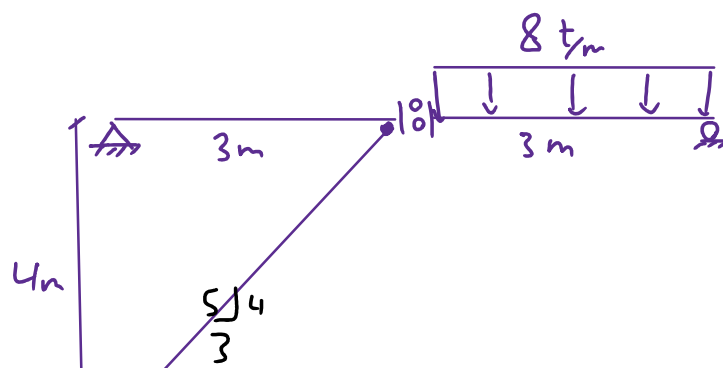
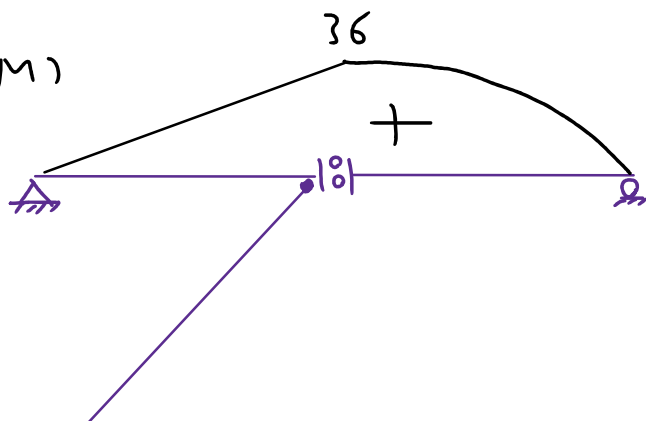
(V)

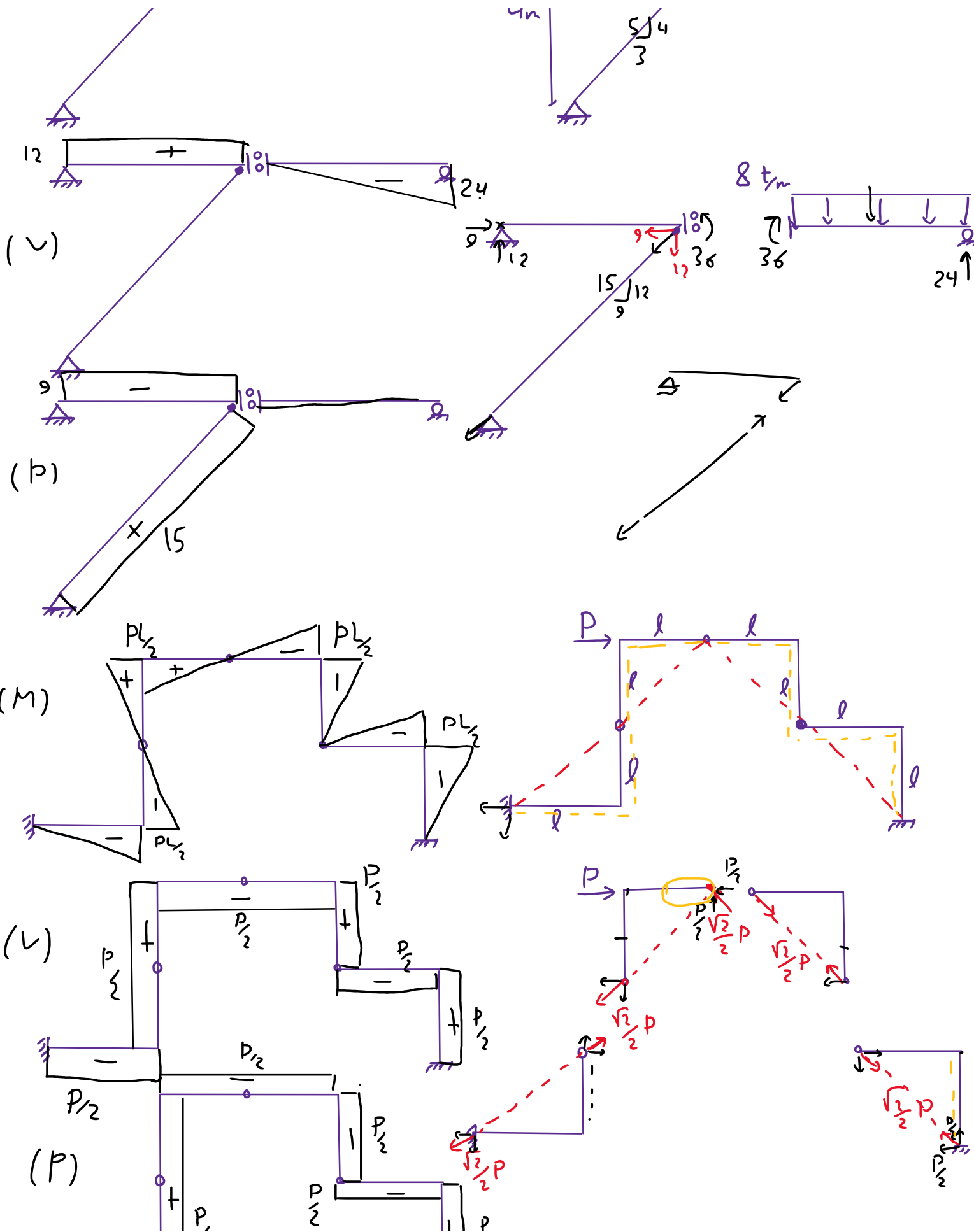


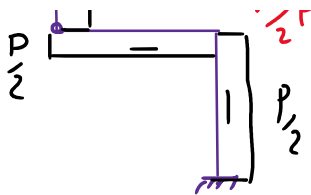
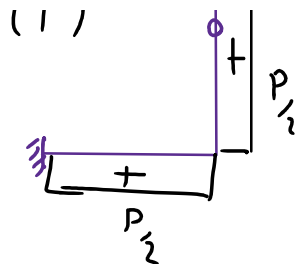
(P)



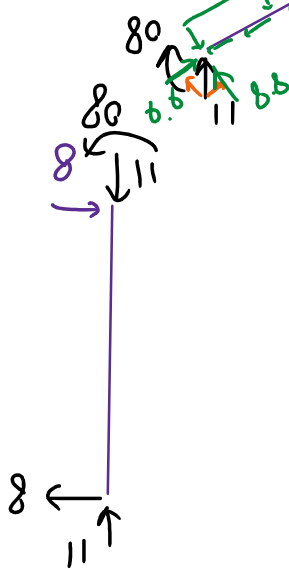
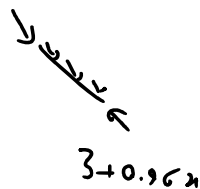
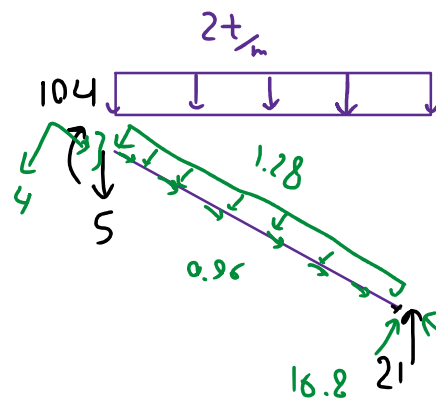
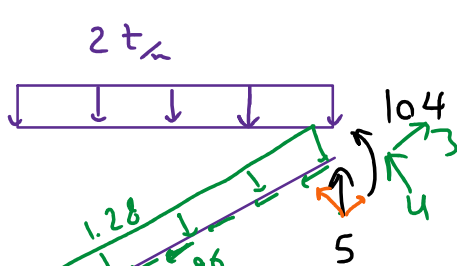
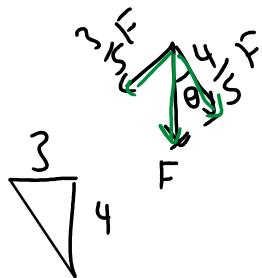
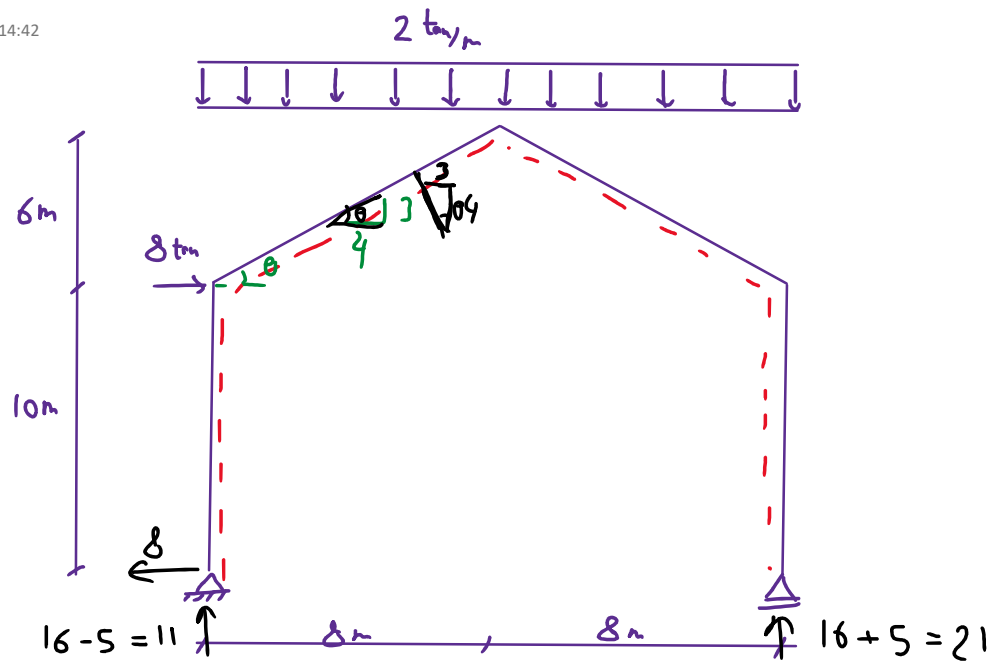
(M)







1/2



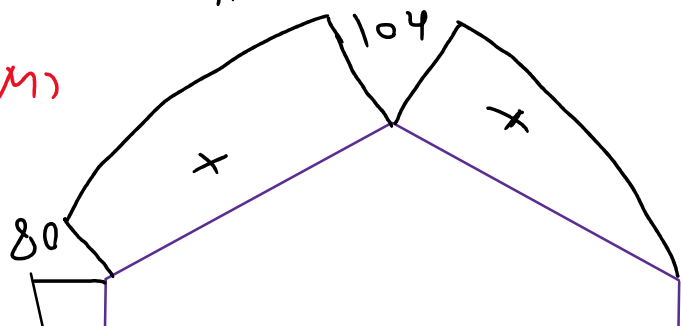
$$\frac{2 \times 8}{8} = 2 \cos \theta = 2 \times \frac{4}{5} = 1.6$$

$$8 \times \frac{4}{5} = 1.28$$

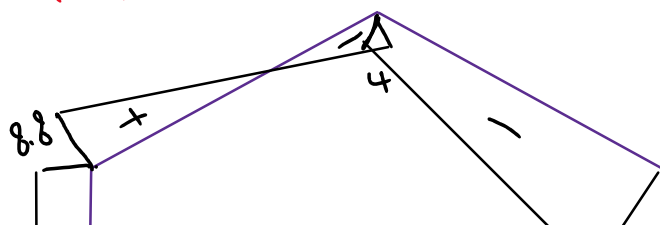
$$8 \times \frac{3}{5} = 0.96$$

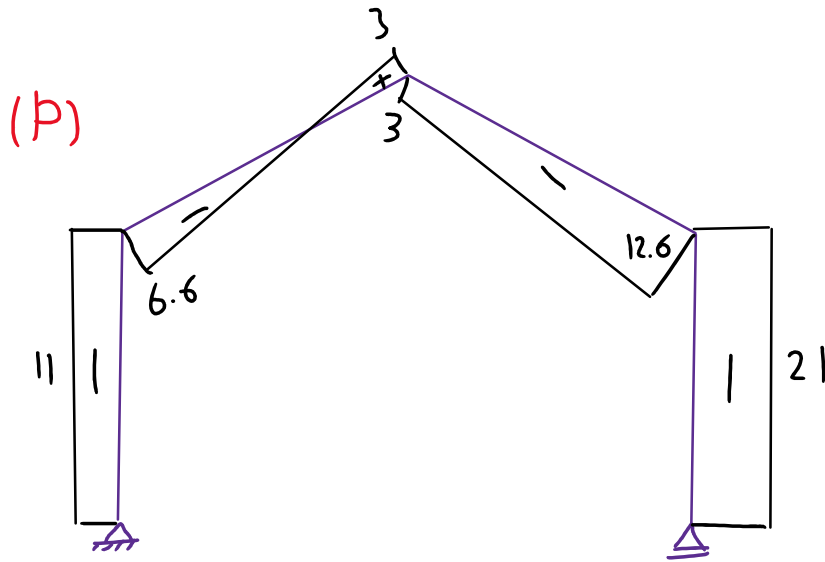
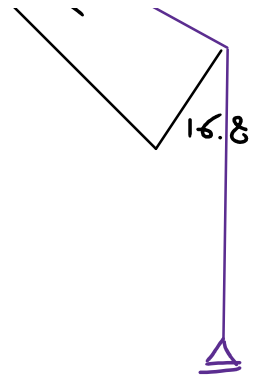
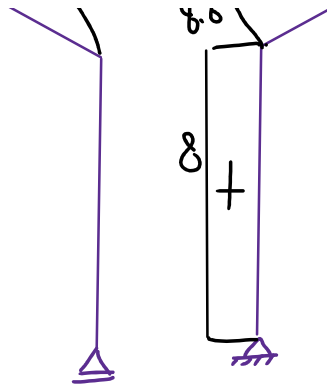
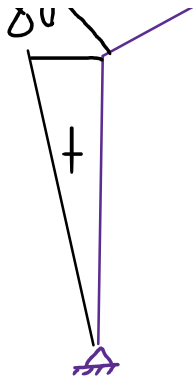


(M)



(V)

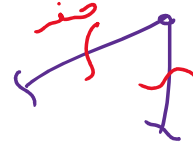
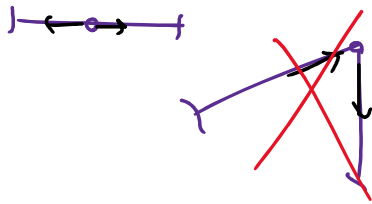




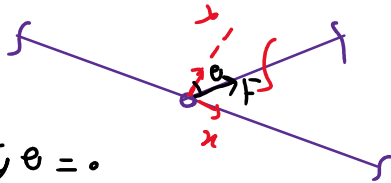
تحليل خراسا معین

روش تبدیل خراسا معین } روش گزین (روش منبرگ)
 ① روش گزین
 ② روش مقطع
 ③ روش عضو جایگزین

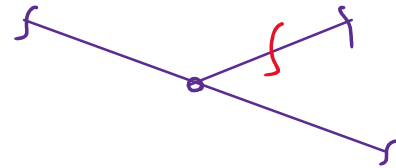
اعضای صفر نیردی



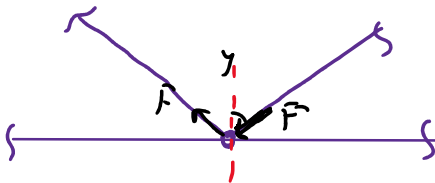
①



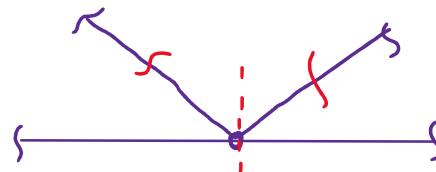
$$F \cos \theta = 0$$



②



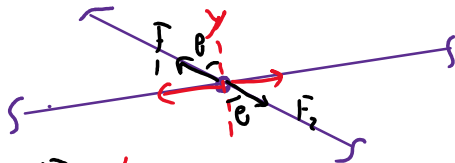
محررتان



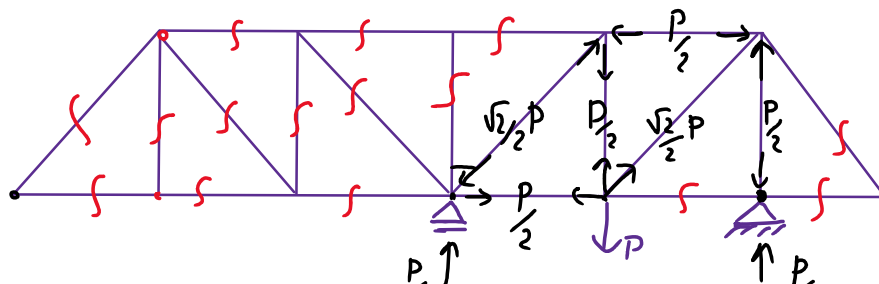
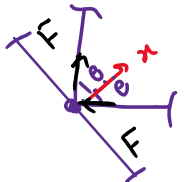
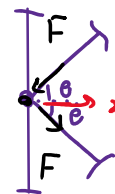
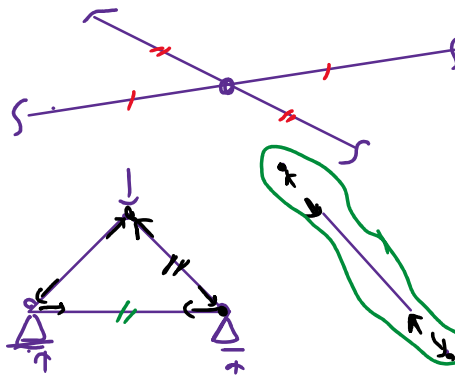
محررتان

③

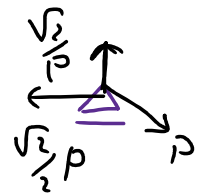
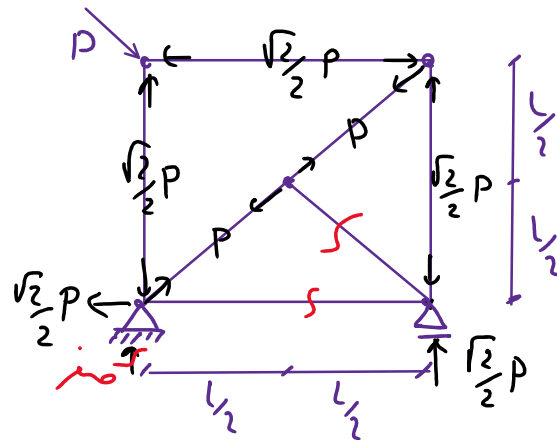
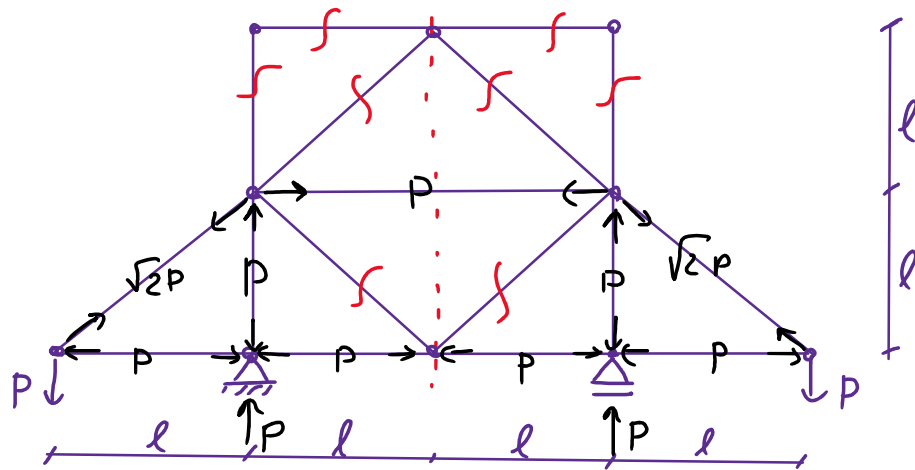
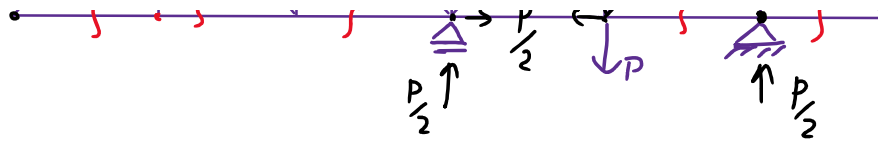
اعضای هم نیرو

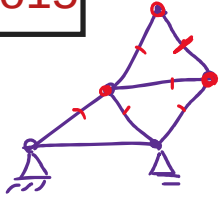


$$F_1 \cos \theta = F_2 \cos \theta$$



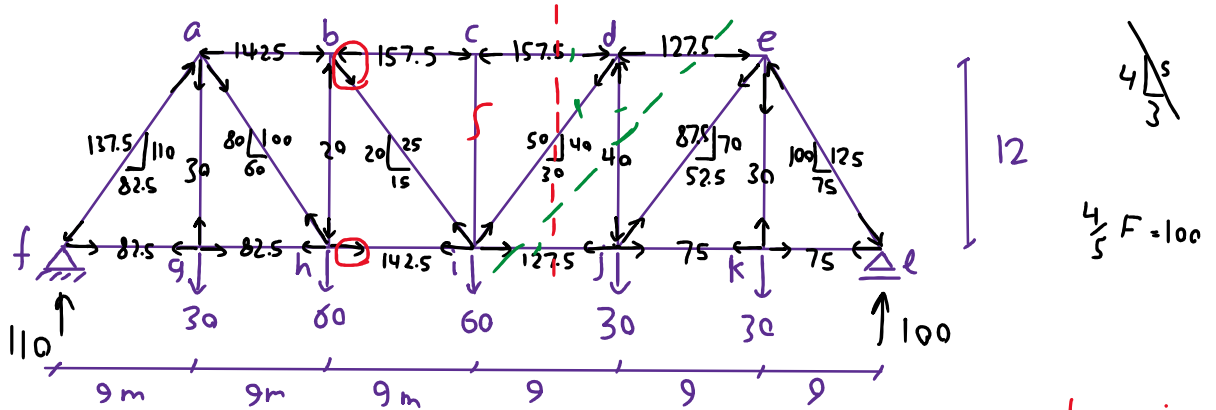
$\frac{P}{2}$
 P





روش تعادل گره

در این روش معادلات تعادل گره ($\sum F_x = 0$ و $\sum F_y = 0$) برابر بدست آوردن دو مجهول در یک گره استفاده می شود. بنابراین باید در آن گره ها دو مجهول دارد. شروع کنیم و پس با معلوم شدن سایر مجهولات گره ها دیگر، در مجهول باقی ماند، در آن گره ها را حل کنیم.



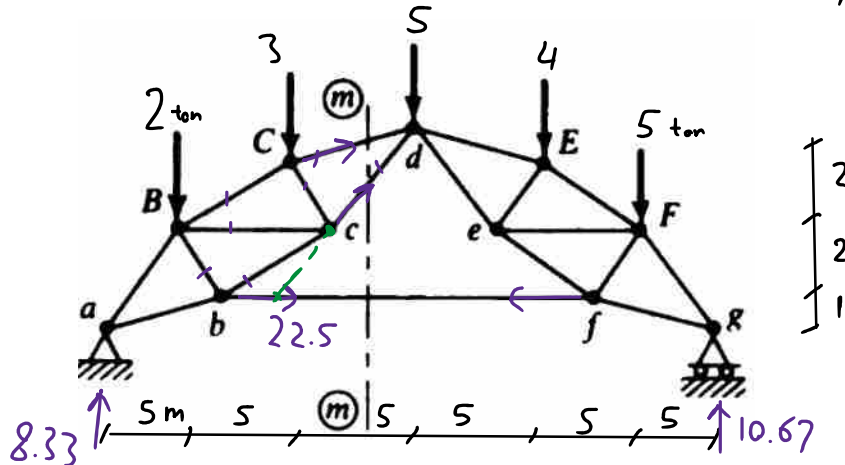
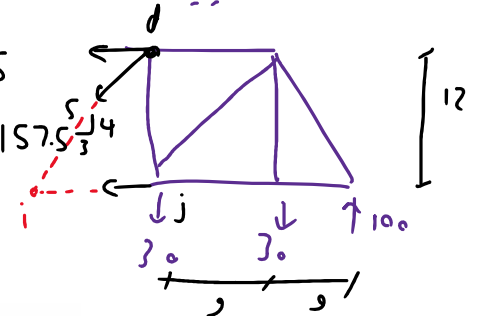
روش مقطع

① برابر بدست آوردن نیروی عضو خاص در خراب شده (راحت تر از روش گره)
 ② وقتی آسان تر باشد و وجود ندارد. (در خراب می شود یا بجا بیاید لازم است)

$$\sum M_d = 0 \Rightarrow F_{ij}(12) + 30(9) - 100(18) = 0 \rightarrow F_{ij} = 127.5$$

$$\sum M_i = 0 \rightarrow -F_{cd}(12) + 30(9) + 30(18) - 100(27) = 0 \rightarrow F_{cd} = -157.5$$

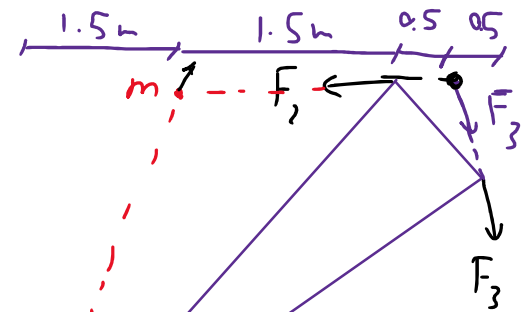
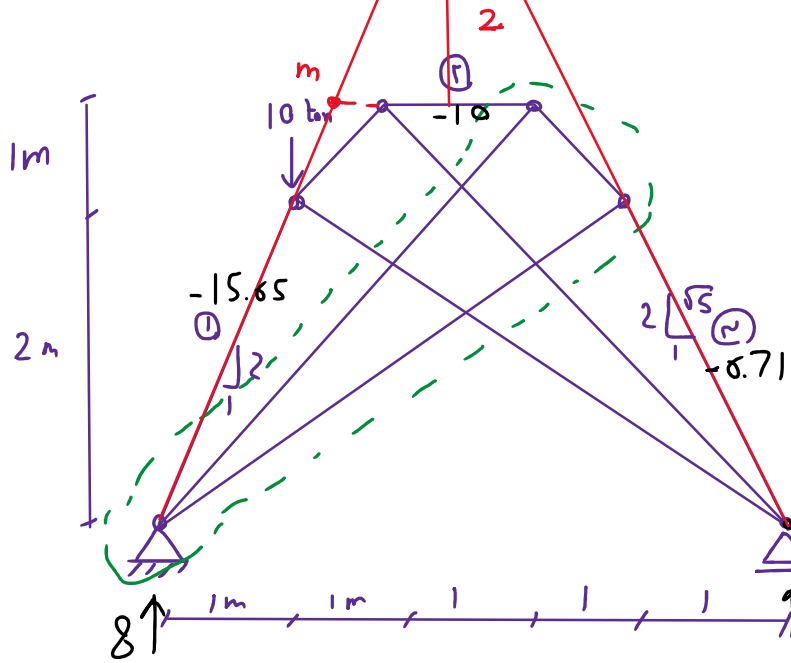
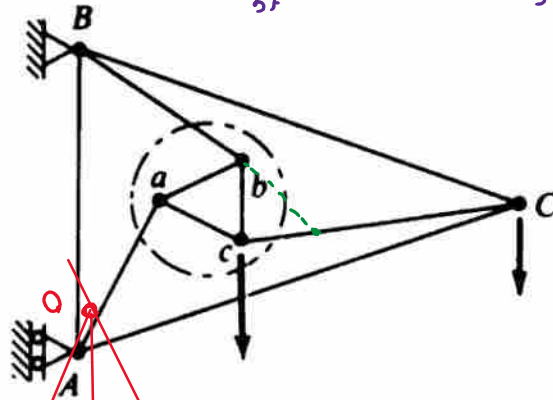
$$\sum F_y = 0 \rightarrow -\frac{4}{5}F_{di} - 30 - 30 + 100 = 0 \rightarrow F_{di} = 50$$



$$\sum M_d = 0 \rightarrow 8.33 \times 15 - 2 \times 10 - 3 \times 5 - F_{bf} \times 4 = 0 \rightarrow F_{bf} = 22.5$$

↖ B

$$\sum M_B = 0 \rightarrow 0.5 \times 15 - 2 \times 10 - 1 \times 5 - F_{bf} \times 4 = 0 \rightarrow F_{bf} = -2.5$$

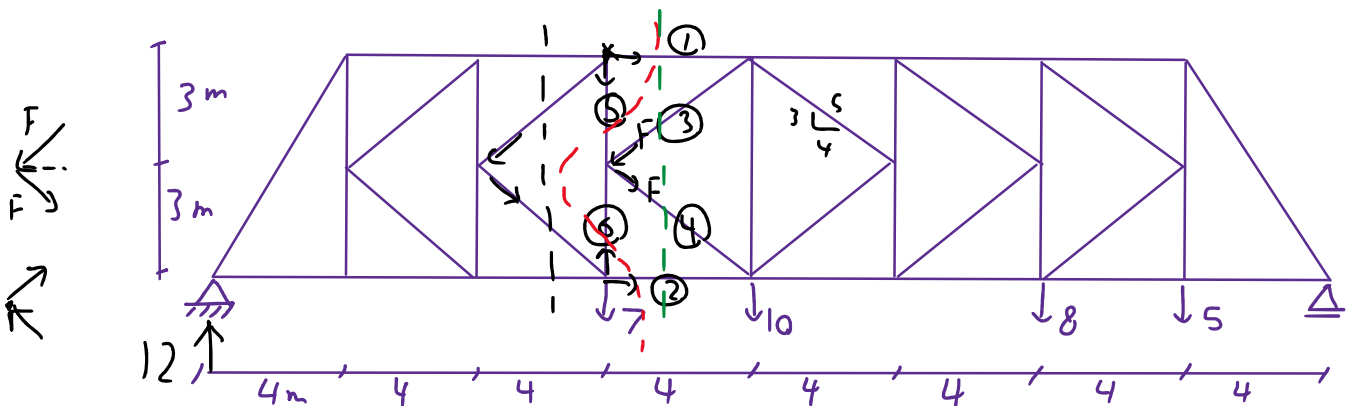


$$\sum M_a = 0 \rightarrow 8(2.5) + F_2(2) = 0 \rightarrow F_2 = -10$$

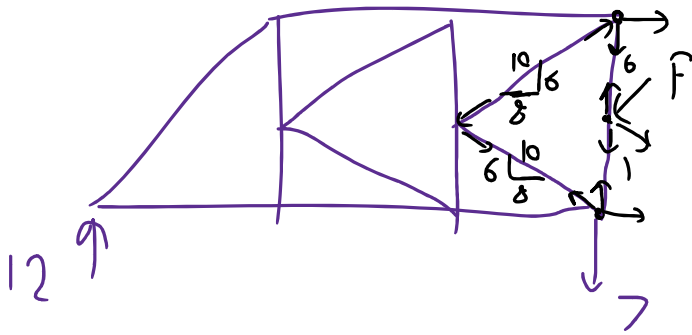
$$\sum M = 8(1.5) + \frac{2}{\sqrt{5}} F_3(2) = 0$$

$$\sum F_y = \frac{2}{\sqrt{5}} F_1 + 8 + \frac{2}{\sqrt{5}} \times 0.71 = 0$$

$$F_1 = -15.65$$



$$12 \times 12 - F(18) = 0 \rightarrow F = 24$$

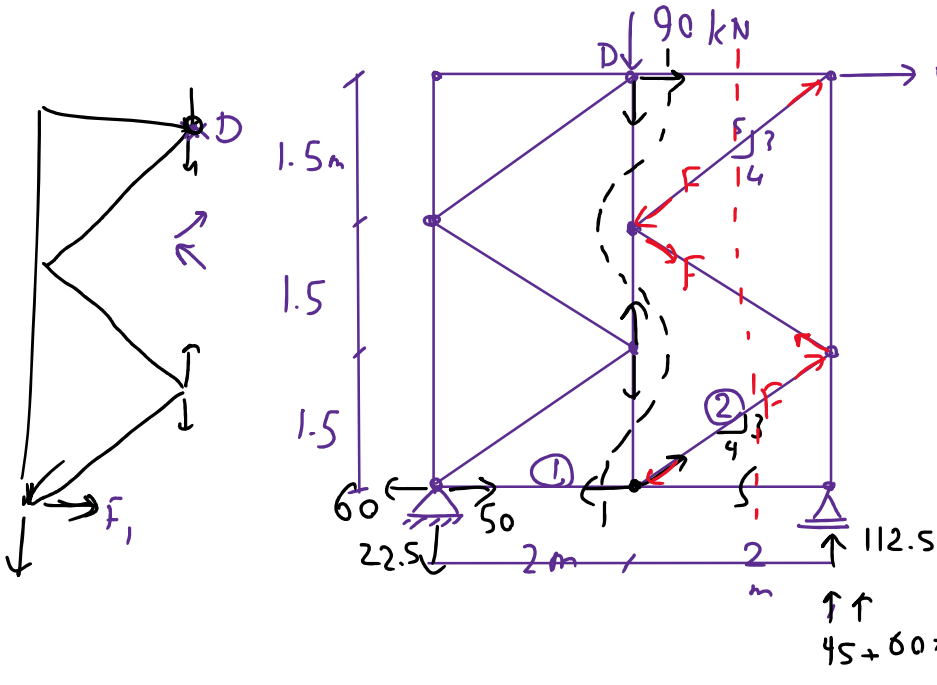


$$12 \times 12 - F_2(18) = 0 \rightarrow F_2 = 24$$

$$\sum F_x = 0 \rightarrow F_1 = -24$$

$$\sum F_y = 0 \rightarrow 2 \times \frac{3}{5} F - 12 + 7 = 0 \rightarrow F_3 = F_4 = 4.1 \text{ كتن}$$

$$F_5 = 8 \quad F_6 = 1$$



$$60 \text{ kN} \left(\sum M_0 = 0 \right) \quad F_1 = 50$$

$$22.5(2) - 60(4.5) + F_1(4.5) = 0$$

$$\frac{4}{5} F = 50 \rightarrow F_2 = \underline{62.5}$$

$$\sum F_y = 0 \rightarrow F_2 = -62.5$$

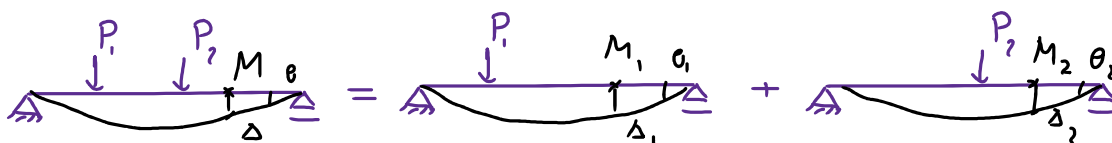
$$112.5 + 3 \times \frac{3}{5} F = 0 \rightarrow$$

$$45 + 60 \times \frac{4.5}{4} =$$

روش عضو جابجایی (روش هینبرگ)

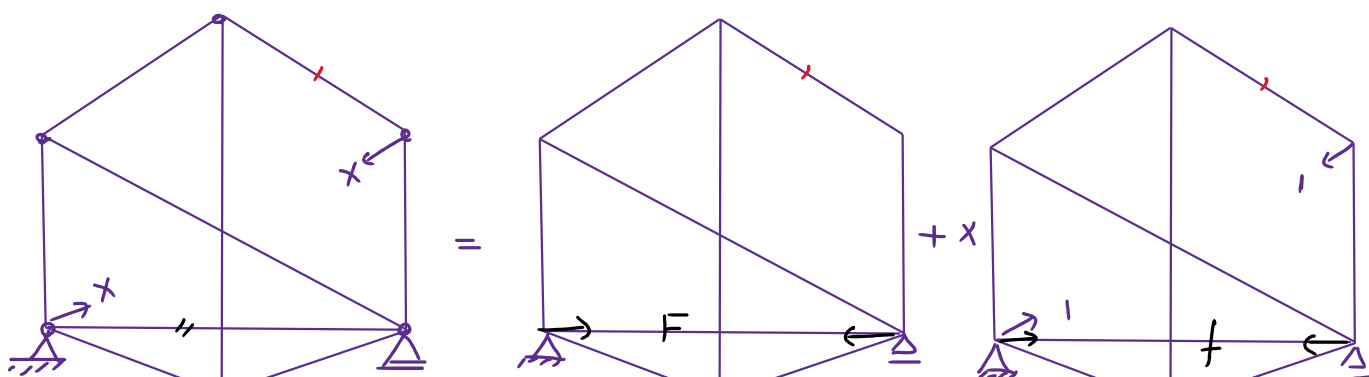
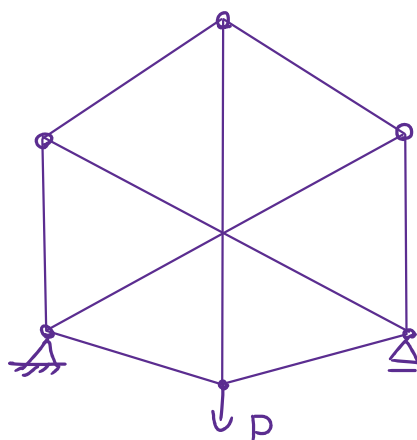
روش عضو جابجایی در سال ۱۸۸۶ بر روی تیرهای سنجیده توسط هینبرگ ارائه شد. این روش بر اصل جمع آثار قرار دارد (اصل سوبرپوزیشن) است.

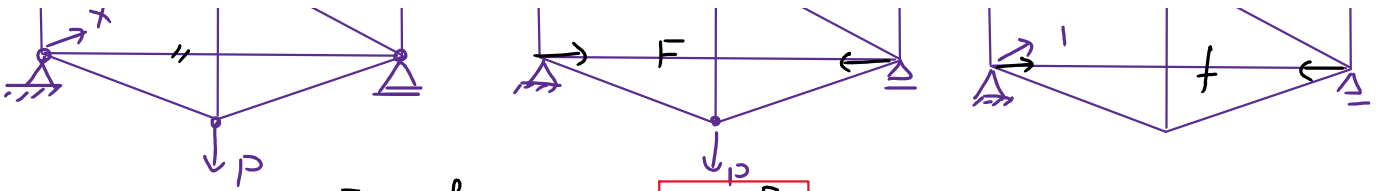
اصل جمع آثار قرار (سوبرپوزیشن)



$$\begin{cases} M = M_1 + M_2 \\ \Delta = \Delta_1 + \Delta_2 \\ v = v_1 + v_2 \\ \vdots \end{cases}$$

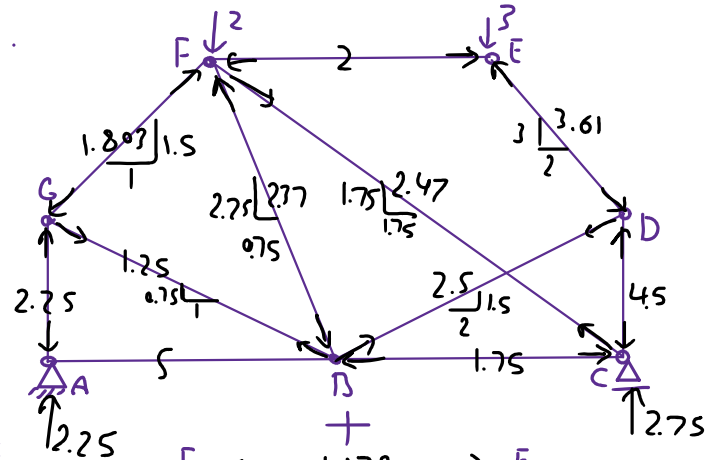
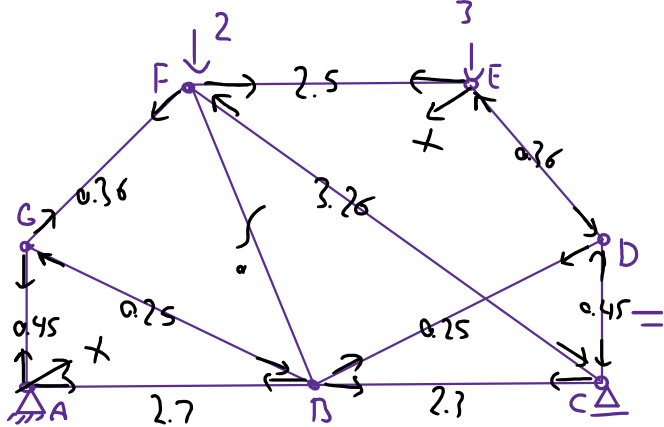
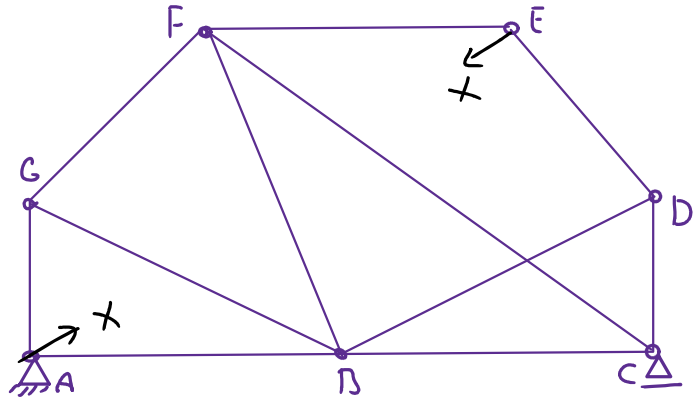
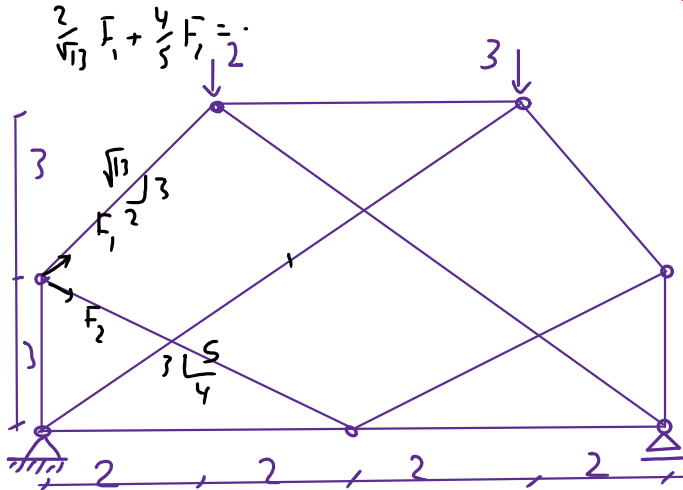
* فقط در محدوده خطی صحت دارد.





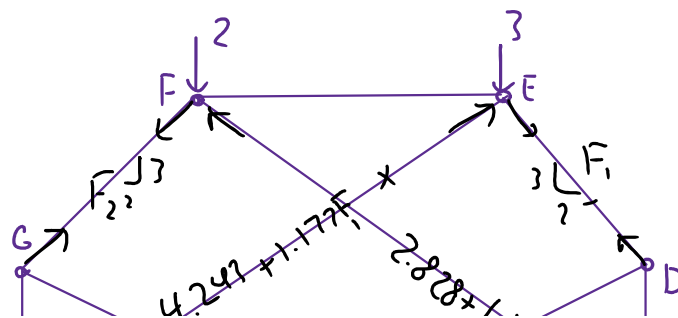
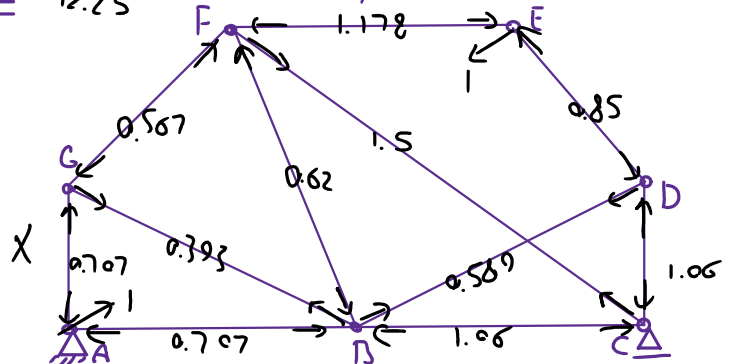
$$F + X f = 0 \rightarrow X = -\frac{F}{f}$$

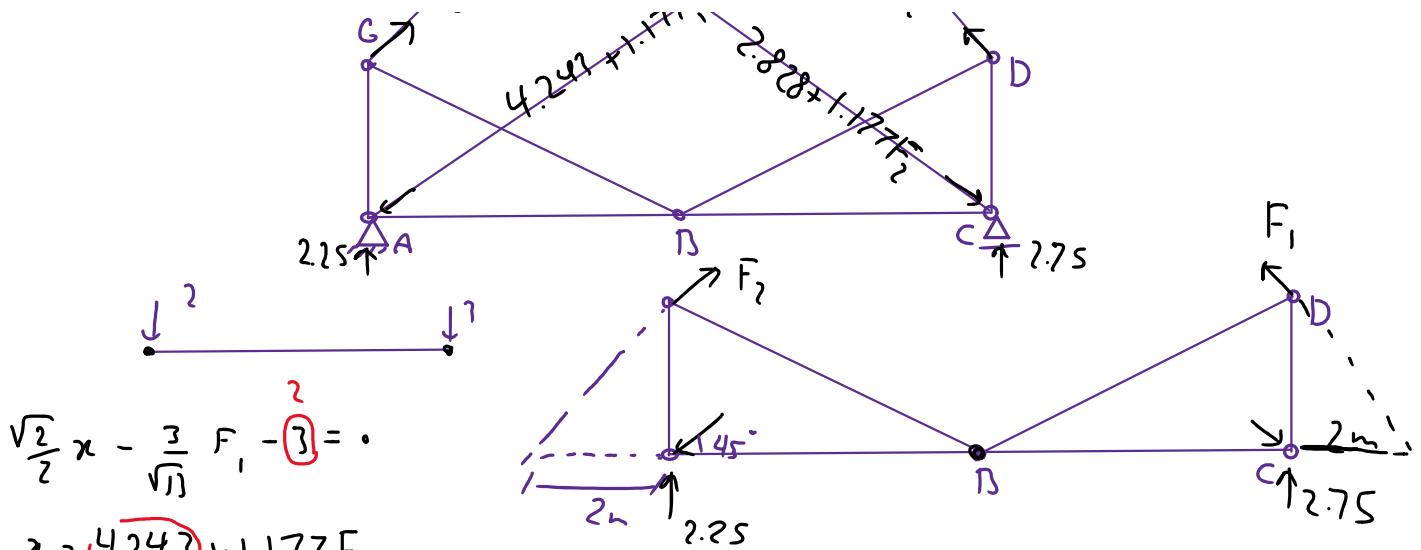
مثال:



$$-2.37 + X(-0.62) = 0$$

$$X = -3.823$$





$$\frac{\sqrt{2}}{2} x - \frac{3}{\sqrt{13}} F_1 - \overset{2}{3} = 0$$

$$x = \overset{2.828}{4.243} + 1.177 F_1$$

$$\star 2.75 \times 4 - \frac{\sqrt{2}}{2} (2.828 + 1.177 F_2) \times 4 + \frac{3}{\sqrt{13}} F_1 \times 6 = 0$$

$\sum M_D = 0$

$$2.25 \times 4 - \frac{\sqrt{2}}{2} (4.243 + 1.177 F_1) \times 4 + \frac{3}{\sqrt{13}} F_1 \times 6 = 0$$

$$\begin{cases} 3.329 F_2 - 4.992 F_1 = 3.001 \\ -3.329 F_2 + 4.992 F_1 = -3.001 \end{cases}$$

$$\rightarrow (-3.329^2 + 4.992^2) F_1 =$$

$$3.001 (3.329 - 4.992) \rightarrow F_1 = -0.361$$

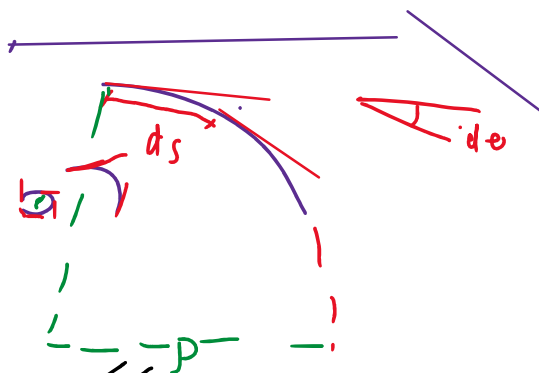
$$F_2 = 0.361$$

فصل سوم: محاسب تغییر شکل ناشی از جفتش بر روی سازه غیر انحرافی

انتگرال مضاعف (مقارنت معادله)

① کثرت سطح

② تغییر زوج



رابطه کثرت-انحناء (moment-curvature)

تعریف هندسی

$$\frac{1}{\rho} = \frac{d\theta}{ds} = \frac{dy'}{dx} = y'' \quad (1)$$

تعریف مقارنت معادله

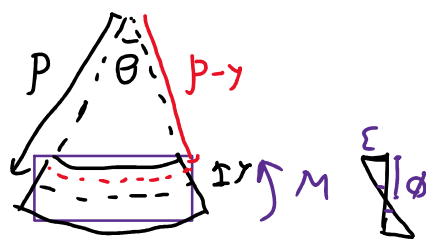
$$\frac{1}{\rho} = \frac{\epsilon}{y} = \frac{\sigma}{Ey} = \frac{My}{EI} = \frac{M}{EI} \quad (2)$$

$$y'' = \frac{M}{EI}$$

$$\epsilon = \frac{\Delta L}{L} = \frac{(p-y)\theta - p\theta}{p\theta}$$

$$\epsilon = -\frac{y}{\rho}$$

$$\frac{1}{\rho} = \frac{\epsilon}{y} \quad \sigma = \frac{My}{I}$$



$$\frac{dy}{dx} = w \quad \frac{dM}{dx} = v \quad M = \int \sigma y dA = \int \frac{y}{c} \sigma_{max} y dA$$

$$M = \frac{\sigma_{max}}{c} \left(\int y^2 dA \right) I$$

$$\theta = y' = \int \frac{M}{EI} dx$$

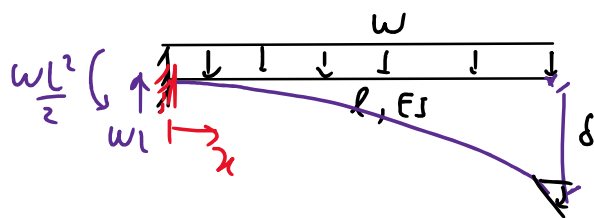
$$y = \int \theta dx$$

$$v = \int w dx$$

$$M = \int v dx$$

$$\theta = \int \frac{M}{EI} dx = \frac{1}{EI} \int M dx$$

$$\theta = \frac{1}{EI} \left(-\frac{wL^2}{2} x + wLx^2 - \frac{w}{6} x^3 \right)$$



مثال:



$$\theta = \frac{1}{EI} \left(-\frac{wL^2}{2}x + \frac{wL}{2}x^2 - \frac{w}{6}x^3 + C_2 \right)$$

$$x=0 \rightarrow \theta=0$$

$$y = \int \theta dx$$

$$y = \frac{1}{EI} \left(-\frac{wL^2}{4}x^2 + \frac{wL}{6}x^3 - \frac{w}{24}x^4 + C_2x \right)$$

$$x=0 \rightarrow y=0 \quad \frac{-6+4-1}{24} = -\frac{3}{24}$$

$$\frac{wL^2}{2} \int \frac{1}{x} dx \rightarrow \frac{wL^2}{2} \ln x$$

$$M(x) = -\frac{wL^2}{2} + wLx - \frac{w}{2}x^2$$

$$\theta = -\frac{wL}{6EI} \quad \delta = -\frac{wL^4}{8EI}$$

روش لنکر-سطح Moment-area

فرمول بند نسبت جابجایی در ۱۸۷۳ میلادی

$$y'' = \frac{M}{EI} \rightarrow \frac{dy'}{dx} = \frac{d\theta}{dx} = \frac{M}{EI}$$

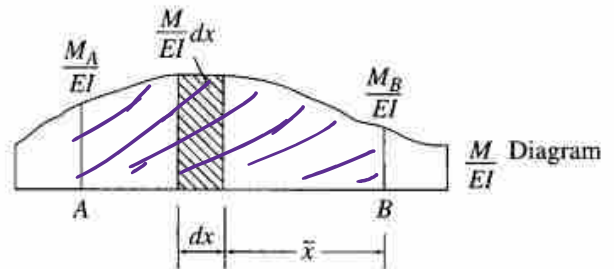
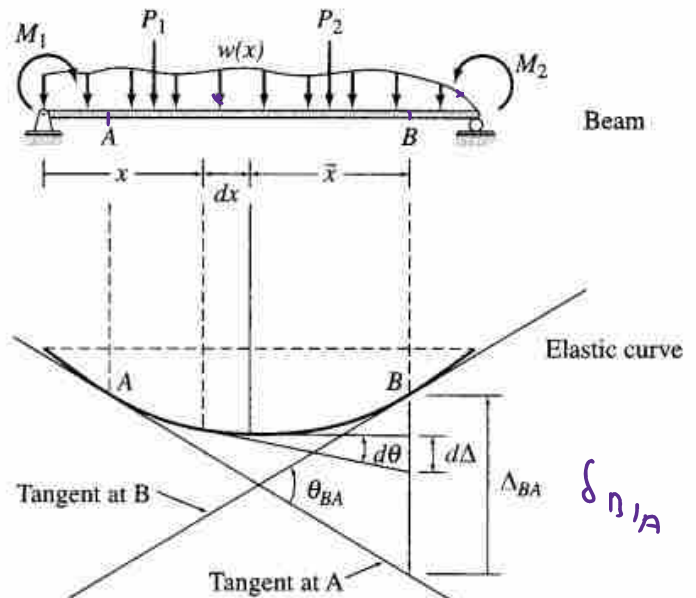
$$d\theta = \frac{M}{EI} dx$$

$$\int_{\theta_A}^{\theta_B} d\theta = \int_{x_A}^{x_B} \frac{M}{EI} dx$$

$$\theta_{B/A} = \int_{x_A}^{x_B} \frac{M}{EI} dx = S \quad \text{تغییر ال}$$

$\theta_{B/A}$: چرخش نقطه B نسبت به نقطه A (برای س
جت نسبت به A)

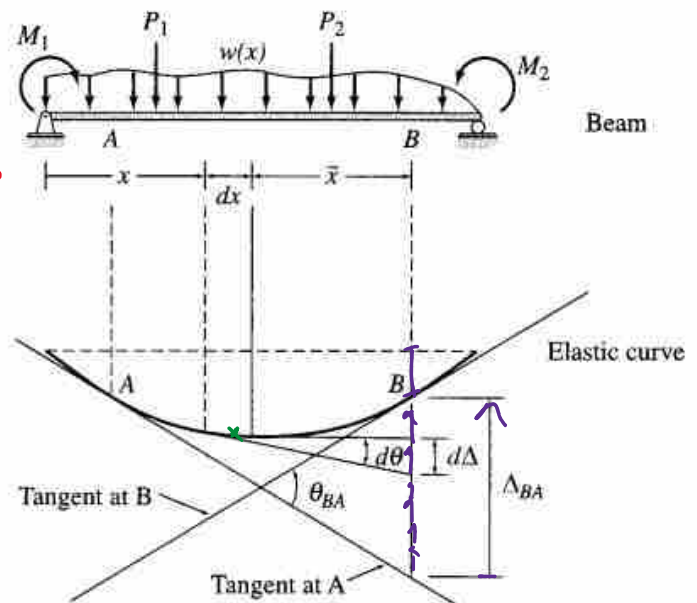
S: مساحت زیر منحنی $\frac{M}{EI}$



$$d\delta = \bar{x} d\theta = \bar{x} \frac{M}{EI} dx$$

$$\delta_{B/A} = \int_{x_A}^{x_B} \bar{x} \frac{M}{EI} dx = \int \bar{x}$$

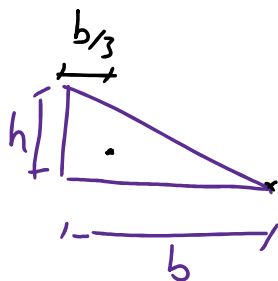
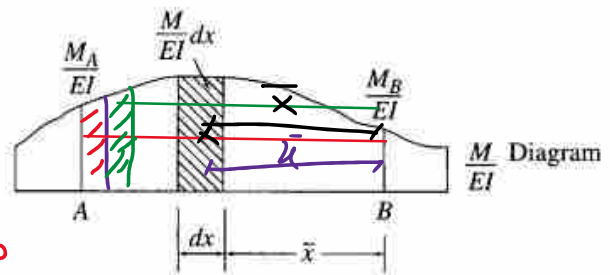
قضیه دوم



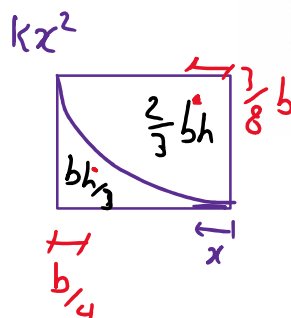
$\delta_{B/A}$: انحراف نقطه B از محاس بر A (برای س بالا مثبت)

S: مساحت زیر نمودار $\frac{M}{EI}$

$\bar{x}$: فاصله مرکز سطح M/EI تا B



$$S = b \frac{h}{2}$$



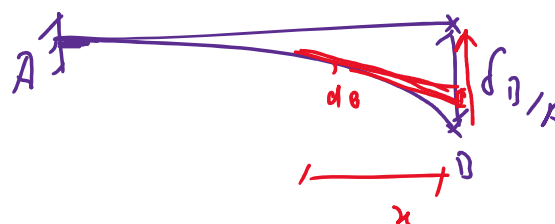
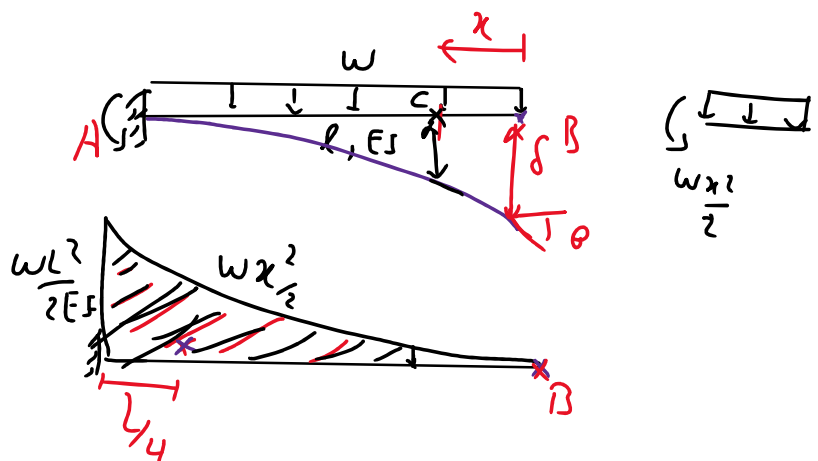
$$\theta_{B/A} = \theta_B - \theta_A = S =$$

$$\frac{1}{3} \left(\frac{wL^2}{2EI} \right) L$$

$$\theta_B = \frac{wL^3}{6EI}$$

$$\delta_{B/A} = \delta_B = \left(\frac{wL^3}{6EI} \right) \left(\frac{3}{4} L \right)$$

$$\delta_B = \frac{wL^4}{8EI}$$



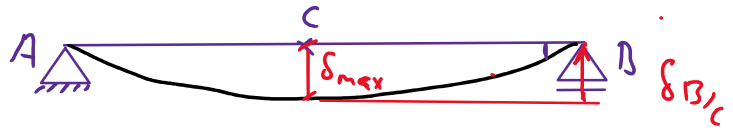
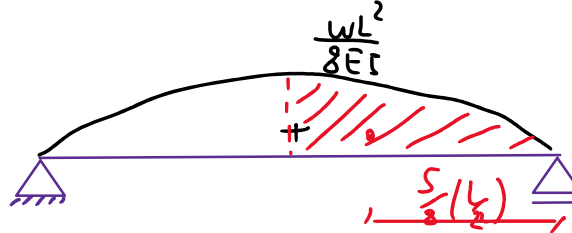
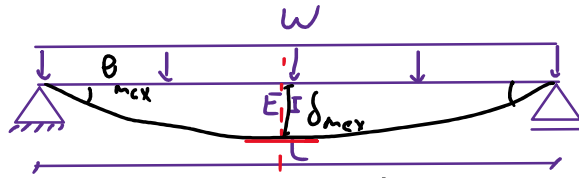
$$\delta_1 = \frac{w l^4}{8 E I}$$



مثال: δ_{max} , θ_{max}

$$\theta_{max} = \theta_{B/C} = \theta_B - \theta_C = \frac{2}{3} \left(\frac{wL^2}{8EI} \right) \left(\frac{L}{2} \right) = \frac{wL^3}{24EI}$$

$$\delta_{max} = \delta_{B/C} = \frac{wL^3}{24EI} \left(\frac{5}{8} \times \frac{L}{2} \right) = \frac{5wL^4}{384EI}$$



مثال: δ_{max}

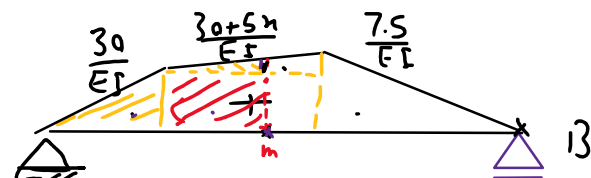
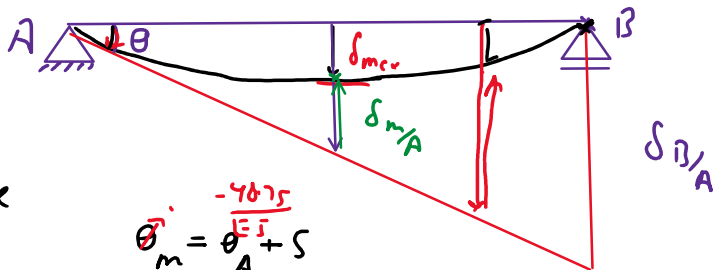
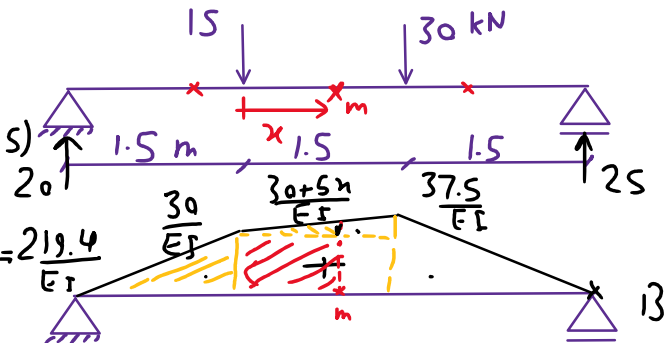
$$\delta_{B/A} = \frac{1}{2} \left(\frac{30}{EI} \right) (1.5)(3.5) + \left(\frac{30}{EI} \right) (1.5)(2.25)$$

$$+ \frac{1}{2} \left(\frac{7.5}{EI} \right) (1.5)(2) + \frac{1}{2} \left(\frac{37.5}{EI} \right) (1.5)(1) = \frac{219.4}{EI}$$

$$\theta_A = \frac{\delta_{B/A}}{L} = \frac{48.75}{EI}$$

$$\theta_m = \theta_A - \theta_B = \frac{1}{2} \left(\frac{30}{EI} \right) (1.5) + \frac{1}{2} \left(\frac{30}{EI} + \frac{30+5x}{EI} \right) x$$

$$-26.25 + 30x + 2.5x^2 = 0 \rightarrow x = 0.819 \text{ m}$$



$$\delta_{max} = L\theta_A + \delta_{m/A} = 2.319 \times \frac{48.75}{EI} + \frac{1}{2} \left(\frac{30}{EI} \right) (1.5)(1.319) + \left(\frac{30}{EI} \right) \left(\frac{0.819^2}{2} \right) + \frac{1}{2} \left(\frac{30+5 \times 0.819}{EI} \right) (0.819)(1.319) = 72.85$$

$$\Delta_{m, \Delta} = \Delta_{0A} + \Delta_{m/A} = \frac{1}{2} \left(\frac{PL}{EI} \right) (1.5)(1.5)(1.5) + \left(\frac{PL}{EI} \right) \left(\frac{0.819}{2} \right) + \frac{1}{2} \left(\frac{5 \times 0.819}{EI} \right) \left(\frac{0.819^2}{3} \right) = \frac{72.85}{EI}$$

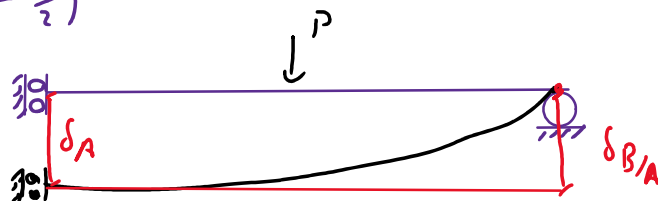
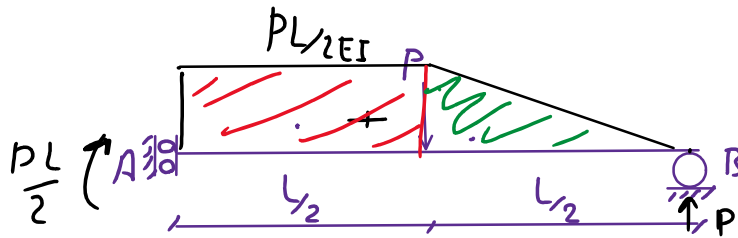
$\delta_A = ?$: مثال

$$\delta_A = \delta_{B/A} =$$

$$\left(\frac{PL}{2EI} \right) \left(\frac{L}{2} \right) \left(\frac{3L}{4} \right) + \frac{1}{2} \left(\frac{PL}{2EI} \right) \left(\frac{L}{2} \right) \left(\frac{2}{3} \cdot \frac{L}{2} \right)$$

$$\frac{3 \times 3 + 2}{48}$$

$$\delta_A = \frac{11}{48} \frac{PL^3}{EI}$$



$$EI : k_g \cdot \frac{1}{\text{cm}^2} \cdot \text{cm}^4 = k_g \cdot \text{cm}^2$$

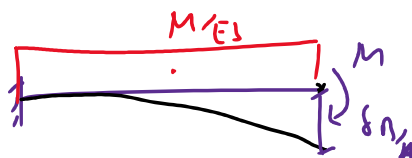
$$\frac{k_g \cdot \text{cm}^3}{k_g \cdot \text{cm}^2} = \text{cm}$$

$$\theta = \frac{PL^2}{EI}$$

$$\frac{WL^4}{EI}$$

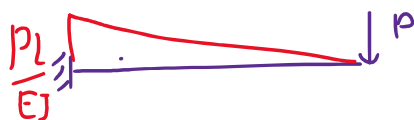
$$k_g$$

$$\theta = \frac{WL^3}{EI}$$



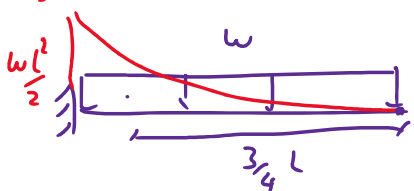
$$\theta = \frac{ML}{EI}$$

$$\delta = \frac{ML^2}{2EI}$$



$$\theta = \frac{PL^2}{2EI}$$

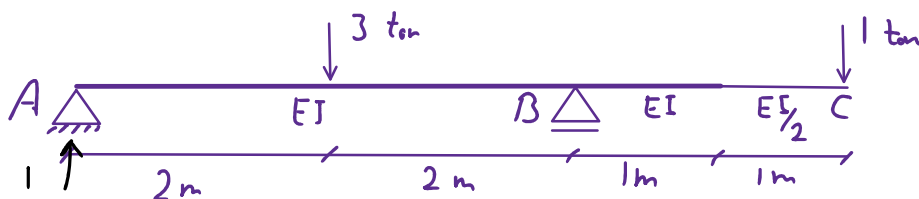
$$\delta = \frac{PL^3}{3EI}$$



$$\theta = \frac{WL^3}{6EI}$$

$$\delta = \frac{WL^4}{8EI}$$

$\delta_c = ?$: مثال



$$\delta_{A/B} = \frac{1}{2} \left(\frac{2}{EI} \right) (2) \left(\frac{4}{3} \right) + \frac{1}{2} \left(\frac{2}{EI} \right) (1) \left(\frac{2}{3} \right)$$

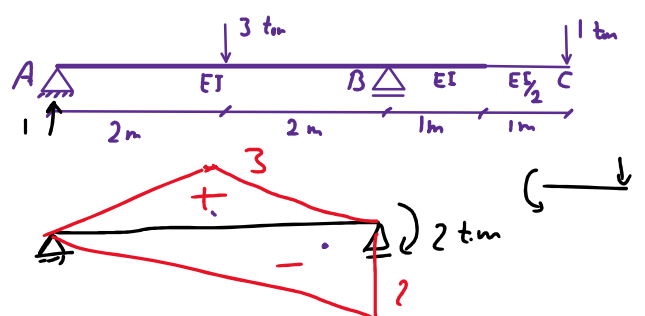
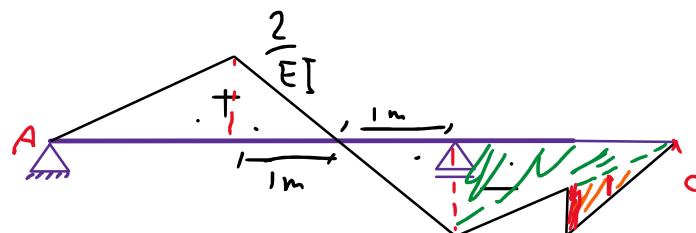
$$- \frac{1}{2} \left(\frac{2}{EI} \right) (1) \left(\frac{11}{3} \right) = \frac{4}{3EI}$$

$$\theta_B = \frac{\frac{4}{3EI}}{\frac{4}{3}} = \frac{1}{3EI}$$

$$\delta_c = 2\theta_B + \delta_{C/B} = 2 \left(\frac{1}{3EI} \right) + \frac{1}{2} \left(\frac{-2}{EI} \right) (2) \left(\frac{4}{3} \right) + \frac{1}{2} \left(\frac{-1}{EI} \right) (1) \left(\frac{2}{3} \right)$$

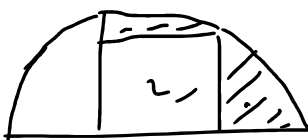
$$\frac{2}{3EI} - \frac{3}{EI}$$

$$\delta_c = \frac{-7}{3EI}$$

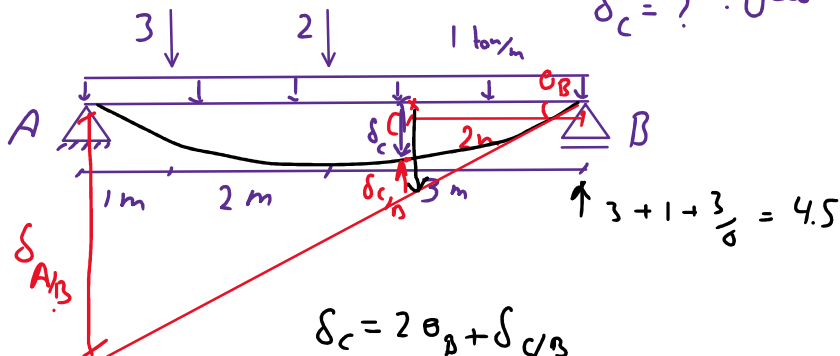


$$\delta_{A/B} = \frac{1}{2} \left(\frac{3}{EI} \right) (4) \left(\frac{2}{3} \right) + \frac{1}{2} \left(\frac{-2}{EI} \right) (4) \left(\frac{8}{3} \right)$$

$$= 12 + \left(-\frac{32}{3} \right) = \frac{36-32}{3} = \frac{4}{3EI}$$



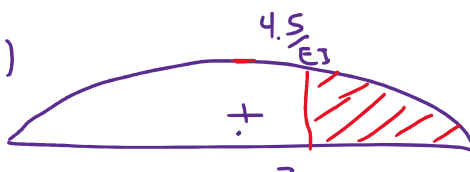
$\delta_c = ?$: مثال



$$\delta_c = 2\theta_B + \delta_{C/B}$$

$$\delta_{A/B} = \frac{2}{3} \left(\frac{4.5}{EI} \right) (6) \left(\frac{3}{3} \right) + \frac{1}{2} \left(\frac{3}{EI} \right) (6) \left(\frac{3}{3} \right)$$

$$+ \left[\frac{1}{2} (2.5) (1) \left(\frac{1}{3} \right) + \frac{1}{2} (2.5) (5) \left(\frac{1}{8} \right) \right]$$

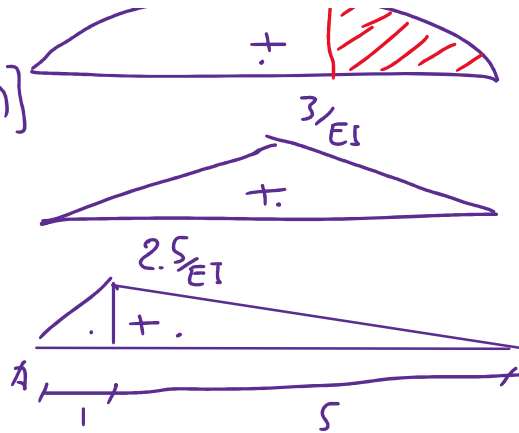


A/B $\frac{3}{EI}$ $\frac{2.5}{EI}$

$$+ \left[\frac{1}{2} \left(\frac{2.5}{EI} \right) (1) \left(\frac{2}{3} \right) + \frac{1}{2} \left(\frac{2.5}{EI} \right) (5) \left(\frac{8}{3} \right) \right]$$

$$\delta_{A/B} = \frac{98.5}{EI}$$

$$\theta_B = \frac{\delta_{A/B}}{6} = \frac{16.42}{EI}$$

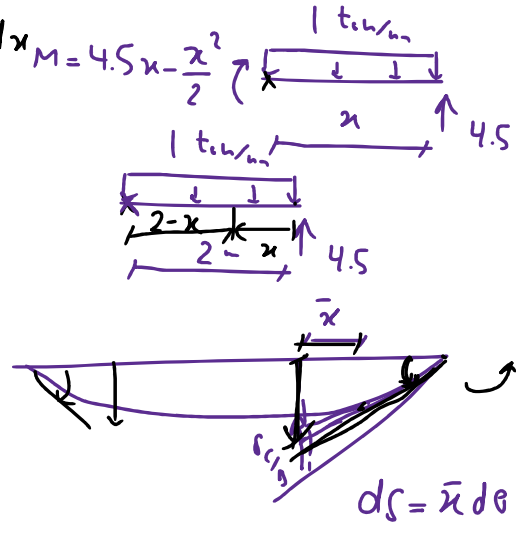


$$\delta_{C/B} = \int_0^2 \frac{M}{EI} \bar{x} dx = \frac{1}{EI} \int_0^2 (4.5x - \frac{x^2}{2})(2-x) dx$$

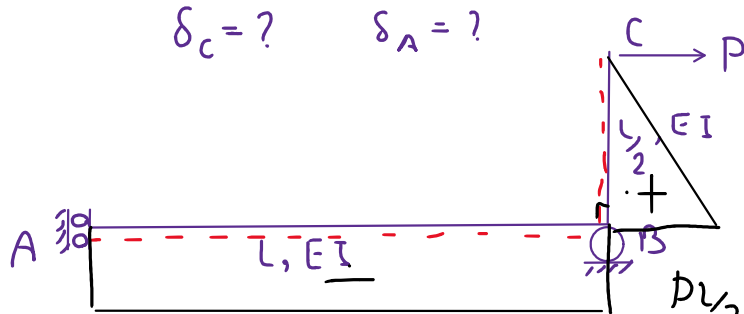
$$\int_0^2 (9x - 4.5x^2 - x^2 + \frac{x^3}{2}) dx =$$

$$\left[\frac{9}{2}x^2 - \frac{5.5}{3}x^3 + \frac{x^4}{8} \right]_0^2 = \frac{5.33}{EI}$$

$$\delta_C = -2 \left(\frac{16.42}{EI} \right) + \frac{5.33}{EI} = -\frac{11.08}{EI}$$



$$d\delta = \bar{x} d\theta = \bar{x} \left(\frac{M}{EI} dx \right)$$

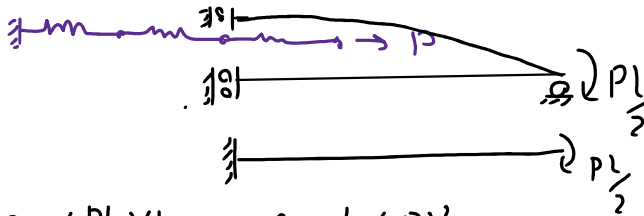


$$\delta_A = \delta_{B/A} = \left(\frac{PL}{2EI} \right) \left(L \right) \left(\frac{L}{2} \right) = \frac{PL^3}{4EI}$$

$$\theta_B = \theta_{B/A} = \left(\frac{PL}{2EI} \right) (L) = \frac{PL^2}{2EI}$$

$$\delta_C = \frac{L}{2} \theta_B + \delta_{C/B} = \frac{L}{2} \left(\frac{PL^2}{2EI} \right) +$$

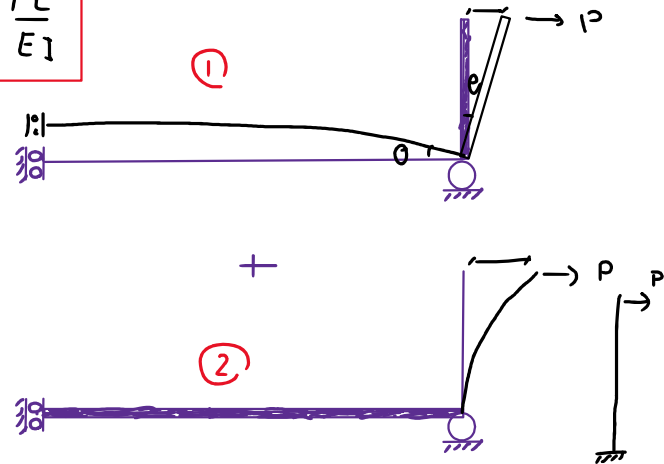
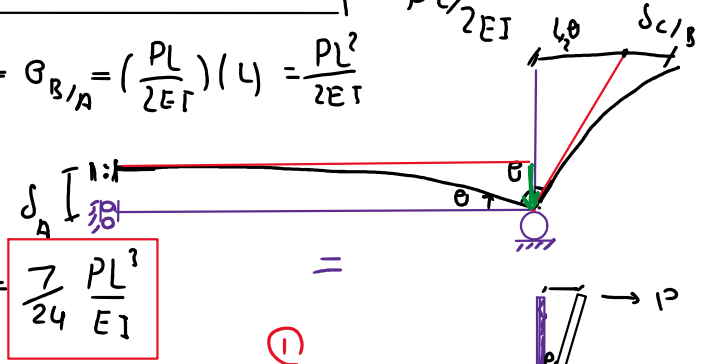
$$\frac{1}{2} \left(\frac{PL}{2EI} \right) \left(\frac{L}{2} \right) \left(\frac{2}{3} \times \frac{L}{2} \right) = \frac{6+1}{24} = \frac{7}{24} \frac{PL^3}{EI}$$



$$\textcircled{1} \theta = \left(\frac{PL}{2} \right) \left(\frac{L}{EI} \right), \delta_C = \frac{L}{2} \left(\frac{PL^2}{2EI} \right)$$

$$\textcircled{2} \delta_C = \frac{P \left(\frac{L}{2} \right)^3}{3EI}$$

$$\textcircled{1} + \textcircled{2} \delta_C = \frac{PL^3}{4EI} + \frac{PL^3}{24EI} = \frac{7}{24} \frac{PL^3}{EI}$$

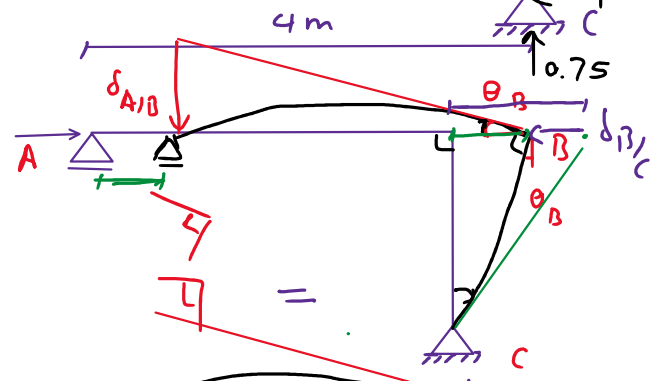
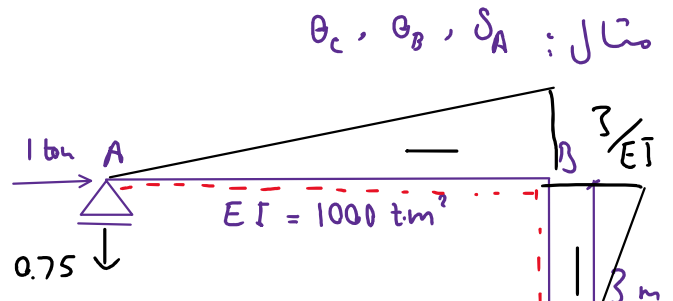


$$\delta_{A/B} = \frac{1}{2} \left(\frac{3}{EI} \right) (4) \left(\frac{2}{3} \times 4 \right) = \frac{16}{EI}$$

$$\theta_B = \frac{16}{EI} = \frac{4}{EI}$$

$$\theta_{C/B} = \theta_C - \theta_B = \frac{1}{2} \left(\frac{3}{EI} \right) (3) = \frac{4.5}{EI}$$

$$\theta_C - \left(\frac{4}{EI} \right) = \frac{4.5}{EI} \rightarrow \theta_C = \frac{8.5}{EI}$$



$$\theta_c - \left(-\frac{4}{EI}\right) = -\frac{4.5}{EI} \rightarrow \theta_c = -\frac{8.5}{EI}$$

$$\delta_A = 3\left(\frac{8.5}{EI}\right) - \frac{1}{2}\left(\frac{3}{EI}\right)(3)(1) = \frac{21}{EI}$$

$$\textcircled{1} \quad \theta_B = \frac{4}{EI}$$

$$\delta_A = \frac{12}{EI}$$

$$\theta_c = \frac{4}{EI}$$

$$\textcircled{2} \quad \theta_B = 0$$

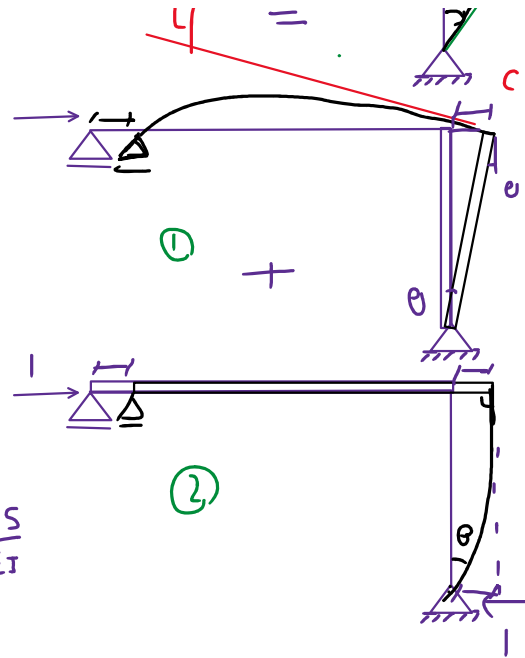
$$\delta_A = \frac{1 \times 3^3}{3EI} = \frac{9}{EI}$$

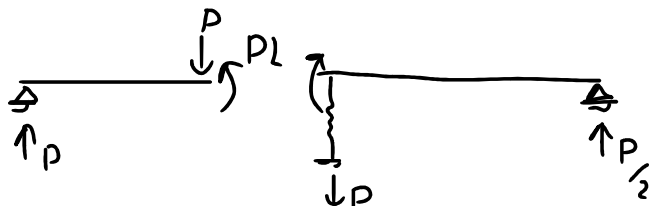
$$\theta_c = \frac{1 \times 3^2}{2EI} = \frac{4.5}{EI}$$

$$\textcircled{1} + \textcircled{2} \quad \theta_B = \frac{4}{EI}$$

$$\delta_A = \frac{21}{EI}$$

$$\theta_c = \frac{8.5}{EI}$$





$$\delta_{BR} = \frac{P}{2} \frac{l^3}{EI} = \frac{Pl^3}{EI}$$

$$\delta_{B/c} = \frac{1}{2} \left(\frac{Pl}{EI} \right) (l) \left(\frac{2l}{3} \right) = \frac{2}{3} \frac{Pl^3}{EI}$$

$$\theta_c = \frac{1}{2l} \left(\frac{Pl^3}{EI} - \frac{2}{3} \frac{Pl^3}{EI} \right) = -\frac{Pl^2}{6EI}$$

$$\theta_{c/B} = \theta_c - \theta_B = \frac{Pl^2}{EI} \rightarrow \theta_B = -\frac{7}{6} \frac{Pl^2}{EI}$$

$$\theta_{B/A} = \theta_B - \theta_A = \frac{Pl^2}{2EI} \rightarrow \theta_A = -\frac{5}{3} \frac{Pl^2}{EI}$$

نکته - سطح

$$\theta_A = \frac{\delta_{B/A}}{L} = \frac{1}{L} \int_0^L \frac{M}{EI} \bar{x} dx$$

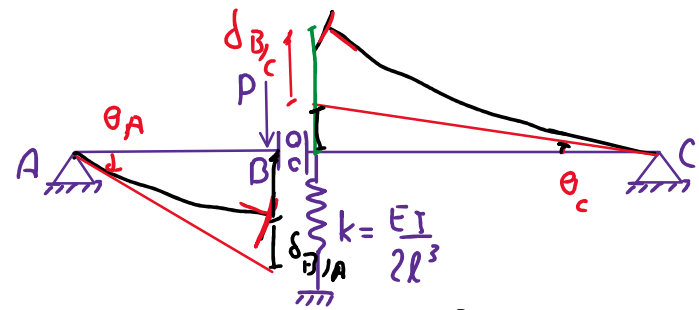
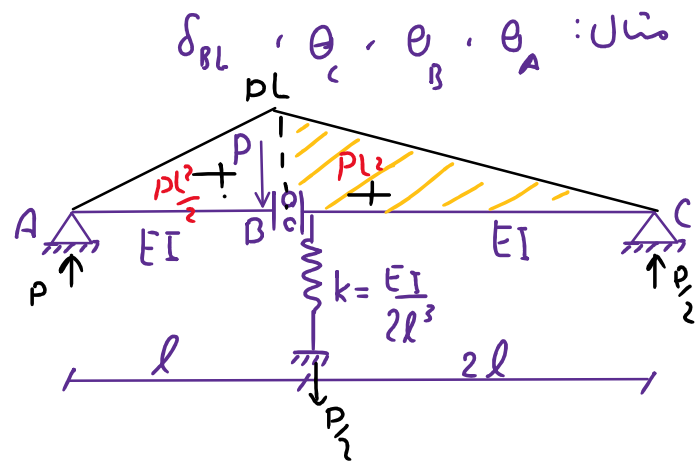
معادله تعادل تیرمانند

$$\sum M_B = 0$$

$$R_A(l) + \int_0^l \frac{M}{EI} \bar{x} dx \rightarrow R_A = -\frac{1}{l} \int_0^l \frac{M}{EI} \bar{x} dx$$

$$R_A = \theta_A$$

نکته - سطح

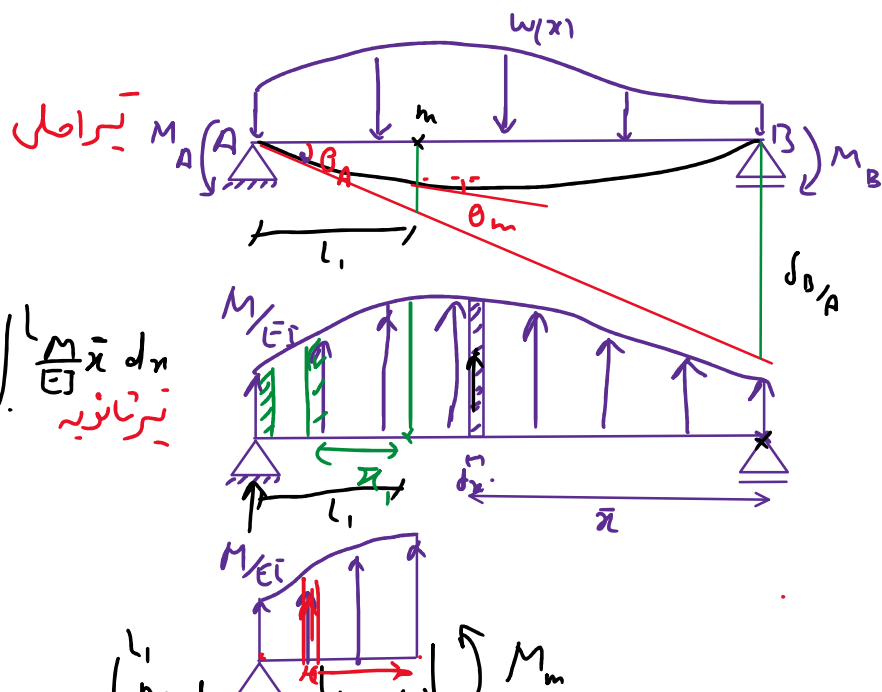


$$\delta_{B/A} = \left(\frac{Pl^2}{2EI} \right) \left(\frac{l}{3} \right) = \frac{Pl^3}{6EI}$$

$$\delta_{BL} = l \left(-\frac{5}{3} \frac{Pl^2}{EI} \right) + \frac{Pl^3}{6EI} = -\frac{3}{2} \frac{Pl^3}{EI}$$

روش بارالاستیک

حالت خاص از روش تیر مزدوج است (برای تیر دوسریعی)



کند-سطح

$$\theta_{m/A} = \theta_m - \theta_A = \int \frac{M}{EI} dx \rightarrow \theta_m = \theta_A + \int \frac{M}{EI} dx$$

برین در تیر اند

$$\sum F_y = 0 \rightarrow \theta_A + \int \frac{M}{EI} dx - V_m = 0 \rightarrow V_m = \theta_A + \int \frac{M}{EI} dx$$

$$V_m = \theta_m$$

تیر اصل تیر اند

کند-سطح

$$\delta_m = L_1 \theta_A + \delta_{m/A} = L_1 \theta_A + \int \frac{M}{EI} \bar{x}_1 dx$$

کند در تیر اند

$$M = \delta_m$$

تیر اصل تیر اند

$$\delta_{m \text{ اصل}} = M_{m \text{ تیر اند}}$$

$$\sum M_0 = 0$$

$$-4 \theta_A + \frac{1}{2} \left(\frac{30}{EI} \right) (1.5) (3.5) + \left(\frac{30}{EI} \right) (1.5) (2.25)$$

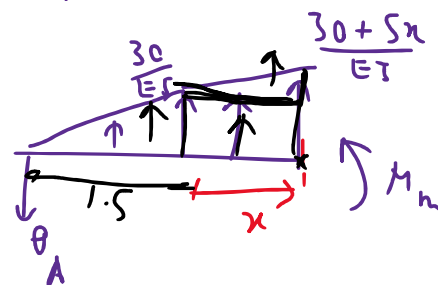
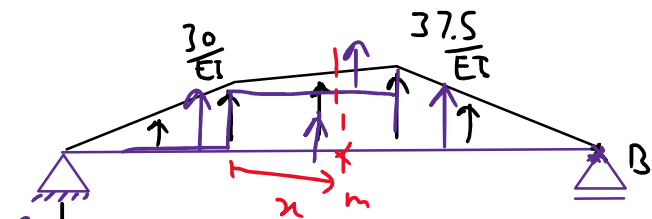
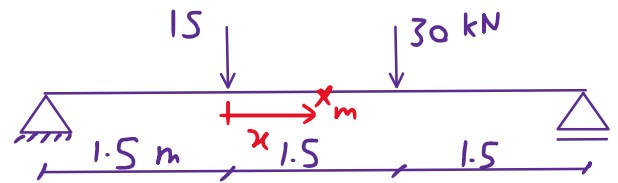
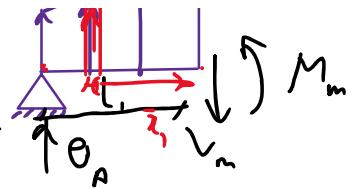
$$+ \frac{1}{2} \left(\frac{7.5}{EI} \right) (1.5) (2) + \frac{1}{2} \left(\frac{37.5}{EI} \right) (1.5) (1) = 0$$

$$\theta_A = \frac{48.75}{EI}$$

$$\sum F_y = 0$$

$$-\frac{48.75}{EI} + \frac{1}{2} \left(\frac{30}{EI} \right) (1.5) + \frac{1}{2} \left(\frac{30}{EI} + \frac{30+5x}{EI} \right) x = 0 \rightarrow x = 0.819$$

$$M_m = -\frac{48.75}{EI} \times 2.319 + \frac{22.5}{EI} \times 1.319 + \frac{30}{EI} \frac{0.819^2}{2} + \frac{1}{2} \left(\frac{5 \times 0.819}{EI} \right) \times \frac{0.819^2}{3} = \frac{-72.85}{EI}$$



روش تیر مزدوج

بررسی معادلات دینامیک تیرها

$$y'' = \frac{M}{EI}$$

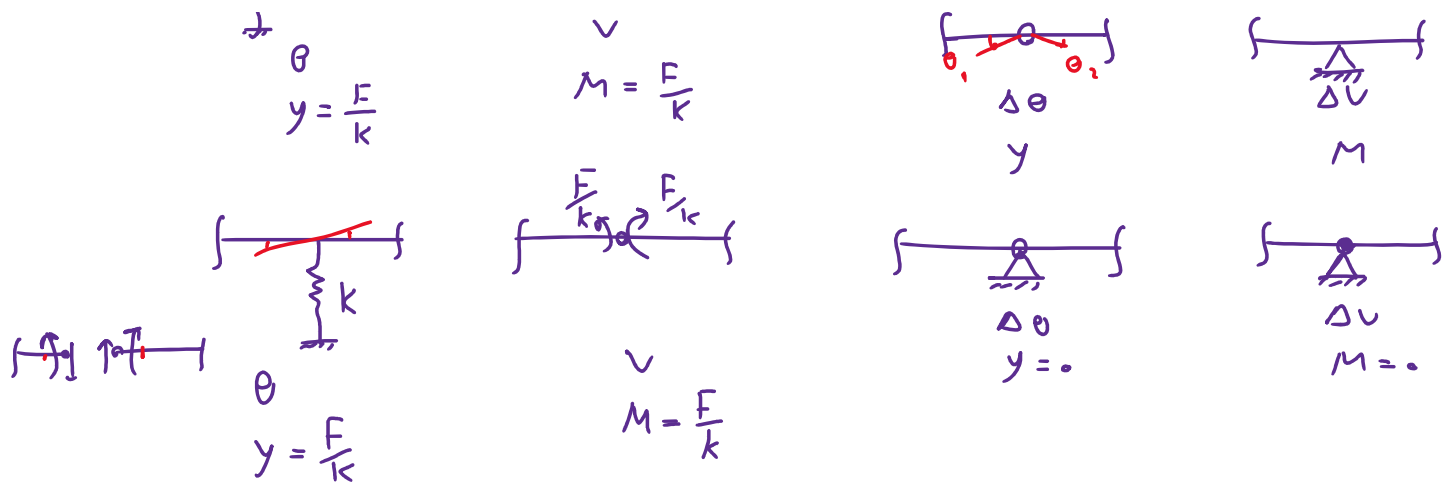
$$\begin{cases} \frac{dV}{dx} = w \\ \frac{dM}{dx} = V \end{cases} \quad \begin{cases} \frac{d\theta}{dx} = \frac{M}{EI} \\ \frac{dy}{dx} = \theta \end{cases}$$

* از آنجا که معادلات دینامیک مستقیماً یکسان است، می توان زمین زد که تیرها نیز وجود دارد که تحت بار ثابت $\frac{M}{EI}$ قرار می گیرند در آن (تیر مزدوج)، برش (V) و لنگر (M) برابر است با چرخش (θ) و تغییر مکان (y) در تیر اصلی.

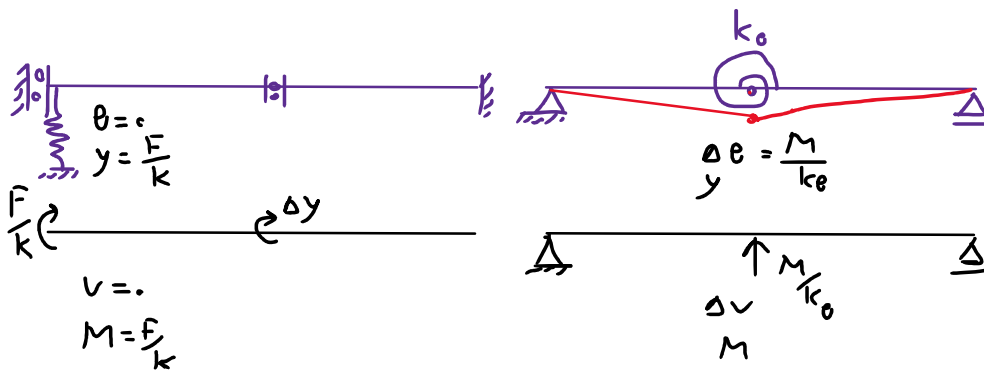
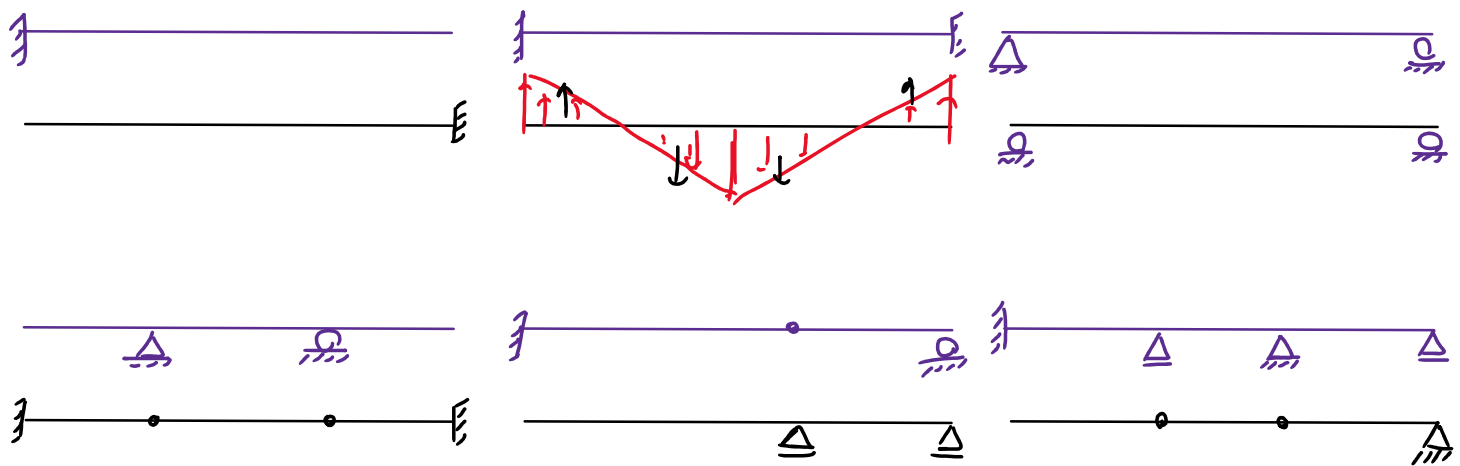
* برای تعیین صحت این شباهت، علاوه بر معادلات دینامیک، شرایط مرزی نیز باید ارضا شود.

بنابراین داریم:

تیر اصلی	تیر مزدوج	تیر اصلی	تیر مزدوج
$\theta = 0$ $y = 0$	$V = 0$ $M = 0$	$\theta = 0$ $y = 0$	$V = 0$ $M = 0$
$\theta = \Delta\theta$ Δy	$V = \Delta V$ ΔM	$\theta = 0$ $y = 0$	$V = 0$ $M = 0$
$\theta = \frac{M}{k_\theta}$ $y = 0$	$V = \frac{M}{k_\theta}$ $M = 0$	$\theta = 0$ $y = 0$	$V = 0$ $M = 0$
$\theta = \frac{F}{k}$ $y = 0$	$V = \frac{F}{k}$ $M = 0$	$\theta = 0$ $y = 0$	$V = 0$ $M = 0$
$\theta = \frac{F}{k}$ $y = 0$	$V = \frac{F}{k}$ $M = 0$	$\theta = 0$ $y = 0$	$V = 0$ $M = 0$



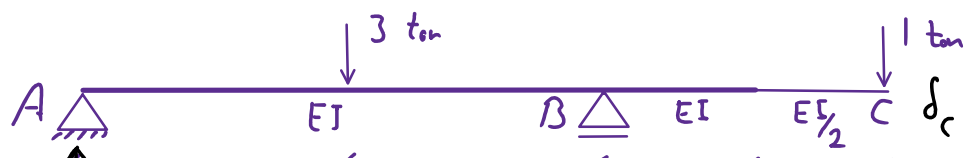
مثال: تیر مزدوج مرکب از تیرهای شکل زیر را رسم نمایید.

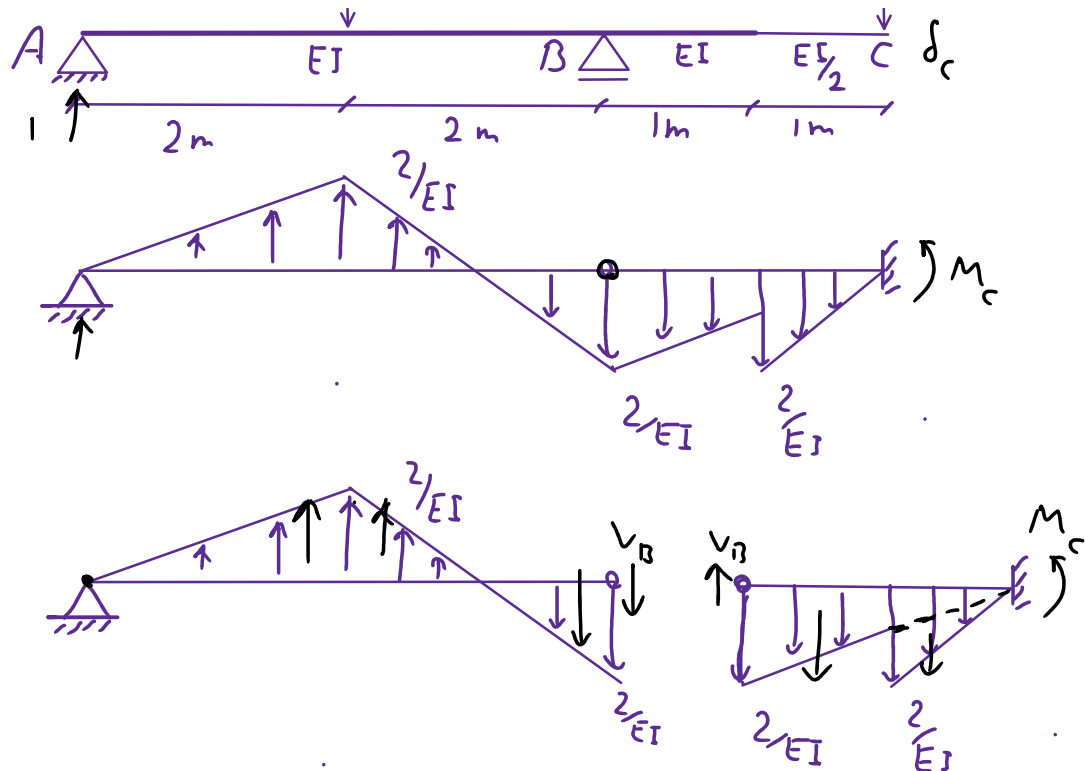


* چنانکه معلوم شود، بعضی از تیرهای مزدوج نابایار مستوی نت بارگذاری خاص $\frac{M}{EI}$ باید از بین بروند.

مثال: $\delta_c = ?$

$$\delta_c = \frac{-7}{3EI}$$





$$(\sum M_A = 0 \rightarrow \frac{1}{2} \left(\frac{2}{EI} \right) (2) \left(\frac{2}{3} \times 2 \right) + \frac{1}{2} \left(\frac{2}{EI} \right) (1) \left(\frac{7}{3} \right) - \frac{1}{2} \left(\frac{2}{EI} \right) (1) \left(\frac{11}{3} \right) - V_B (4) =$$

$$\theta_B = V_B = \frac{1}{3EI}$$

$$\sum M_C = 0 \rightarrow M_C = \left(\frac{1}{3EI} \right) (2) - \frac{1}{2} \left(\frac{2}{EI} \right) (2) \left(\frac{2}{3} \times 2 \right) - \frac{1}{2} \left(\frac{1}{EI} \right) (1) \left(\frac{2}{3} \times 1 \right) = \frac{-7}{3EI}$$

$$\delta_c = M_C = \frac{-7}{3EI}$$

$$\delta_c = 2\theta_B + \delta_{c/B} = 2 \left(\frac{1}{3EI} \right) + \frac{1}{2} \left(\frac{-2}{EI} \right) (2) \left(\frac{4}{3} \right) + \frac{1}{2} \left(\frac{-1}{EI} \right) (1) \left(\frac{2}{3} \right)$$

مثال: $\delta_D = ?$ $\theta_C = ?$ $\Delta\theta_B = ?$

$$M = -12 + 7x - x^2$$

$$\sum M_i = 0 \rightarrow$$

$$\int_0^6 \frac{1}{EI} (-12 + 7x - x^2) (6 - x) dx + R_B (2) = 0$$

$$-\frac{72}{EI} + R_B (2) = 0 \rightarrow \Delta\theta = R_B = \frac{36}{EI}$$

نیم تغییرات

$$\sum F_y = 0$$

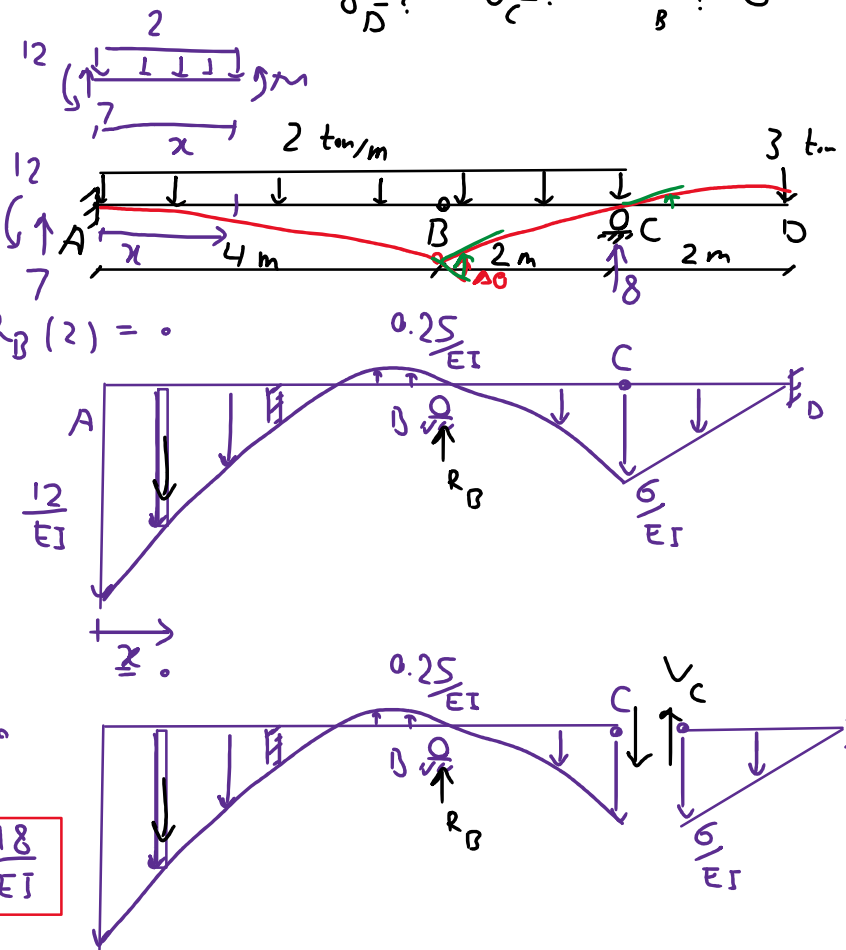
$$\int_0^6 \frac{1}{EI} (-12 + 7x - x^2) dx + \frac{36}{EI} - V_C = 0$$

$$-\frac{18}{EI} + \frac{36}{EI} - V_C = 0 \rightarrow \theta = V_C = \frac{18}{EI}$$

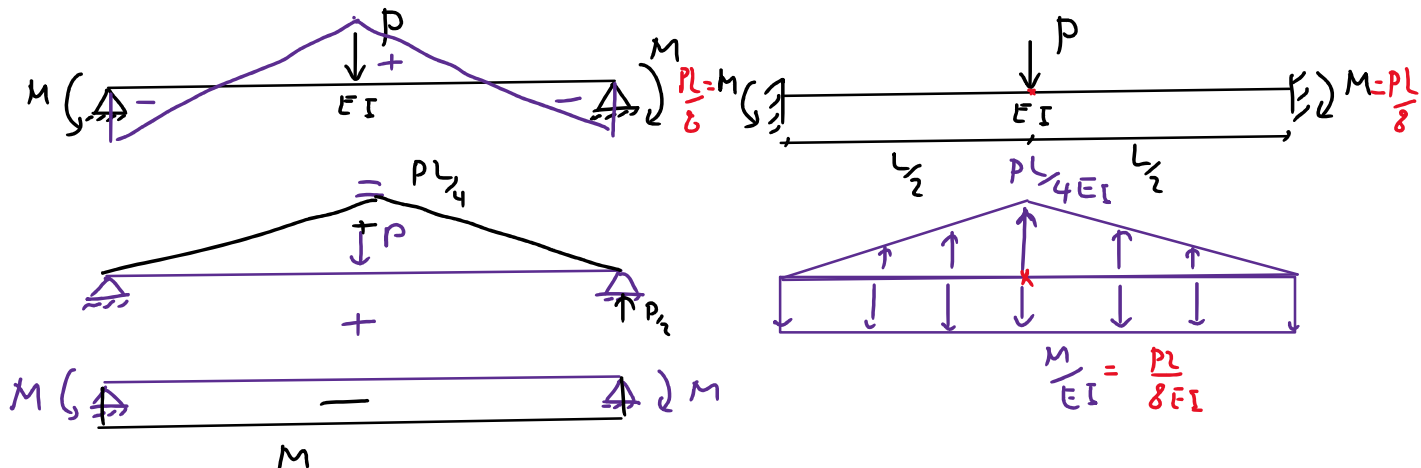
برای C

$$M_D = \left(\frac{18}{EI}\right)(2) - \frac{1}{2} \left(\frac{6}{EI}\right)(2) \left(\frac{2}{3} \times 2\right) = \frac{28}{EI}$$

$$\delta_D = M_D = \frac{28}{EI} \uparrow$$

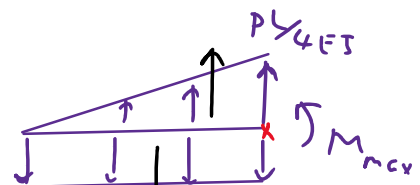


مثال: تغییر مکان ماکزیم بر شکل زیر را بدست آورید.



$$\sum F_y = 0 \rightarrow \frac{1}{2} \left(\frac{PL}{4EI}\right)(L) - \left(\frac{M}{EI}\right)(L) = 0 \rightarrow M = \frac{PL}{8}$$

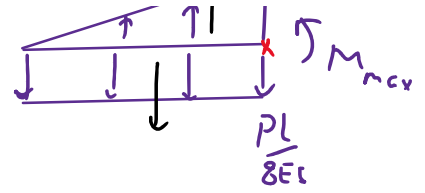
$$M = \frac{1}{2} \left(\frac{PL}{4EI}\right)(L) - \left(\frac{M}{EI}\right)(L) =$$



$$M_{max} = \frac{1}{2} \left(\frac{PL}{4EI} \right) \left(\frac{L}{2} \right) \left(\frac{1}{3} \times \frac{L}{2} \right) - \left(\frac{PL}{8EI} \right) \left(\frac{L}{2} \right) \left(\frac{L}{4} \right) =$$

$$\frac{2-3}{32 \times 3 \times 2} \frac{PL^3}{EI}$$

$$\delta_{max} = M_{max} = \frac{-1}{192} \frac{PL^3}{EI}$$



$$\delta_{BR} = \frac{P}{\frac{EI}{2l}} = \frac{PL^3}{EI}$$

$$\sum M_B = 0$$

$$R_C (2l) + \left(\frac{PL^2}{EI} \right) \left(\frac{1}{3} \times 2l \right) - \frac{PL^3}{EI} = 0$$

$$\theta_C = \psi_C = -\frac{1}{6} \frac{PL^2}{EI}$$

$$\sum f_y = 0$$

$$R_A + \frac{PL^2}{2EI} + \frac{PL^2}{EI} + \frac{1}{6} \frac{PL^2}{EI} = 0$$

$$\theta_A = \psi_A = -\frac{10}{6} \frac{PL^2}{EI}$$

$$\psi_B = -\frac{10}{6} \frac{PL^2}{EI} + \frac{PL^2}{2EI} \rightarrow \theta_B = \psi_B = -\frac{7}{6} \frac{PL^2}{EI}$$

$$\delta_{BL} = \left(-\frac{10}{6} \frac{PL^2}{EI} \right) (L) + \left(\frac{PL^2}{2EI} \right) \left(\frac{L}{3} \right) \rightarrow \delta_{BL} = -\frac{3}{2} \frac{PL^2}{EI}$$

مثال: اگر در تیر شکل زیر تکیه گاه C به اندازه 4 سانت و تکیه گاه A به اندازه 1 دران بادا بکند داشته باشد، مقدار θ_B و δ_D چندرست؟

$$\sum M_B = 0$$

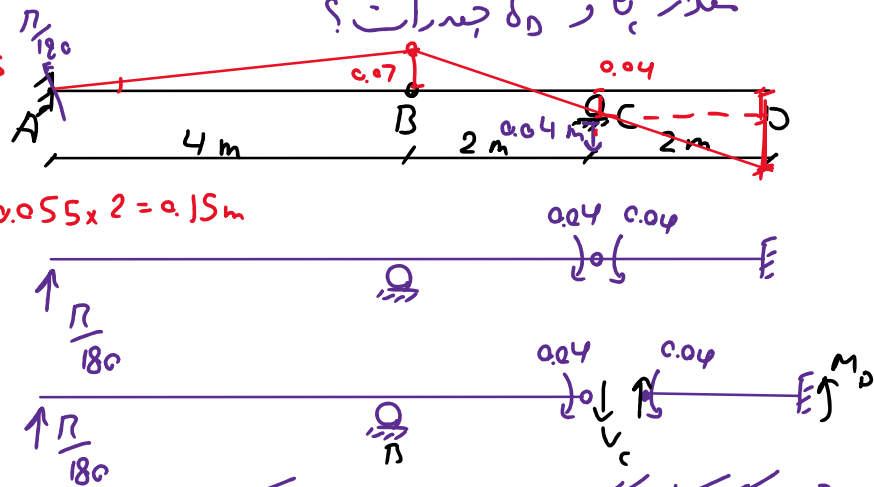
$$\theta_C = \frac{0.07 + 0.04}{2} = 0.055$$

$$\left(\frac{R}{180} \right) (4) + 0.04 + \psi_C (2) = 0$$

$$\theta_C = \psi_C = -0.055 \text{ rad}$$

$$M_B = (-0.055)(2) - 0.04 = -0.15 \text{ m}$$

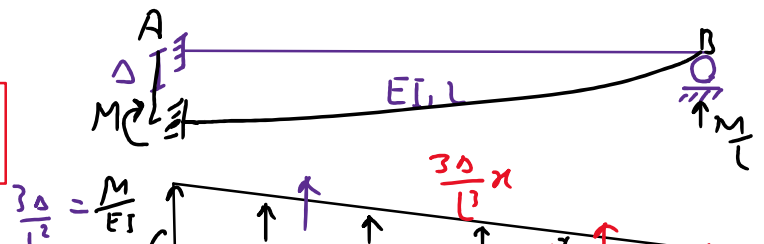
$$\delta_D = M_B = -0.15 \text{ m}$$



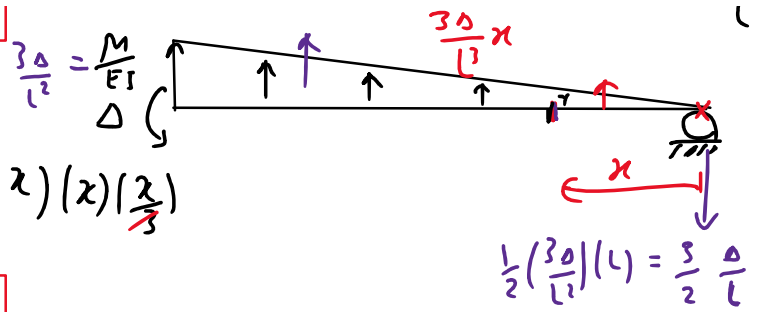
مثال: اگر تکیه گاه گیردار A به اندازه 5 سانت کند؛ لنگر انحنای یاردار و تابع تغییر شکل تیر را بدست آورید.

$$\sum M_B = 0$$

$$\frac{1}{2} \left(\frac{M}{EI} \right) (L) \left(\frac{2}{3} L \right) - \Delta = 0 \rightarrow M = \frac{3EI}{L^2} \Delta$$



L

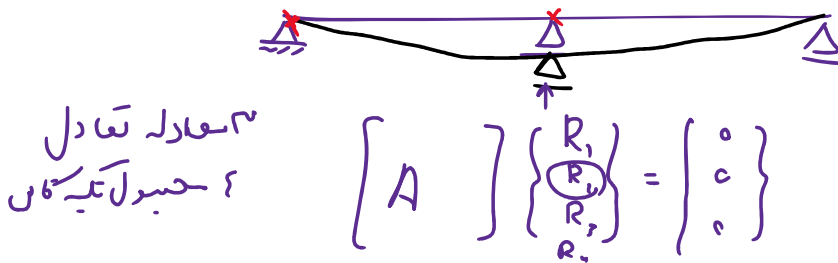
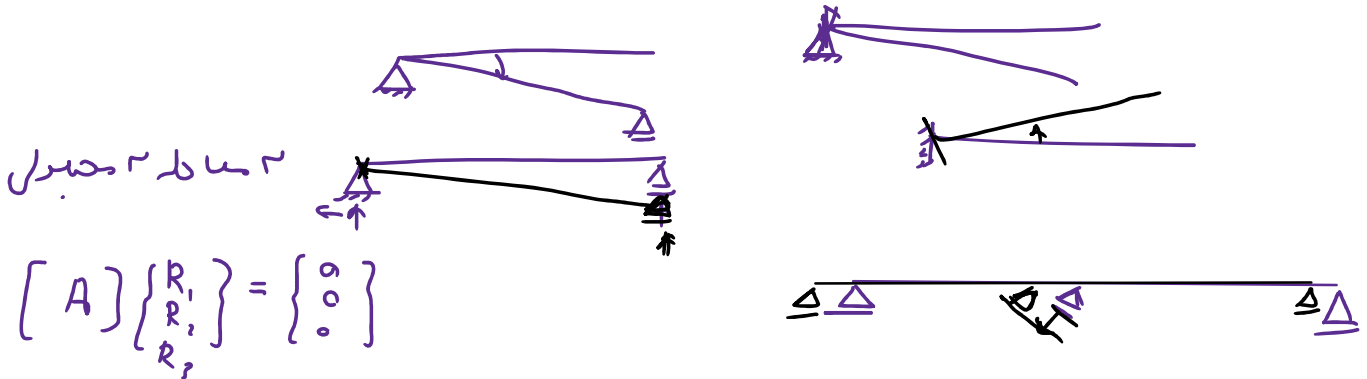


$$\delta(x) = M(x) = -\frac{3}{2} \left(\frac{\Delta}{L} \right) x + \frac{1}{2} \left(\frac{3\Delta}{L^2} x \right) (x) \left(\frac{x}{3} \right)$$

تبدیل
تبدیل

$$\delta(x) = \left(-\frac{3}{2} \frac{\Delta}{L} \right) x + \left(\frac{1}{2} \frac{\Delta}{L^2} \right) x^3$$

* نتایج و تغییرات
سازمانی و تغییرات
سازمانی و تغییرات



$$\begin{cases} a_1 R_1 + c_1 R_2 + c_2 R_3 + c_3 R_4 = 0 \\ b_1 R_1 + \dots = 0 \\ c_1 R_1 + \dots = 0 \end{cases}$$

محاسبه تغییر شکل با به روشی های انرژی

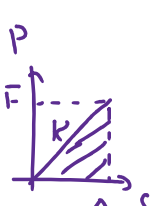
روش های انرژی برای محاسبه تغییر شکل ها عبارتند از:

- روش کار حقیقی

- روش بار واحد (روش کار مجازی)

- روش کاستیلانو

- قانون بیتا و مانوئل ماکسول



$$U_{1 \rightarrow 2} = \Delta T + \Delta V_s$$

$$\frac{1}{2} F \delta = \frac{1}{2} k \delta^2$$



روش کار حقیقی

بنابراین برای انرژی از دست کار انجام شده توسط بارها خارجی برابری با کار انجام شده توسط نیروهای داخلی (انرژی کرنشی ذخیره شده در اعضا ساز)

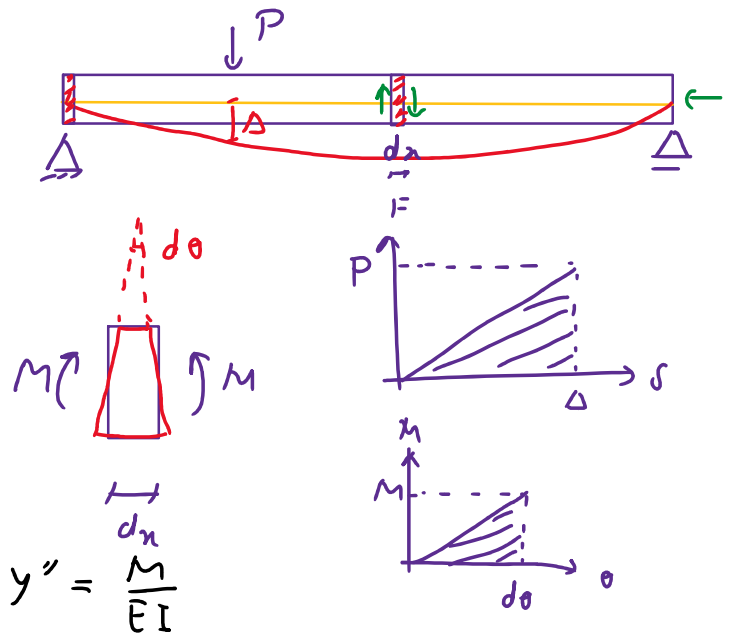
$$W_{ext} = (W_{int} = U)$$

به طور مثال برای یک تیر کار خاکی و انرژی کرنشی ذخیره شده را می توان به صورت زیر نوشت:

$$W_{ext} = \frac{1}{2} P \Delta$$

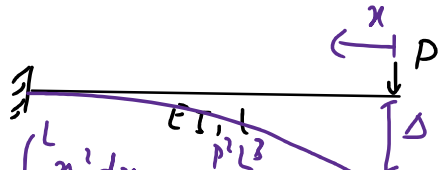
$$dW_{int} = \frac{1}{2} M d\theta = \frac{1}{2} M \left(\frac{M}{EI} dx \right)$$

$$W_{int} = \frac{1}{2} \int_0^L \frac{M^2}{EI} dx$$



$$\frac{d\theta}{dx} = y'' = \frac{M}{EI}$$

مثال: تغییر مکان قائم سر آزاد تیر زیر بار دلتا آورده.

$$\left\{ \begin{array}{l} W_{ext} = \frac{1}{2} P \Delta \\ W_{int} = \int_0^L \frac{M^2}{EI} dx = \int_0^L \frac{(-Px)^2}{EI} dx = \frac{P^2}{EI} \int_0^L x^2 dx = \frac{P^2 L^3}{6EI} \end{array} \right.$$


$$w_{int} = U = \frac{1}{2} \int_0^L \frac{M^2}{EI} dx = \frac{1}{2} \int_0^L \frac{(-Px)^2}{EI} dx = \frac{P^2}{2EI} \int_0^L x^2 dx = \frac{P^2 L^3}{6EI}$$

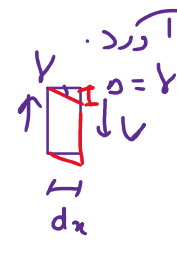
$$\frac{1}{2} P \Delta = \frac{P^2 L^3}{6EI} \rightarrow \Delta = \frac{PL^3}{3EI}$$


* انرژی کرنشی ناشی از سایر نیروها (نیروهای محوری، نیروهای برشی و گشتاورهای پیچشی) را نیز می‌توان به روش مشابه

$$U = U_{کشش} + U_{نیرومحوری} + U_{برشی} + U_{پیچشی}$$

$$U = \frac{1}{2} \int_0^L \frac{M^2}{EI} dx + \frac{1}{2} \int_0^L \frac{N^2}{EA} dx + \frac{1}{2} k \int_0^L \frac{V^2}{GA} dx + \frac{1}{2} \int_0^L \frac{T^2}{JG} dx$$

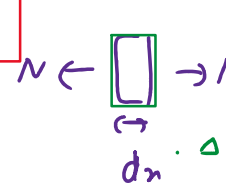
بدست آورد.



$$\sigma = \frac{N}{A}$$

$$\tau = \frac{V}{A}$$

$$\tau = \frac{VQ}{I} \quad \square \triangleright \text{I} \square \quad \tau \approx \frac{V}{A}$$



$$\Delta = \frac{N dx}{EA}$$

* k در مقاطع مستطیلی برابر 1.2، در مقاطع دایره‌ای برابر 10/9 و در مقاطع J شکل برابر 1 است.

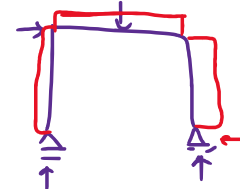
$$\int_0^L \int_0^h \tau d\phi = \frac{T dx}{GJ}$$

$$U = \sum_i \frac{1}{2} \frac{N_i^2 L_i}{E_i A_i}$$

خوابها

$$U = \frac{1}{2} \int_0^L \frac{M^2}{EI} dx$$

بهره‌ها و قاب‌ها



* دامن‌کاربرد در این کارچین، بسیار محدود است و در تحلیل سازه به ندرت استفاده می‌شود. چرا که این روش فقط تغییر مکان زیر بار متمرکز را می‌دهد.



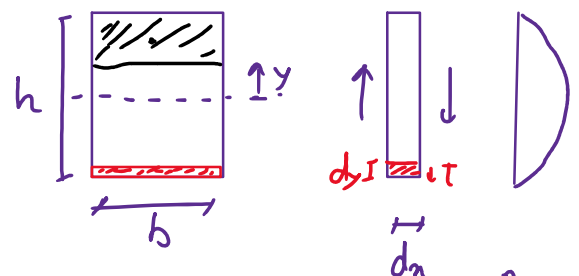
انرژی کرنشی ناشی از برش

$$U = \frac{1}{2} \int_0^L \int_{-h/2}^{h/2} \frac{\tau^2}{G} b dy dx$$

$$Q = b \left(\frac{h}{2} - y \right) \frac{1}{2} \left(\frac{h}{2} + y \right)$$

$$Q = \frac{b}{8} (h^2 - 4y^2)$$

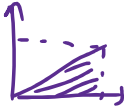
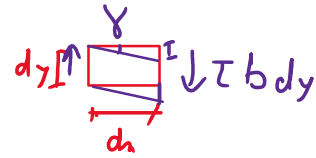
$$U = \frac{1}{2} \int_0^L \left(\frac{VQ}{Ib} \right)^2 \frac{b}{G} dy dx = \frac{1}{2} \frac{V^2 dx}{I^2 b G} \int_{-h/2}^{h/2} Q^2 dy$$



$$Q = \frac{b}{8} (h^2 - 4y^2)$$

b

h
 dy



$$\int_{-h/2}^{h/2} Q^2 dy = \int_{-h/2}^{h/2} \left(\frac{b}{8}\right)^2 (h^2 - 4y^2)^2 dy =$$

$$\frac{b^2}{64} \int (h^4 + 16y^4 - 8h^2y^2) dy =$$

$$\frac{b^2}{64} \left[h^4 y + \frac{16}{5} y^5 - 8h^2 y^3 \right]_{-h/2}^{h/2} = \frac{b^2}{64} \times 2 \left[h^4 \left(\frac{h}{2}\right) + \frac{16}{5} \left(\frac{h}{2}\right)^5 - 8h^2 \left(\frac{h}{2}\right)^3 \right] = \frac{15+3-10}{30} h^5$$

$$\int Q^2 dy = \frac{1}{120} b^2 h^5$$

$$\frac{b^2}{64} \times 2 \times \frac{8}{15} h^5 = \frac{1}{120} b^2 h^5$$

$$U = \frac{V^2 dx}{I^2 b G} \int Q^2 dy = \frac{V^2 dx}{I^2 b G} \times \frac{1}{120} b^2 h^5 = \frac{V^2 dx}{\left(\frac{1}{12} b h^3\right) G} \times \frac{1}{10} = 1.2 \frac{V^2 dx}{G A}$$

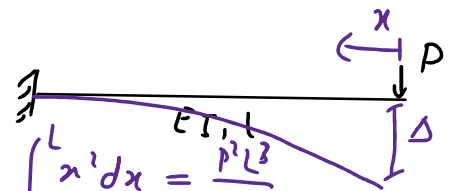
$$U = \frac{1}{2} \int_0^L \frac{1.2 V^2}{G A} dx$$

مثال رابا در نظر رشت تغییر شکل برشی حد نبه .

$$W_{ext} = \frac{1}{2} P \Delta$$

$$W_{int} = U = \frac{1}{2} \int_0^L \frac{M^2}{EI} dx = \frac{1}{2} \int_0^L \frac{(-Px)^2}{EI} dx = \frac{P^2}{2EI} \int_0^L x^2 dx = \frac{P^2 L^3}{6EI}$$

$$\frac{1}{2} P \Delta = \frac{P^2 L^3}{6EI} \rightarrow \Delta = \frac{PL^3}{3EI}$$



$$U = U_{ضری} + U_{برشی}$$

$$U_{برشی} = \frac{1}{2} \int_0^L \frac{1.2 V^2}{G A} dx = \frac{6}{10} \frac{1}{G A} \int_0^L P^2 dx = \frac{6}{10} \frac{P^2 L}{G A}$$

$$G = \frac{E}{2(1+\nu)} = \frac{E}{2(1+0.1)}$$

$$\Delta = \frac{PL^3}{3EI} + \frac{6}{10} \frac{P^2 L}{G A} \rightarrow \Delta = \frac{PL^3}{3EI} + \frac{6}{10} \frac{P^2 L}{G A} = \frac{PL^3}{3EI} \left(1 + \frac{18}{EI} \frac{1}{2(1+0.1)} \frac{1}{h^2} \right)$$

$$\frac{1}{2} \cancel{P} \Delta = \frac{\cancel{P} L^3}{6 E I} + \frac{6}{10} \frac{\cancel{P} L}{G A} \rightarrow \Delta = \frac{P L^3}{3 E I} + \frac{6}{5} \frac{P L}{G A} = \frac{P L^3}{3 E I} \left(1 + \frac{18}{5} \frac{E I}{G A} \times \frac{1}{L^2} \right)$$

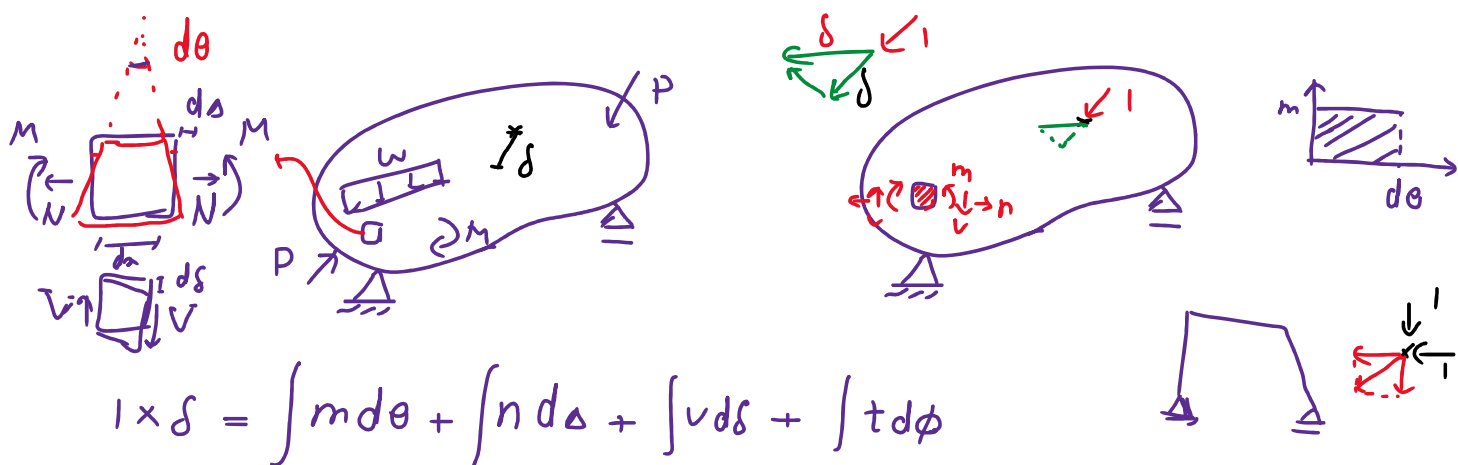
^{2.0}
 ^{$\frac{1}{12} h^2$}

$$\Delta = \frac{P L^3}{3 E I} \left(1 + 0.78 \frac{h^2}{L^2} \right)$$

روش بار واحد

این روش عمدتاً برای روش محاسبه تغییر شکل هادساز است که تقریباً بر کلیه محدودیت های روش دیگر فائق می آید و زیر مجموعه روش های کار مجازی است.

در این روش ابتدا یک بار مجازی واحد در محل و جهت تغییر شکل خواسته به بررسی سازه قرار می گیرد. سپس کلیه بارهای خارجی، نتایج گاه، تغییر دما و نقص عضو به سازه اعمال می شود. در این صورت کار انجام شده توسط بار واحد برابر با کار انجام شده توسط نیروهای مجازی داخلی است از بار واحد.



$$1 \times \delta = \int m d\theta + \int n d\Delta + \int v d\delta + \int t d\phi$$

$$1 \times \delta = \int \frac{mM}{EI} dx + \int \frac{nN}{EA} d\Delta + k \int \frac{vV}{GA} dx + \int \frac{tT}{GJ} d\phi$$

m, n, v, t : گسترختی، یزد و جرد، یزد و جردی و گسترختی ناحی از بار واحد

M, N, V, T : ناشی از بارگذاری خارجی

$$1 \times \delta = \sum_i \frac{n_i N_i L_i}{E_i A_i}$$

غیرها

* کار داخلی انجام شده توسط بار واحد با انتقال گیر بررسی کلیه اعضاء سازه به دست می آید.

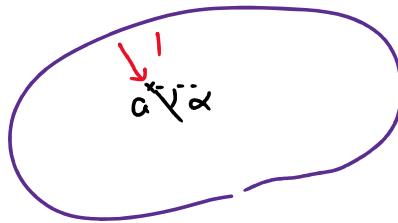
$$1 \times \delta = \int \frac{mM}{EI} dx$$

یضا و جها

روش اعمال بار واحد

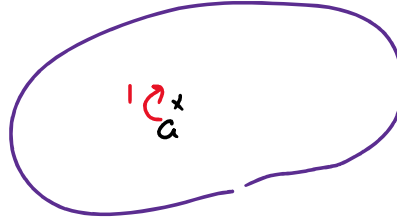
روش اتمال بارها حد

① تغییر مکان نقطه a در امتداد α



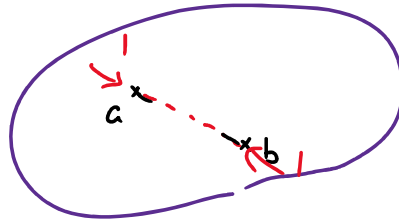
$$1 \times \delta_a = U$$

② چرخش نقطه a



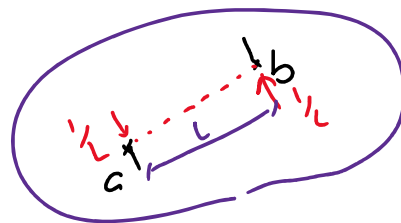
$$1 \times \theta_a = U$$

③ مقدار نز و بس و درجه در نقطه c و b



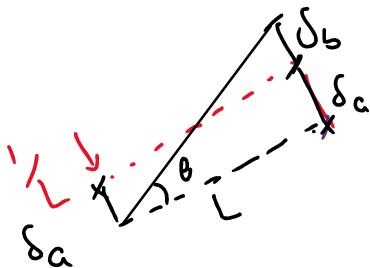
$$1 \times \delta_a + 1 \times \delta_b = U$$

④ دران عضو ab یا چرخش در نقطه c و b نسبت به a



$$\frac{1}{L} \delta_a + \frac{1}{L} \delta_b = U$$

$$\theta = \frac{1}{L} (\delta_a + \delta_b)$$



کار مجازی

بیز در مجازی

$$1 \Delta = \sum u \cdot dl$$

$$\frac{1}{2} (1) \delta + \frac{1}{2} P_1 \Delta_1 + \frac{1}{2} P_2 \Delta_2 + 1 \cdot \Delta$$

$$= \frac{1}{2} \sum u \cdot dL_1 + \frac{1}{2} \sum S \cdot dL + \sum u \cdot dL$$

بیز در مجازی

$$1 \times \delta = \int \frac{m \cdot m}{EI} dx$$

$$= \int \frac{m \cdot m}{EI} d\theta$$

تغییر مکان واقعی

مثال: تغییر مکان دیتب انتقال اکو ادتیر را بدست آورید.

$$1 \times \Delta = \int_0^L \frac{m M}{EI} dx$$

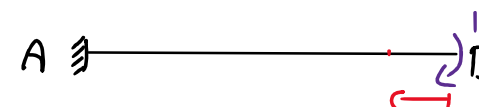
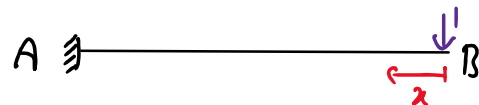
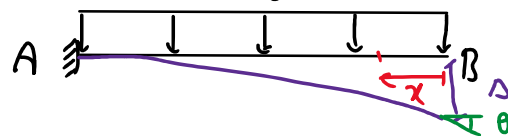
$$1 \times \Delta = \frac{1}{EI} \int_0^L (-x) \left(-\frac{w x^2}{2} \right) dx =$$

$$1 \times \Delta = \frac{w}{2EI} \left. \frac{x^4}{4} \right|_0^L = \frac{w L^4}{8EI}$$

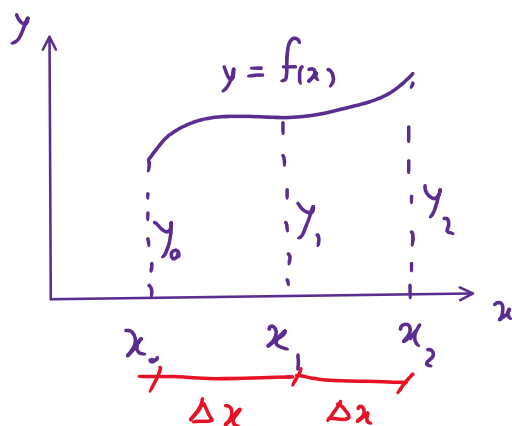
$$M = -\frac{w x^2}{2}$$

$$m = -x$$

$$m = -1$$



$$1 \times \theta = \int_0^L \frac{m M}{EI} dx = \frac{1}{EI} \int_0^L (-1) \left(-\frac{w x^2}{2} \right) dx = \frac{w}{2EI} \int_0^L x^2 dx = \frac{w L^3}{6EI}$$



محاسبه عددی انتگرال $\int m M dx$

① روش پرنمودار

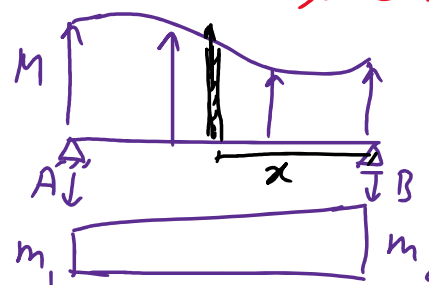
$$\int_{x_0}^{x_2} f(x) dx = \frac{\Delta x}{3} (y_0 + 4y_1 + y_2)$$

$$\text{پسوند} \quad \frac{\Delta x}{3} (y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \dots + y_n)$$

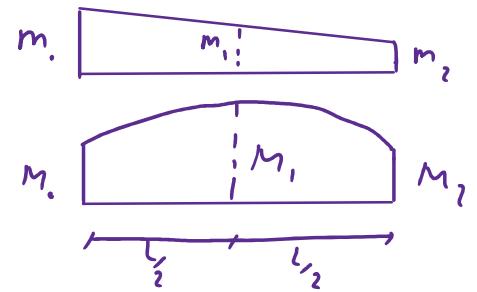
* اگر درجه عبارت زیر انتگرال ۳ یا کمتر باشد، جواب روش پرنمودار دقیق است.

$$\int m M dx = m_1 A + m_2 B$$

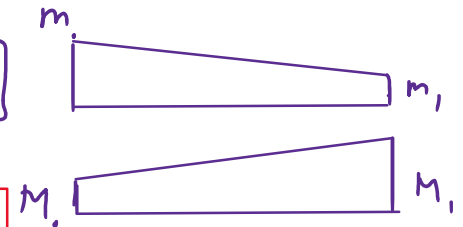
② روش مور



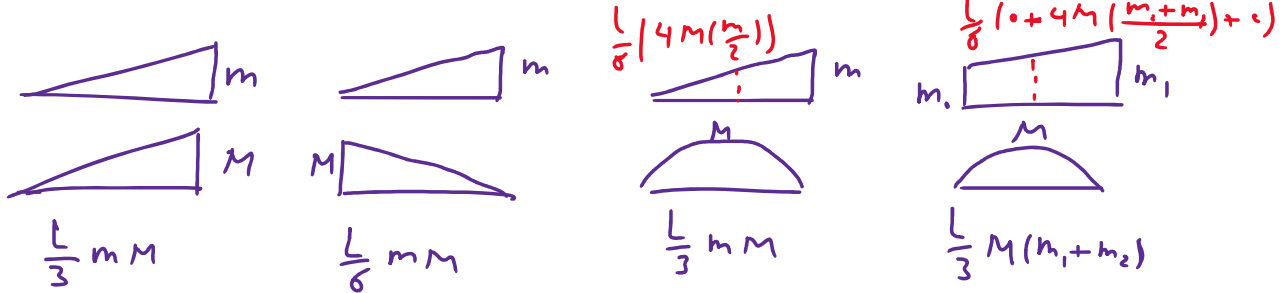
$$\int m M dx = \frac{L}{8} [m_1 M_1 + 4 m_1 M_1 + m_2 M_2]$$



$$\int m M dx = \frac{L}{8} [m_1 M_1 + 4 \left(\frac{m_1 + m_2}{2} \right) \left(\frac{M_1 + M_2}{2} \right) + m_2 M_2]$$



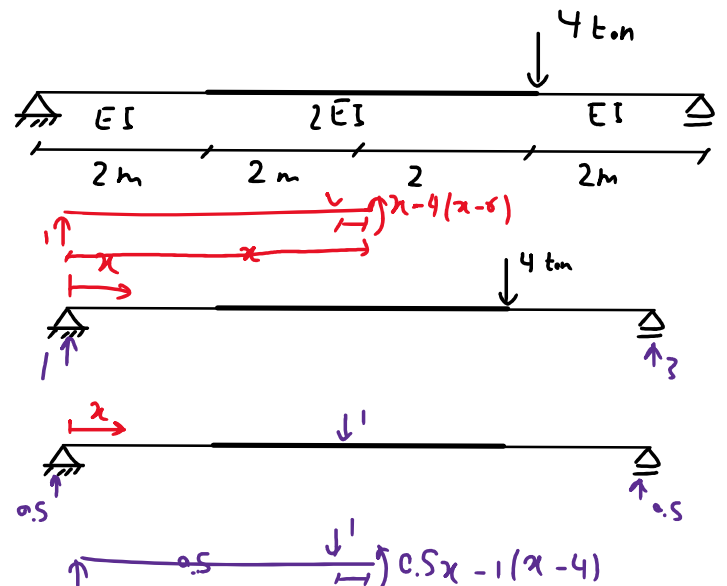
$$\int m M dx = \frac{L}{8} [2 m_1 M_1 + m_2 M_1 + m_1 M_2 + 2 m_2 M_2]$$



$$I \times \delta = \int \frac{m M}{EI} dx$$

$$I \times \delta = \frac{(4 \text{ ton})^2}{8 \times m^2} m$$

مثال: تغییر مکان قائم وسط دهانه تیر راب دست آدرید ($EI = 600 \text{ ton}^2$)



$$M = \begin{cases} x & x \leq 2 \\ -3x + 24 & 2 < x < 8 \end{cases}$$

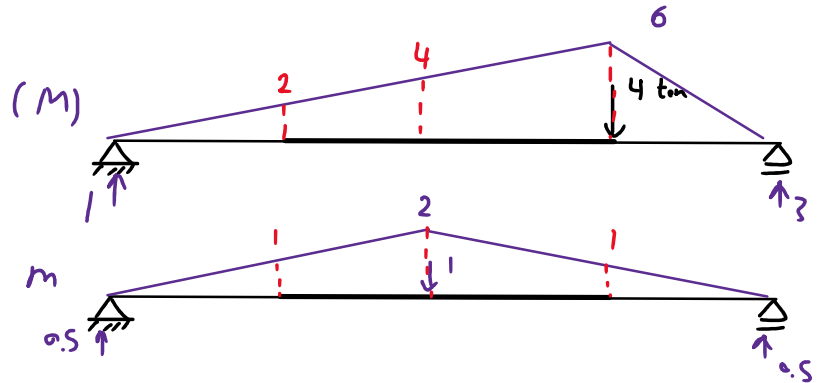
$$m = \begin{cases} \frac{1}{2} x & x \leq 4 \\ -\frac{1}{2} x + 4 & 4 < x \leq 8 \end{cases}$$

$$I \times \delta = \frac{1}{EI} \int_0^2 \left(\frac{x}{2} \right) (x) dx + \frac{1}{2EI} \int_2^4 \left(\frac{x}{2} \right) (x) dx + \frac{1}{2EI} \int_4^6 \left(-\frac{x}{2} + 4 \right) (x) dx + \frac{1}{EI} \int_6^8 \left(-\frac{x}{2} + 4 \right) (-3x + 24) dx$$

$$I \times \delta = \frac{1}{EI} \left[\frac{x^3}{6} \Big|_0^2 + \frac{x^3}{6} \Big|_2^4 + \frac{1}{2} \left(-\frac{x^3}{6} + 2x^2 \right) \Big|_4^6 + \left(\frac{x^3}{6} - 12x^2 + 96x \right) \Big|_6^8 \right]$$

$$1 \times \delta = \frac{1}{EI} \left[\frac{x^3}{6} \Big|_0^2 + \frac{1}{2} \frac{x^3}{6} \Big|_2^4 + \frac{1}{2} \left(-\frac{x^3}{6} + 2x^2 \right) \Big|_4^6 + \left(\frac{x^3}{2} - 12x^2 + 96x \right) \Big|_6^8 \right]$$

$$1 \times \delta = \frac{1}{EI} [1.33 + 4.67 + 7.33 + 4] = \frac{17.33}{EI} = \frac{17.33}{600} = 0.029 \text{ m} = 2.9 \text{ cm}$$



$$\frac{1}{2EI} \left(\frac{2}{6} \right) (4 \times 2 + 4 \times 5 \times \frac{3}{2} + 6 \times 1)$$

$$1 \times \delta = \int \frac{mM}{EI} dx = \frac{1}{EI} \left(\frac{2}{8} \right) (2)(1) + \frac{1}{2EI} \left(\frac{2}{6} \right) [2(2)(1) + 2(4)(2) + (2)(2) + (4)(1)]$$

$$+ \frac{1}{2EI} \left(\frac{2}{6} \right) [2(4)(2) + 2(6)(1) + (4)(1) + (6)(2)] + \frac{1}{EI} \left(\frac{2}{3} \right) (6)(1)$$

$$1 \times \delta = \frac{1}{EI} [1.33 + 4.67 + 7.33 + 4] = \frac{17.33}{EI} = 2.9 \text{ cm}$$

مثال: تغییر مکان امن رجیستری نقطه D را محاسبه کنید.

$$1 \times \delta_D = \int \frac{mM}{EI} dx$$

$$1 \times \delta_D = \frac{1}{EI} \left(\frac{4}{3} \right) (4) (100) +$$

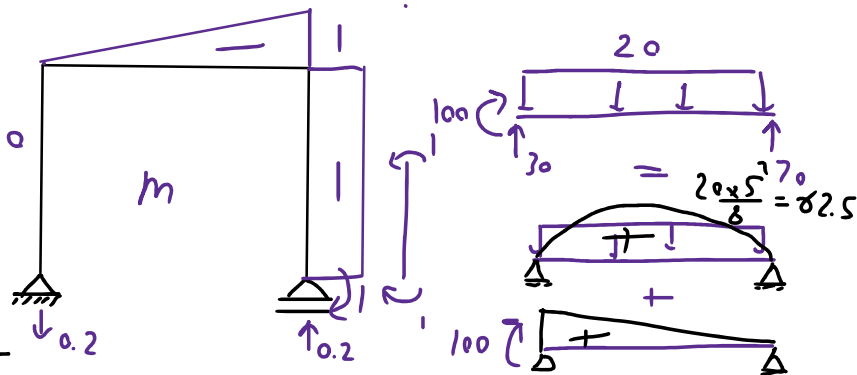
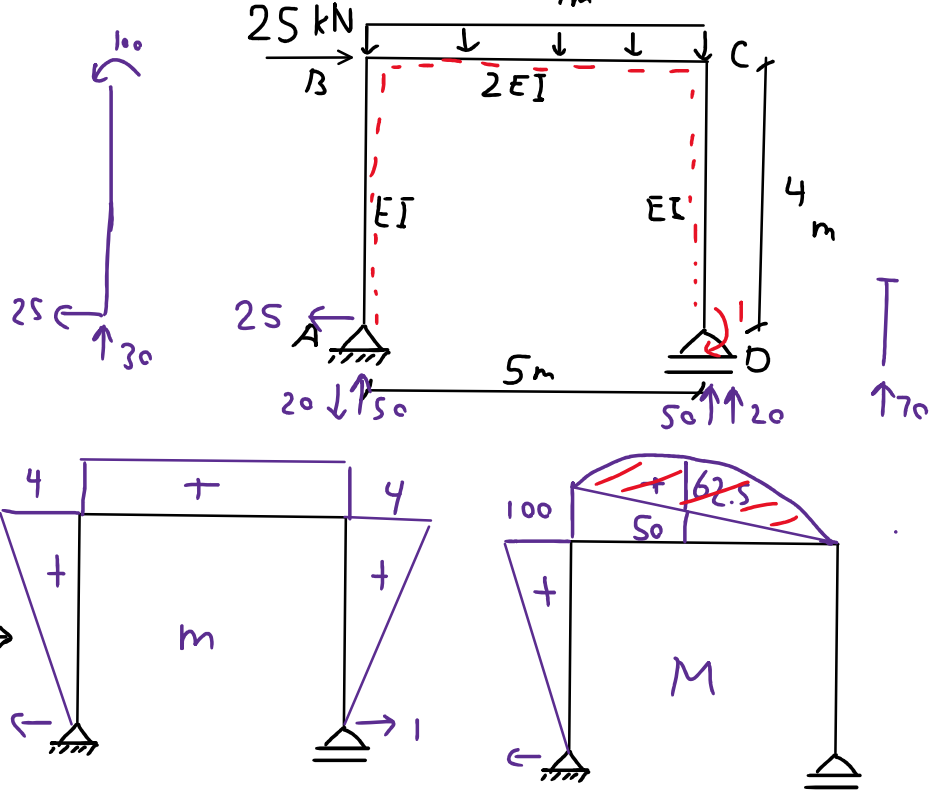
$$\frac{1}{2EI} \left(\frac{5}{6} \right) \left((4) (100) + 4(4) (112.5) + 0 \right) + 0 = \frac{1}{EI} (533.3 + 916.7) = \frac{1450}{EI} \rightarrow$$

$$\delta_D = \frac{1450}{EI} \rightarrow$$

$$1 \times \theta_D = \int \frac{mM}{EI} dx$$

$$1 \times \theta_D = 0 + \frac{1}{2EI} \left[0 + 4(-0.5) (112.5) + 0 \right] + 0$$

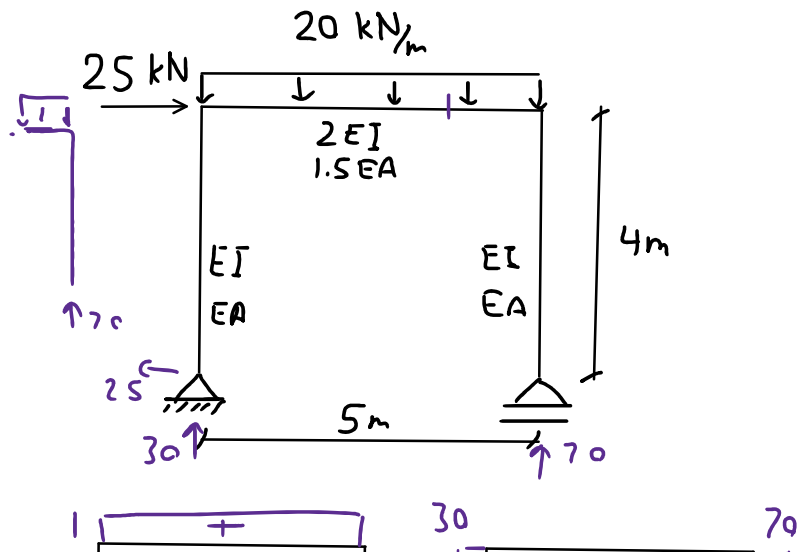
$$\theta_D = \frac{-93.75}{EI} \rightarrow$$



مثال: مثال فوق را با در نظر گرفتن تغییر شکل ماسه در حل کنید.

$$1 \times \delta_D = \int \frac{mM}{EI} dx + \sum \frac{n_i N_i}{EA} L_i$$

$$* \sum \frac{n_i N_i}{EA} L_i = 0$$



$$\frac{1}{EA} \int \frac{M}{EI} dx + \sum \frac{n_i N_i L_i}{EA_i}$$

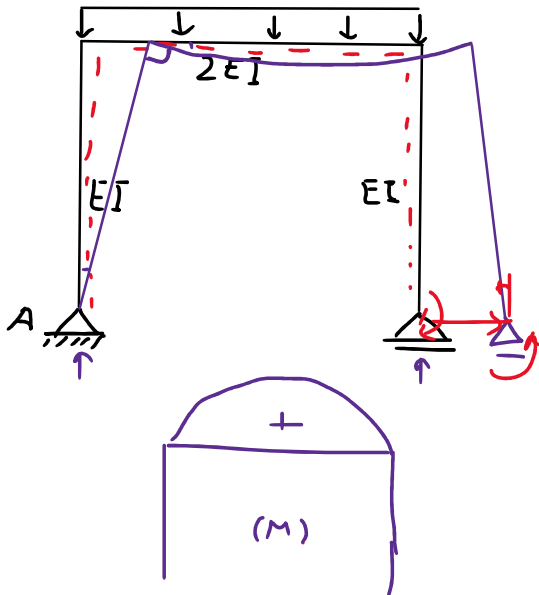
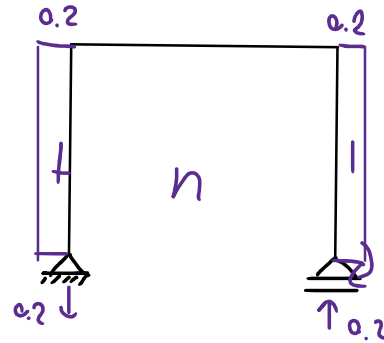
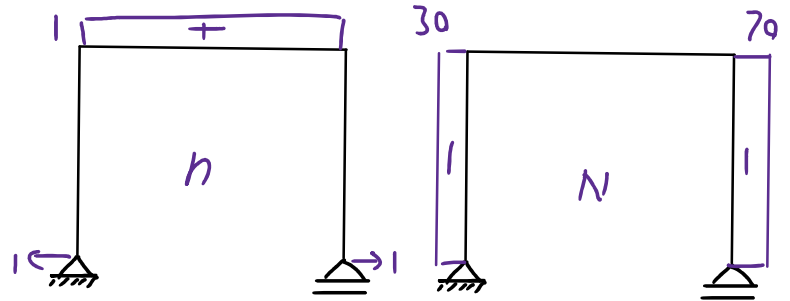
$$\delta_D = \frac{1450}{EI}$$

$$1 \times \theta_D = \int \frac{m M}{EI} dx + \sum \frac{n_i N_i L_i}{EA_i}$$

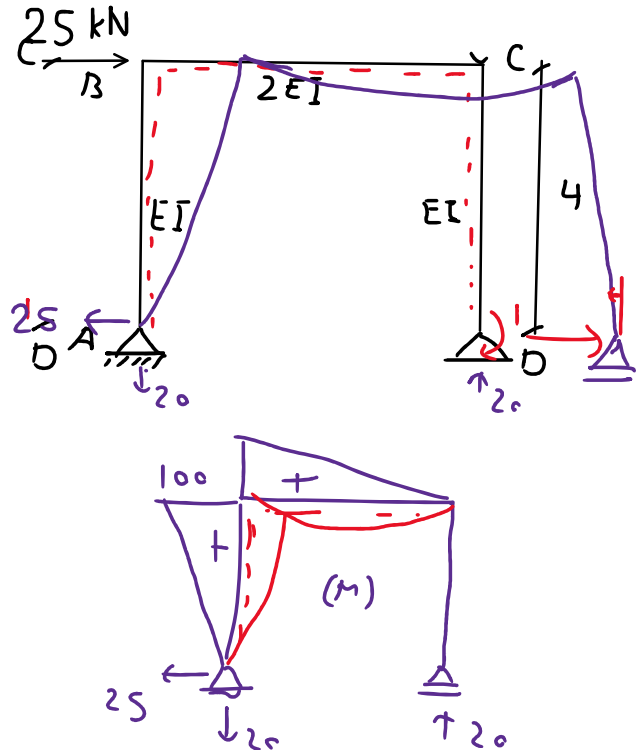
$$\sum \frac{n_i N_i}{EA_i} L_i = \frac{1}{EA} (0.2)(-30)(4) + 0$$

$$+ \frac{1}{EA} (-0.2)(-70)(4) = \frac{32}{EA}$$

$$\theta_D = -\frac{93.75}{EI} + \frac{32}{EA}$$

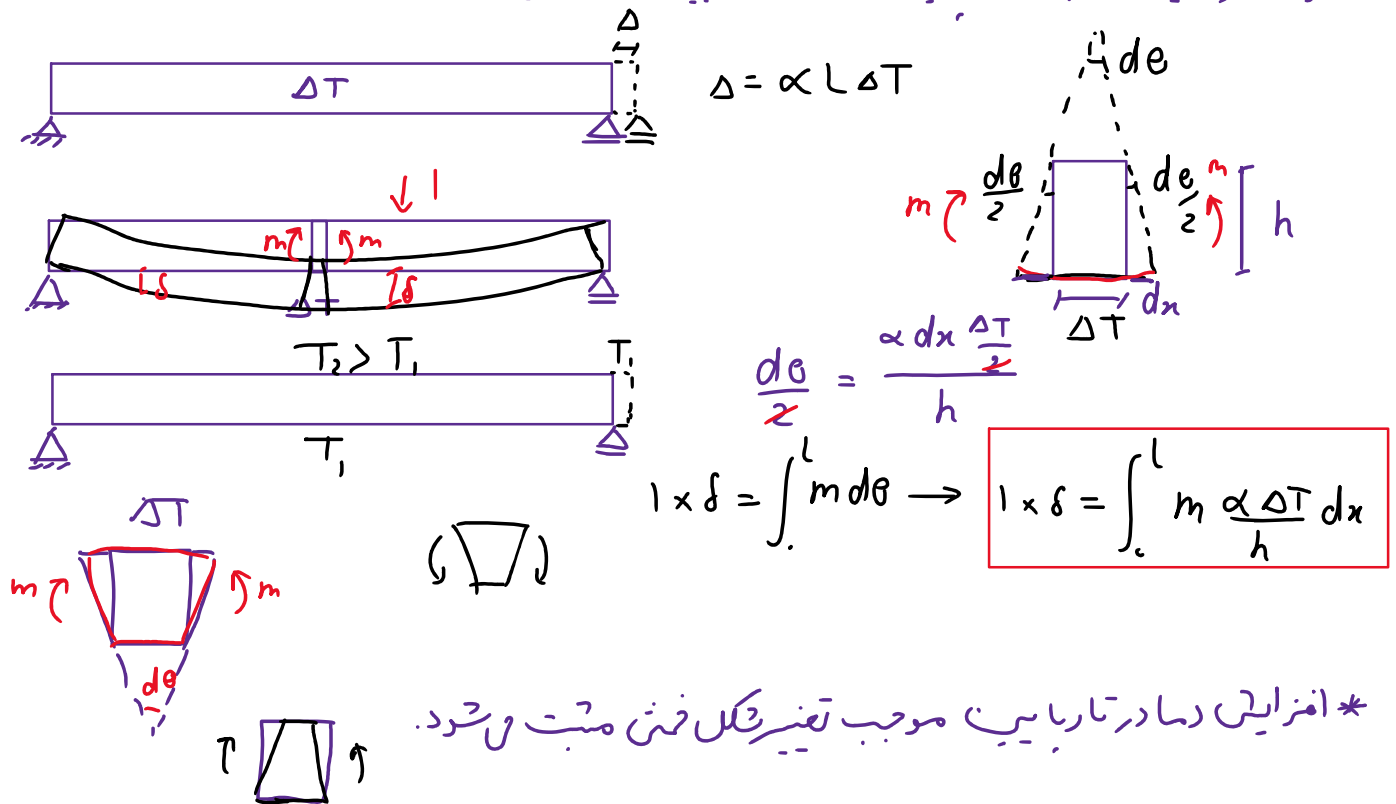


+

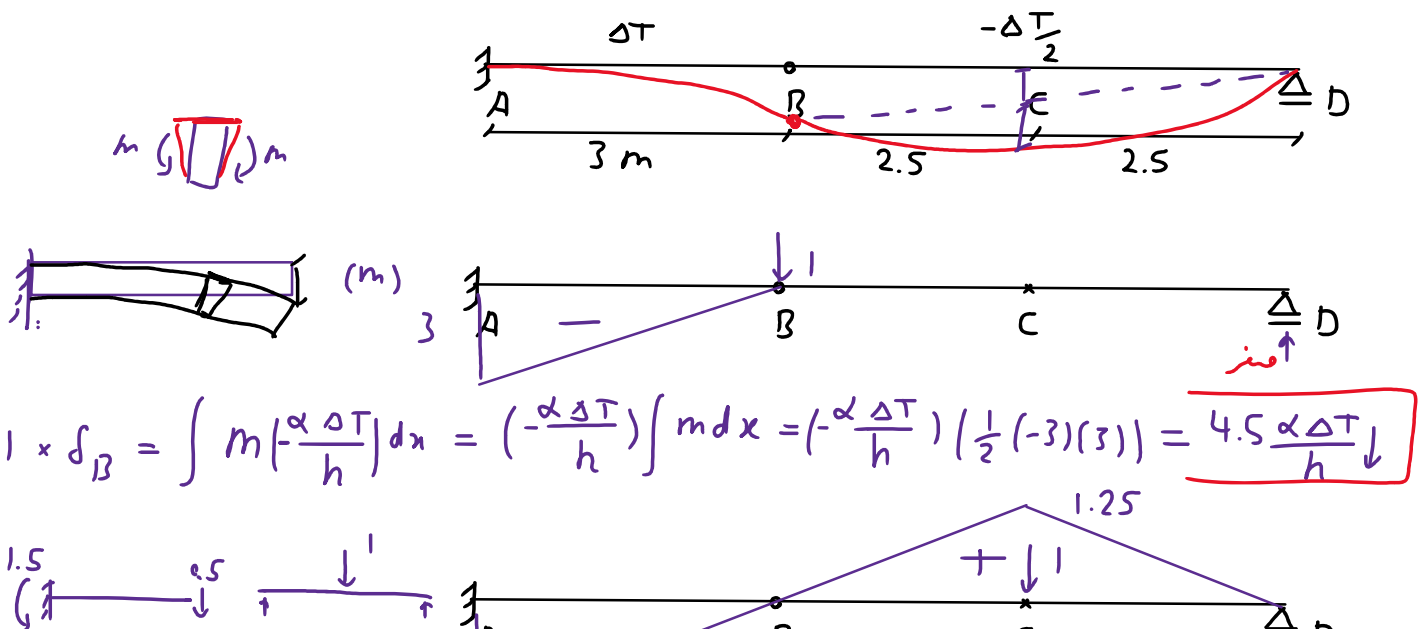


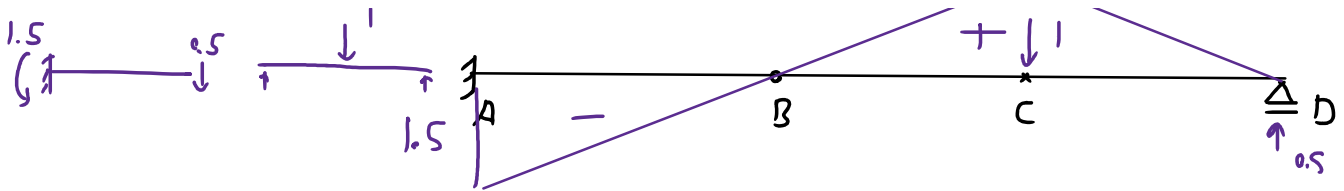
تغییر شکل خمشی تیرها ناشی از تغییرات دما

تغییر دما در مقطع تیر شکل زیر، نقطه موجب انبساط و انقباض محوری تیر می شود ولی تغییر دما تنها در یک طرف تیر (از بالا یا از پایین)، موجب تغییر شکل خمشی تیر می شود.



مثال: تغییر مکان قائم نقاط B و C در اثر تغییر دما در تار بالای تیر را به دست آورید. ($\Delta T > 0$)





$$I \times \delta_c = \int m \frac{\alpha \Delta T}{h} dx = \left(-\frac{\alpha \Delta T}{h} \right) \left(\frac{1}{2} (-1.5)(3) \right) + \left(\frac{\alpha \Delta T}{2h} \right) \left(\frac{1}{2} (1.25)(5) \right) = 3.81 \frac{\alpha \Delta T}{h} \downarrow$$

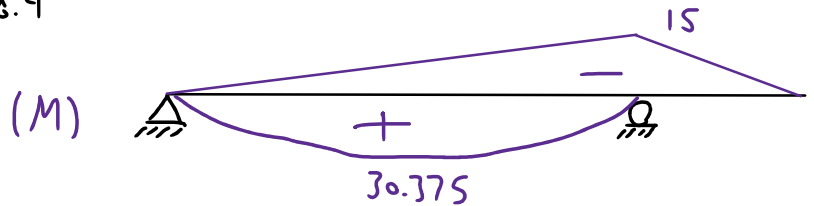
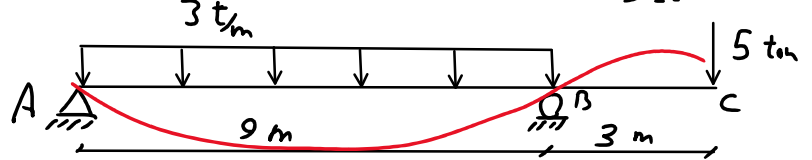
$$I \times \delta_c = \int \frac{m M}{EI} dx$$

$$I \times \delta_c = \frac{1}{EI} \left[\left(\frac{9}{3} \right) (-15) (-3) + \left(\frac{9}{3} \right) (-3) (30.375) + \right. \left. \right] 138.4$$

$$\left[\left(\frac{3}{3} \right) (-3) (-15) \right] = \frac{-93.375}{EI}$$

$$\delta_c = \frac{-93.375}{18000} = -5.2 \times 10^{-3} = 5.2 \text{ mm} \uparrow \quad (m)$$

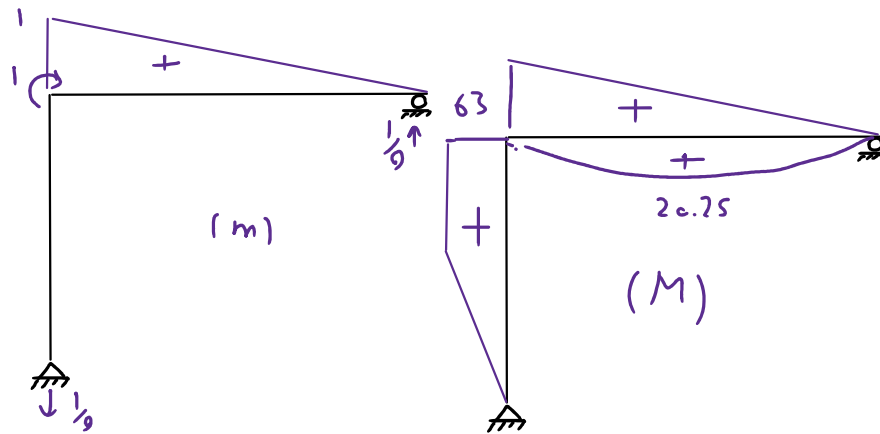
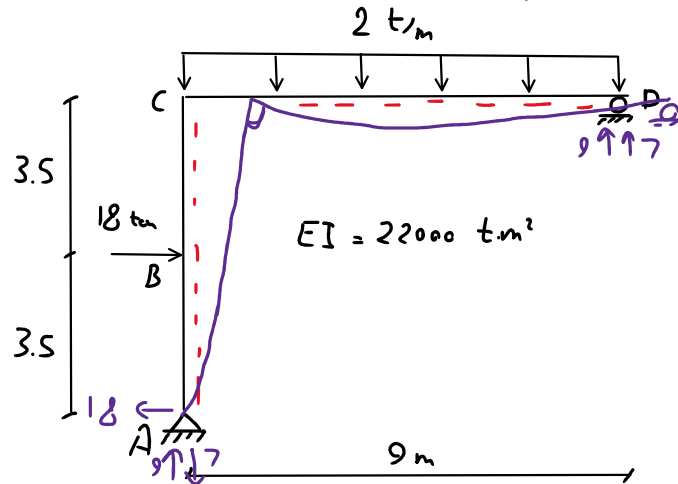
مثال: تغییر مکان تمام c را به دست آورید. ($EI = 18000 \text{ t.m}^2$)



$$\left(\frac{9}{6} \right) (0 + 4(-1.5)(22.875) + (3)(-15)) = -138.4$$

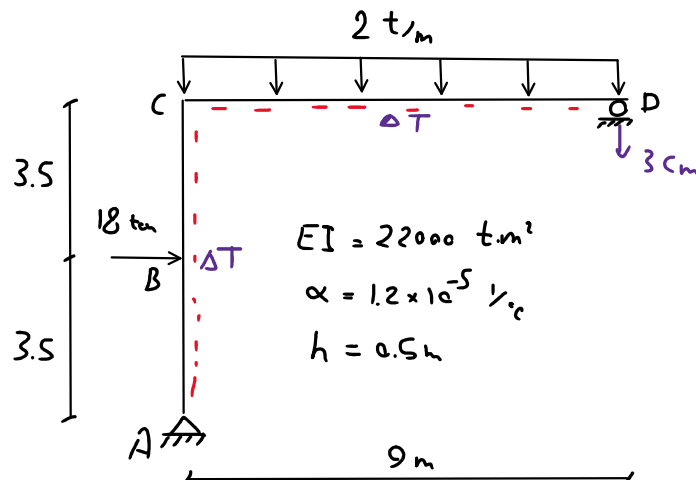
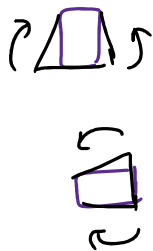
مثال: مطلوب است چرخش در C.

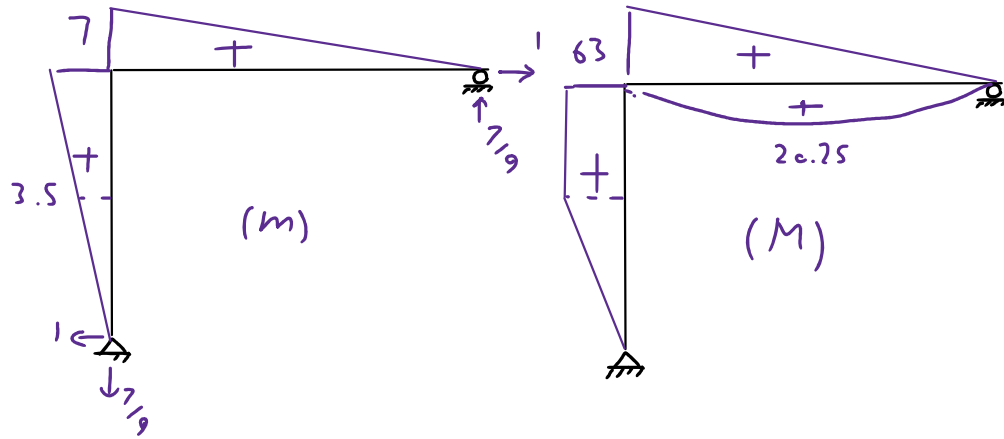
$$1 \times \theta_c = \int \frac{m/m}{EI} dx$$



$$1 \times \theta_c = \frac{1}{EI} \left[\left(\frac{9}{3} \right) (1) (63) + \left(\frac{9}{3} \right) (1) (20.25) \right] = \frac{249.75}{EI} = \frac{249.75}{22000} = 0.011 \text{ rad}$$

مثال: مطلوب است تغییر مکان افقی D ناشی از بارهای خارجی، انزایش دمای داخل ستاب به مقدار 30 و نسبت تلبگاه D به مقدار 3 cm.





$$1 \times \delta_D - \left(\frac{7}{9}\right)(0.03) = \int \frac{mM}{EI} dx + \int m \frac{\alpha \Delta T}{h} dx$$

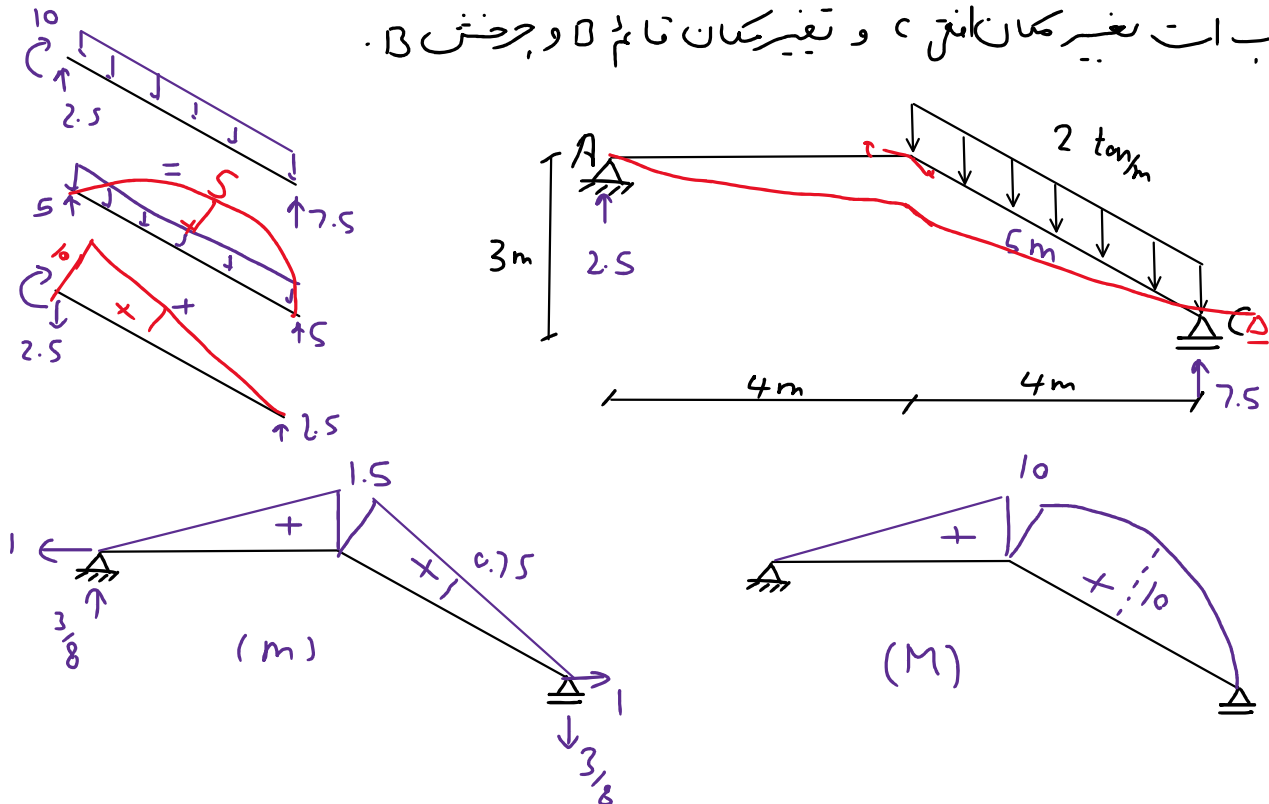
$$\int \frac{mM}{EI} dx = \frac{1}{EI} \left[\left(\frac{9}{3}\right)(7)(63) + \left(\frac{9}{3}\right)(7)(20.25) + \left(\frac{3.5}{3}\right)(3.5)(63) + 63\left(\frac{7+3.5}{2}\right)(3.5) \right]$$

$$\frac{1}{EI} \left(\frac{1748.25}{EI} + \frac{1414.87}{EI} \right) = \frac{3163}{EI} = \frac{3163}{22000} = 0.14 \text{ m} = 14 \text{ cm} \rightarrow$$

$$\int m \frac{\alpha \Delta T}{h} dx = \frac{\alpha \Delta T}{h} \left[\frac{7 \times 9}{2} + \frac{7 \times 7}{2} \right] = 56 \frac{\alpha \Delta T}{h} = 56 \times \frac{1.2 \times 10^{-5} \times 30}{0.5} = 0.04 \text{ m} = 4 \text{ cm} \rightarrow$$

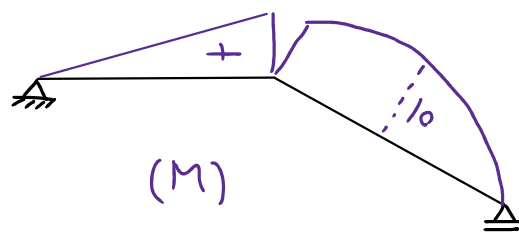
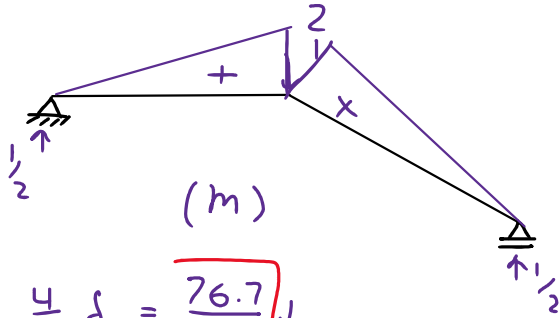
$$1 \times \delta_D - 0.0233 = 0.14 + 0.04 \rightarrow \delta_D = 20.33 \text{ cm} \rightarrow$$

مثال: مطلوب است تغییر مکان انتی c و تغییر مکان تانگ B و عرضش B.

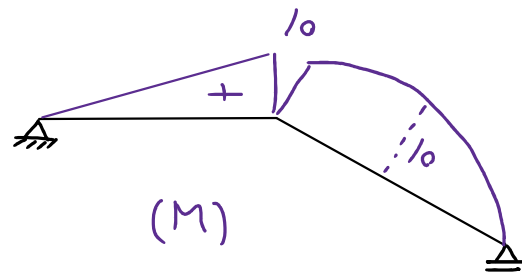
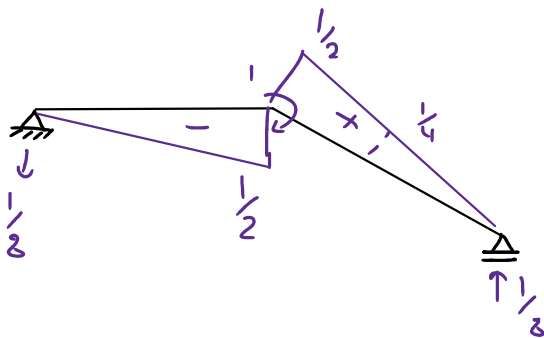


$\sim 3/8$

$$1 \times \delta_c = \int \frac{m M}{EI} dx = \frac{1}{EI} \left[\left(\frac{4}{3} \right) (1.5) (10) + \left(\frac{5}{6} \right) (1.5) (10) + 4 (0.75) (10) + 0 \right] = \frac{57.5}{EI}$$

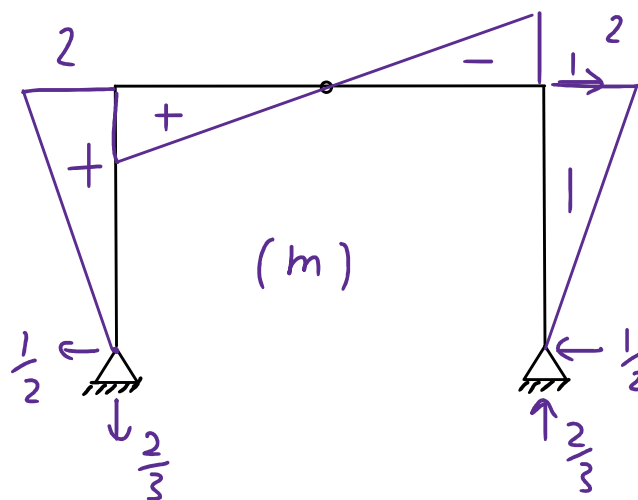
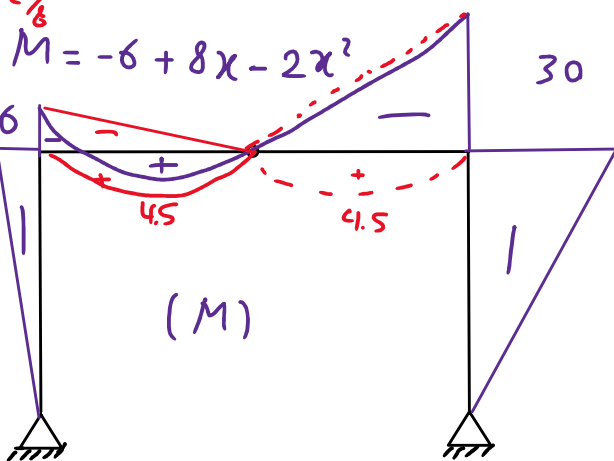
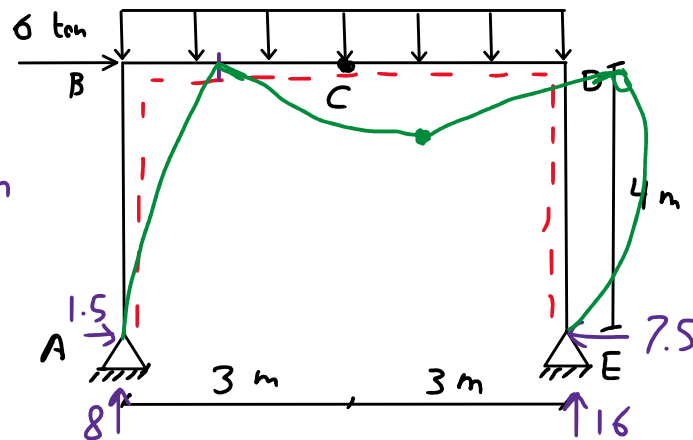
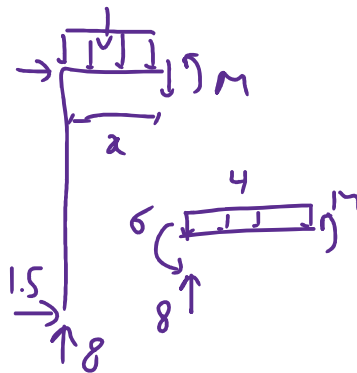
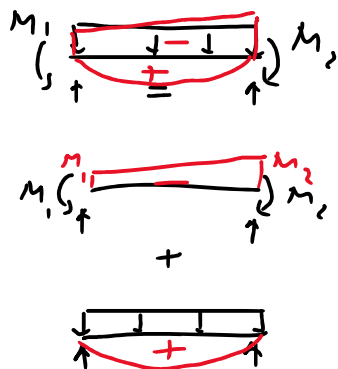


$$\delta_B = \frac{4}{3} \delta_c = \frac{76.7}{EI} \downarrow$$



$$1 \times \theta_B = \frac{1}{EI} \left[\left(\frac{4}{3} \right) \left(-\frac{1}{2} \right) (10) + \left(\frac{5}{6} \right) \left(\frac{1}{2} \right) (10) + 4 \left(\frac{1}{4} \right) (10) + 0 \right] = \frac{5.83}{EI}$$

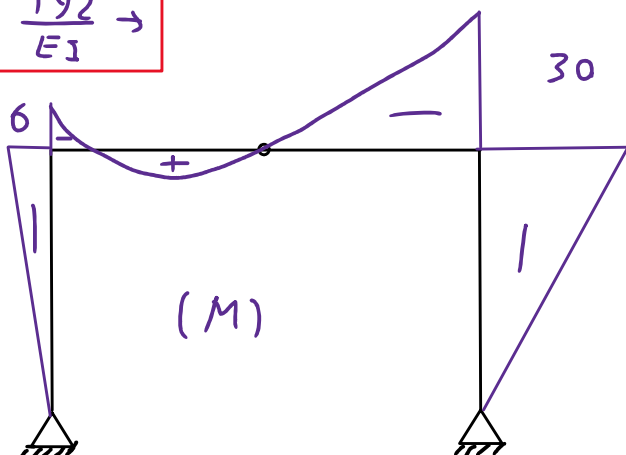
مثال: تغییر مکان افقی و جزیئی D، جزیئی ست چپ فصل C، $\Delta \theta_c$



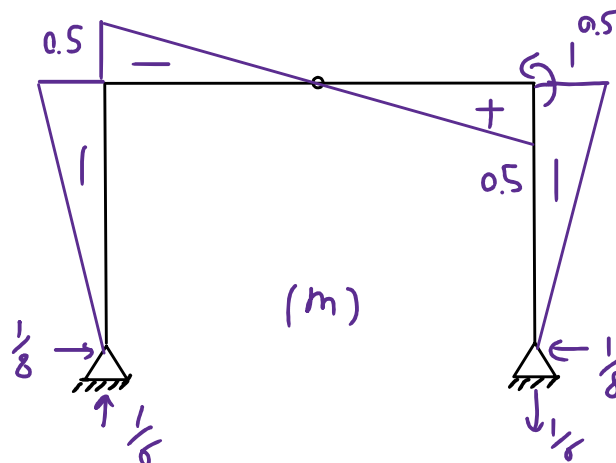
$$1 \times \delta_D = \int \frac{m M}{EI} dx$$

$$EI \delta_D = \left(\frac{4}{3}\right)(2)(-6) + \left(\frac{4}{3}\right)(-2)(-30) + \left(\frac{6}{8}\right)[(2)(-6) + 0 + (-2)(-30)] = 192$$

$$\delta_D = \frac{192}{EI} \rightarrow$$

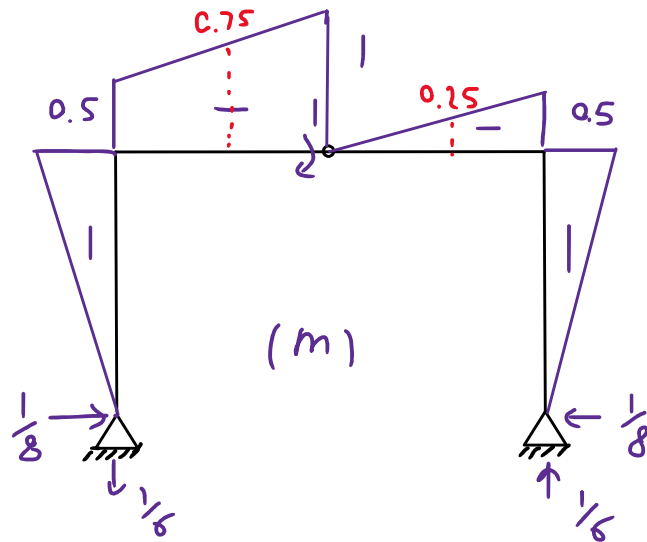
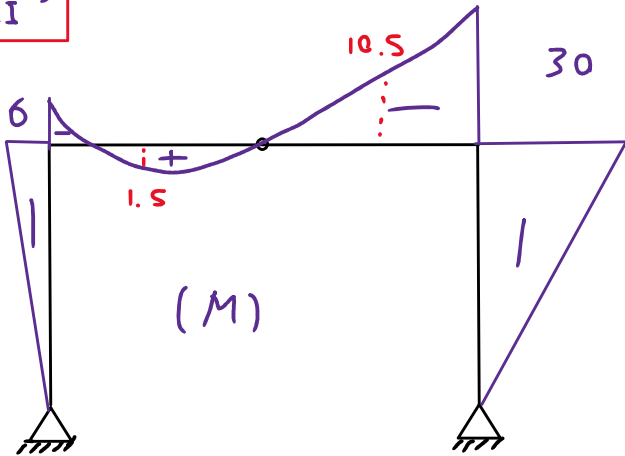


$$1 \times \theta_D = \int \frac{m M}{EI} dx$$



$$EI \theta_0 = \left(\frac{4}{3}\right)(-0.5)(-6) + \left(\frac{4}{3}\right)(-0.5)(-30) + \left(\frac{6}{8}\right) [(-0.5)(-6) + 0 + (0.5)(-30)] = 12$$

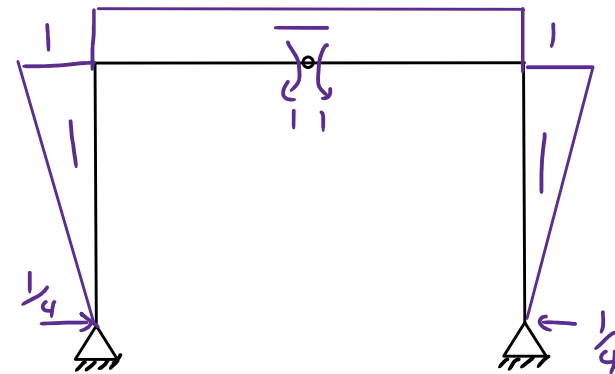
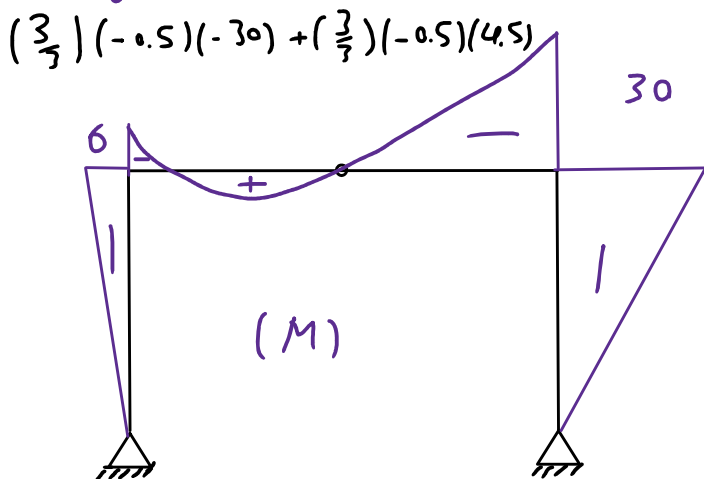
$$\theta_0 = \frac{12}{EI} \curvearrowright$$



$$1 \times \theta_{cl} = \int \frac{m M}{EI} dx$$

$$EI \theta_{cl} = \left(\frac{4}{3}\right)(-0.5)(-6) + \left(\frac{4}{3}\right)(-0.5)(-30) + \left(\frac{3}{8}\right) [(-0.5)(-6) + 4(-0.75)(1.5) + 0] + \left(\frac{3}{8}\right) [0 + 4(-0.25)(-10.5) + (-0.5)(-30)] = 36$$

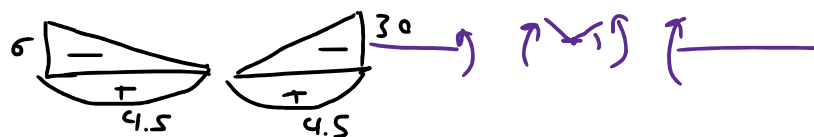
$$\theta_{cl} = \frac{36}{EI} \curvearrowright$$



$$1 \times \Delta \theta_c = \int \frac{m M}{EI} dx$$

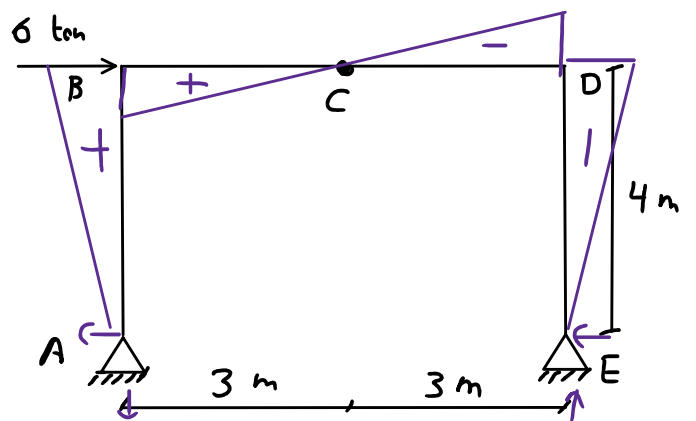
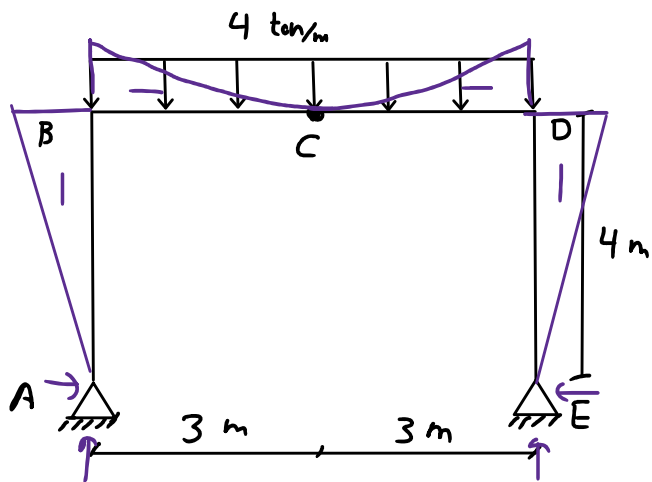
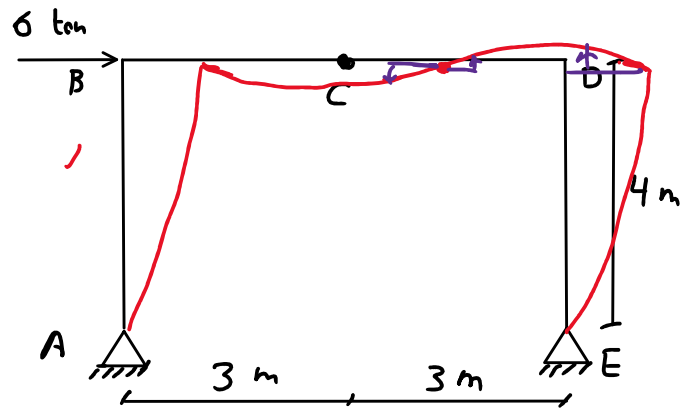
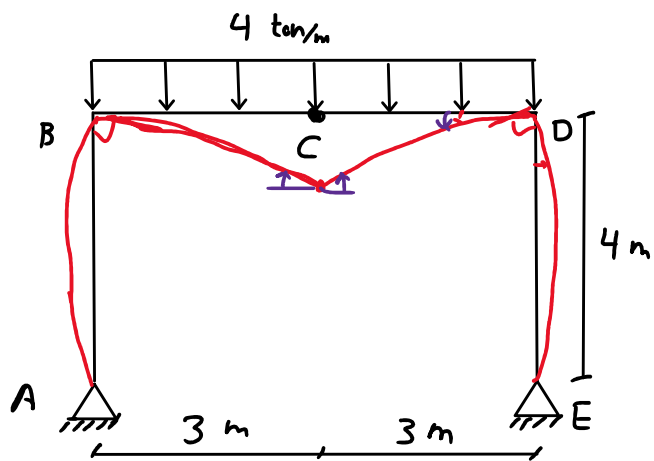
$$EI \Delta \theta_c = \left(\frac{4}{3}\right)(-1)(-6) + \left(\frac{4}{3}\right)(-1)(-30) + \left(\frac{6}{8}\right) [(-1)(-6) + 0 + (-1)(-30)] = 84$$

$$\Delta \theta_c = \frac{84}{EI} \curvearrowright$$



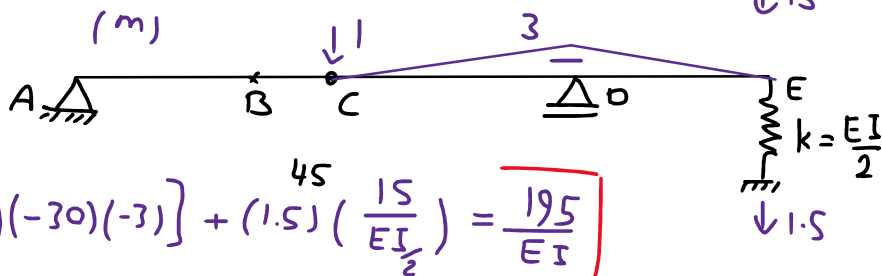
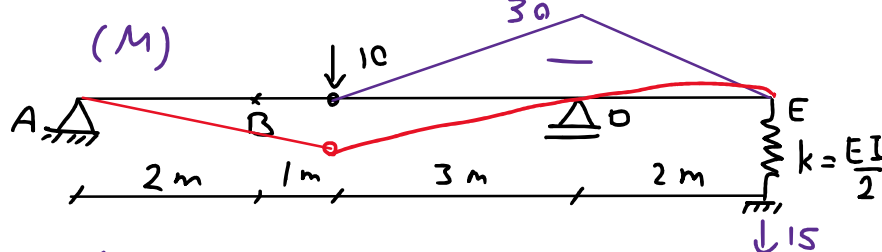
$$\frac{1}{2}(-6)(3) + \frac{1}{2}(-30)(3) + 2 \times \frac{1}{2} \times 4.5 \times 3 = 36$$

$$-9 - 45 + 18$$



$$1 \times \delta = \int \frac{mM}{EI} dx + \sum f \frac{F}{k} + \sum m \frac{M}{k_{\theta}}$$

مثال: تغییر مکان قائم 13 را بدست آورید.



$$1 \times \delta_c = \int \frac{mM}{EI} dx + f \frac{F}{k}$$

$$1 \times \delta_c = \frac{1}{EI} \left[\left(\frac{3}{3} \right) (-30) (-3) + \left(\frac{2}{3} \right) (-30) (-3) \right] + (1.5) \left(\frac{15}{EI/2} \right) = \frac{195}{EI}$$

بار واحد

$$1 \times \delta_B = \int \frac{mM}{EI} dx = 0 + \left(\frac{L}{6} \right) \left(\frac{L}{4} \right) \left(\frac{PL}{4EI} \right) = \frac{PL^3}{96EI}$$

$$\delta_{C/A} = \frac{1}{2} \left(\frac{PL}{4EI} \right) \left(\frac{L}{2} \right) \left(\frac{L}{3} \right) = \frac{PL^3}{48EI}$$

$$\theta_A = \frac{\delta_{C/A}}{L} = \frac{PL^2}{48EI}$$

$$\delta_B = \theta_A \left(\frac{L}{2} \right) = \frac{PL^3}{96EI}$$

تغییر درجه

$$\sum M_c = 0 \rightarrow R_A L - \frac{1}{2} \left(\frac{PL}{4EI} \right) \left(\frac{L}{2} \right) \left(\frac{L}{3} \right) = 0$$

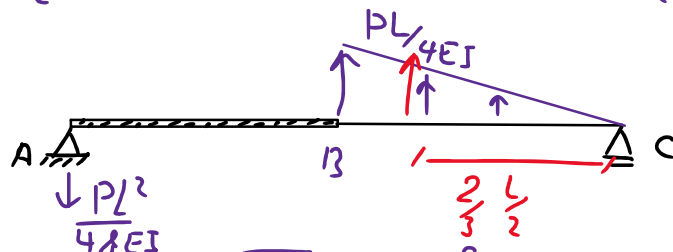
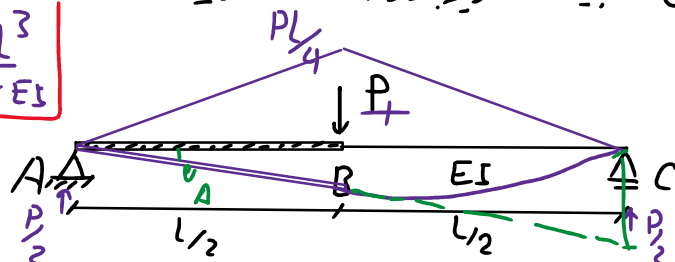
$$M_B = \left(\frac{PL^2}{48EI} \right) \left(\frac{L}{2} \right) = \frac{PL^3}{96EI}$$

$$1 \times \delta_{BL} = \int \frac{mM}{EI} dx + f \left(\frac{F}{k} \right)$$

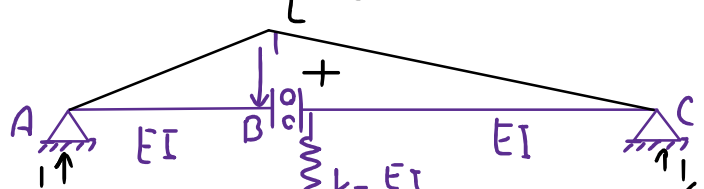
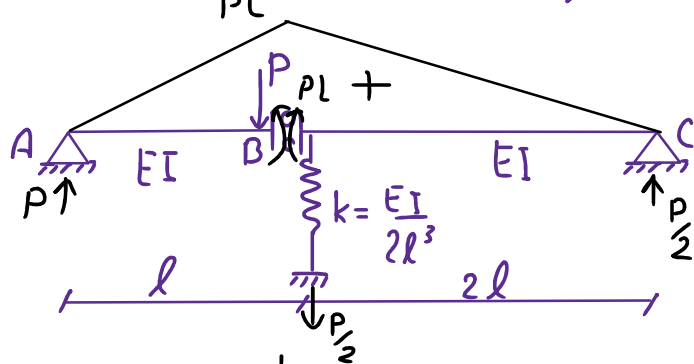
$$1 \times \delta_{BL} = \frac{1}{EI} \left[\left(\frac{3L}{3} \right) (L) (PL) \right] + \left(\frac{1}{2} \right) \left(\frac{P/2}{EI/2L^3} \right) =$$

$$\delta_{BL} = \frac{3}{2} \frac{PL^3}{EI}$$

مثال: تغییر مکان زیر بار را بدست آورید.

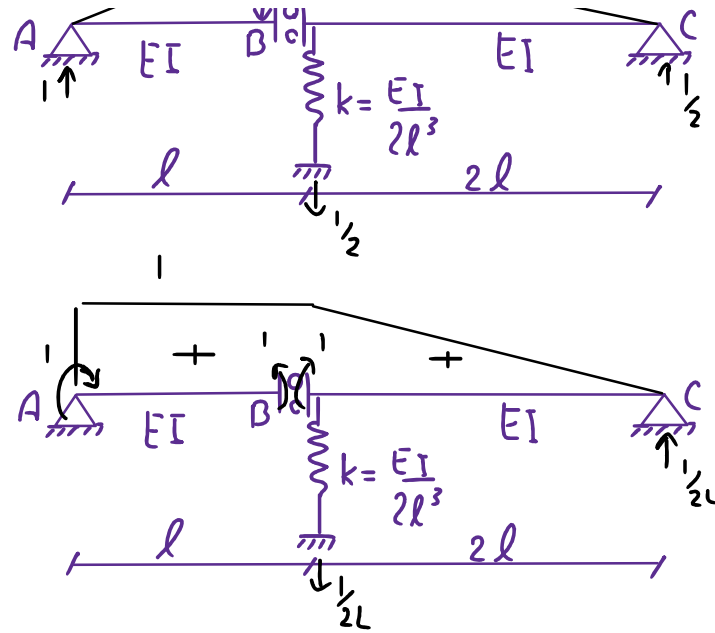


مثال: θ_A و δ_{BL} را بدست آورید.



$$1 \times \theta_A = \int \frac{m M}{EI} dx + f\left(\frac{F}{k}\right)$$

$$1 \times \theta_A = \frac{1}{EI} \left[(1) \frac{1}{2} (PL)(L) + \left(\frac{2L}{3}\right) (1) (PL) \right] + \left(\frac{1}{2L}\right) \left(\frac{P}{EI}\right) = \frac{5}{3} \frac{PL^2}{EI}$$



بار واحد

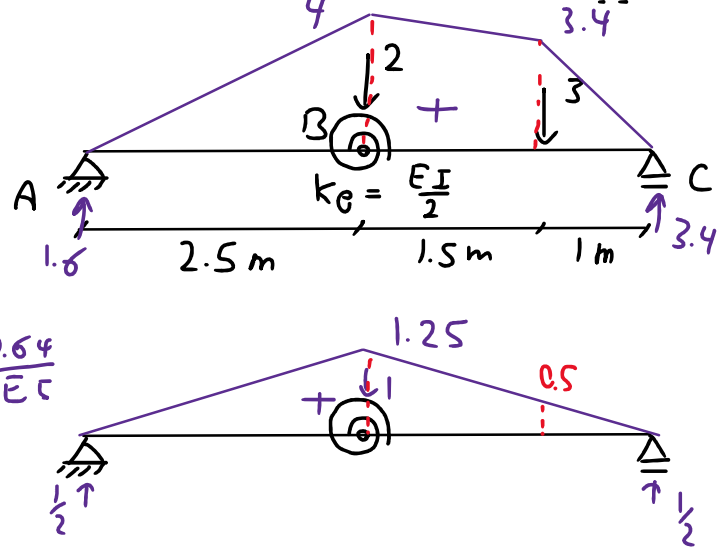
$$1 \times \delta_B = \int \frac{m M}{EI} dx + m \left(\frac{M}{k_\theta}\right)$$

$$\delta_B = \frac{1}{EI} \left[\left(\frac{2.5}{3}\right) (1.25)(4) + \right.$$

$$\left. \left(\frac{1.5}{6}\right) \left((1.25)(4) + 4 \left(\frac{1.75}{2}\right) (3.7) + (0.5)(3.4) \right) + \left(\frac{1}{3}\right) (0.5)(3.4) \right] + (1.25) \left(\frac{4}{EI}\right) = \frac{19.64}{EI}$$

$$\delta_B = \frac{19.64}{EI}$$

مثال: تغییر مکان قائم B را بدست آورید.



تشریح

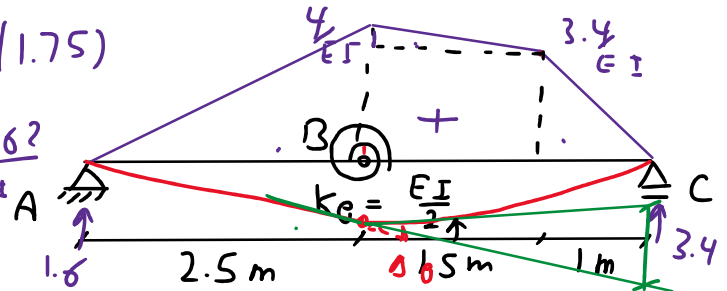
$$\delta_{C/A} = \left(\frac{1}{2}\right) \left(\frac{4}{EI}\right) (2.5)(3.333) + \left(\frac{3.4}{EI}\right) (1.5)(1.75)$$

$$+ \frac{1}{2} \left(\frac{0.6}{EI}\right) (1.5)(2) + \frac{1}{2} \left(\frac{3.4}{EI}\right) (1) \left(\frac{2}{3}\right) = \frac{27.62}{EI}$$

$$\theta_A = \frac{\delta_{C/A}}{5} = \frac{5.52}{EI}$$

$$\delta_B = -2.5 \theta_A + \delta_{B/A} =$$

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$k_\theta = \infty$ θ

$$\delta_B = -2.5 \theta_A + \delta_{B/A} =$$

$$-2.5 \left(\frac{5.52}{EI} \right) + \frac{1}{2} \left(\frac{4}{EI} \right) (2.5) \left(\frac{2.5}{3} \right) = \frac{9.63}{EI}$$

$$\Delta \theta = \frac{M}{k_\theta} = \frac{4}{\frac{EI}{2}} = \frac{8}{EI}$$

$$\delta_B = 2.5 \theta = 2.5 \left(\frac{4}{EI} \right) = \frac{10}{EI}$$

تیر مندرج

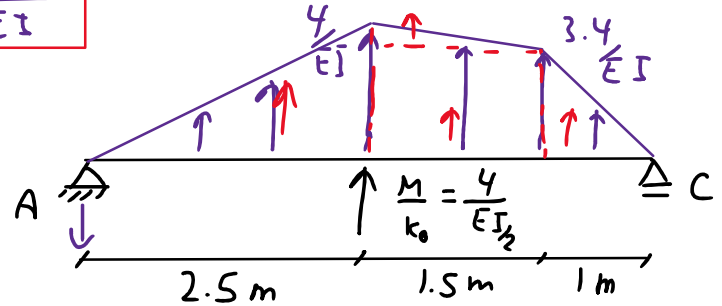
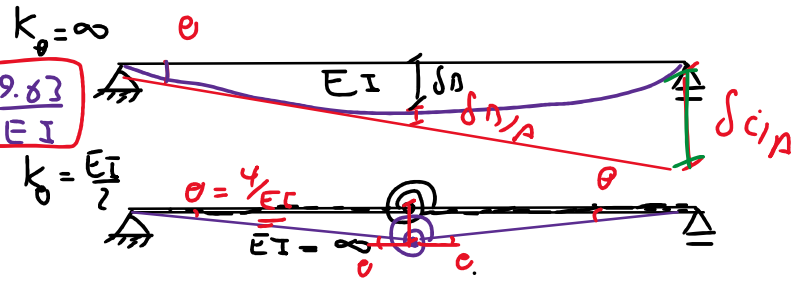
$$\delta_B = \frac{19.63}{EI}$$

$$\sum M_c = 0$$

$$\left(\frac{1}{2} \right) \left(\frac{4}{EI} \right) (2.5) (3.333) + \left(\frac{3.4}{EI} \right) (1.5) (1.75)$$

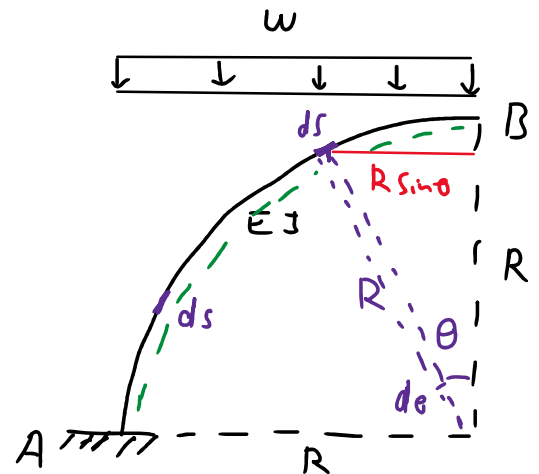
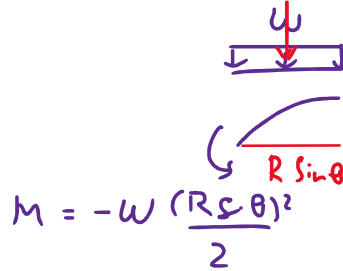
$$+ \frac{1}{2} \left(\frac{0.6}{EI} \right) (1.5) (2) + \frac{1}{2} \left(\frac{3.4}{EI} \right) (1) \left(\frac{2}{3} \right) + \left(\frac{8}{EI} \right) (2.5) = R_A (5) \rightarrow R_A = \frac{9.52}{EI}$$

$$\delta_B = M_p = -\frac{9.52}{EI} (2.5) + \frac{1}{2} \left(\frac{4}{EI} \right) (2.5) \left(\frac{2.5}{3} \right) = \frac{19.64}{EI}$$



مثال: مطلوب است تغییر مکان قائم و افق نقطه B.

$$1 \times \delta_{By} = \int \frac{mM}{EI} ds$$



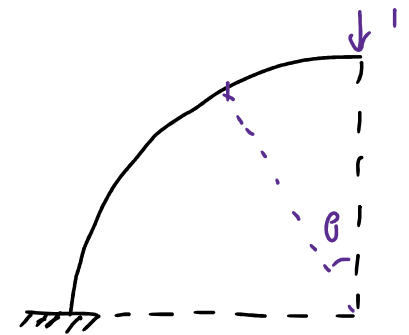
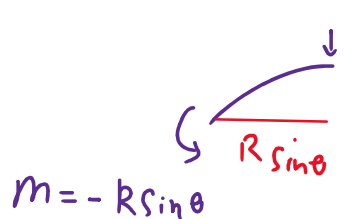
$$1 \times \delta_{By} = \frac{1}{EI} \int_0^{\pi/2} (-R \sin \theta) \left(-\frac{w}{2} R^2 \sin^2 \theta \right) R d\theta$$

$$\delta_{By} = \frac{wR^4}{2EI} \int_0^{\pi/2} \sin \theta (1 - \cos^2 \theta) d\theta$$

$$= \frac{wR^4}{2EI} \left[-\cos \theta + \frac{\cos^3 \theta}{3} \right]_0^{\pi/2}$$

$$= \frac{wR^4}{2EI} \left[0 - \left(-1 + \frac{1}{3} \right) \right]$$

$$\delta_{By} = \frac{wR^4}{3EI}$$



$$1 \times \delta_{Bx} = \int \frac{mM}{EI} ds = \frac{1}{EI} \int_0^{\pi/2} (-R(1 - \cos \theta)) \left(-\frac{w}{2} R^2 \sin^2 \theta \right) R d\theta$$

$$\delta_{Bx} = \frac{wR^4}{2EI} \int_0^{\pi/2} (\sin^2 \theta - \cos \theta \sin^2 \theta) d\theta$$

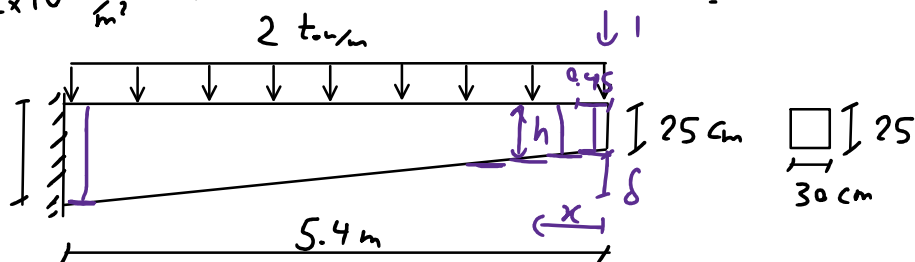
$$\delta_{Bx} = \frac{wR^4}{2EI} \int_0^{\pi/2} \left(\frac{1 - \cos 2\theta}{2} - \cos \theta \sin^2 \theta \right) d\theta$$

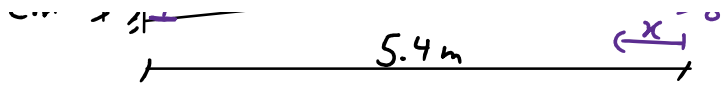
$$\frac{wR^4}{2EI} \left[\frac{\theta}{2} - \frac{1}{4} \sin 2\theta - \frac{\sin^3 \theta}{3} \right]_0^{\pi/2} = \frac{wR^4}{2EI} \left[\left(\frac{\pi}{4} - 0 - \frac{1}{3} \right) - 0 \right]$$

$$\delta_{Bx} = 0.226 \frac{wR^4}{EI}$$

مثال: تغییر مکان سه آزاد تیر را بدست آورید.
 $E = 2 \times 10^6 \frac{\text{t}}{\text{m}^2}$

$$h = 0.25 + \frac{0.05}{5.4} x = 0.25 + \frac{1}{12} x$$





$$I \times \delta = \int \frac{m m}{EI} dx = \int_{-0.25}^{5.4} \frac{(-x)(-x^2)}{E \times \frac{1}{12} \times 0.3 \times \left(\frac{1}{4} + \frac{1}{12}x\right)^3} dx$$

$$I \times \delta = \int_{0.25}^{0.7} \frac{12^3 (u - \frac{1}{4})^3}{2 \times 10^6 \times \frac{0.3}{12} u^3} 12 du =$$

$$\begin{cases} u = \frac{1}{4} + \frac{1}{12}x \\ * x = 12(u - \frac{1}{4}) \\ * dx = 12 du \end{cases}$$

$$\delta = \frac{12^5}{2 \times 10^6 \times 0.3} \int_{0.25}^{0.7} \frac{(u - \frac{1}{4})^3}{u^3} du = 0.41472 \int_{0.25}^{0.7} \frac{u^3 - \frac{3}{4}u^2 + \frac{3}{16}u - \frac{1}{64}}{u^3} du$$

$$\delta = 0.41472 \int_{0.25}^{0.7} \left(1 - \frac{3}{4} \frac{1}{u} + \frac{3}{16} \frac{1}{u^2} - \frac{1}{64} \frac{1}{u^3}\right) du$$

$$\frac{1}{n+1} u^{n+1}$$

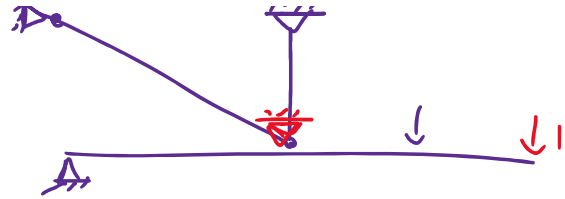
$$\delta = 0.41472 \left(u - \frac{3}{4} \ln u + \frac{3}{16} \frac{-1}{u} - \frac{1}{64} \frac{-\frac{1}{2}}{u^2} \right) \Big|_{0.25}^{0.7}$$

$$\delta = 0.41472 \left[(0.7 - 0.25) - \frac{3}{4} (\ln 0.7 - \ln 0.25) - \frac{3}{16} \left(\frac{1}{0.7} - \frac{1}{0.25} \right) + \frac{1}{128} \left(\frac{1}{0.7^2} - \frac{1}{0.25^2} \right) \right]$$

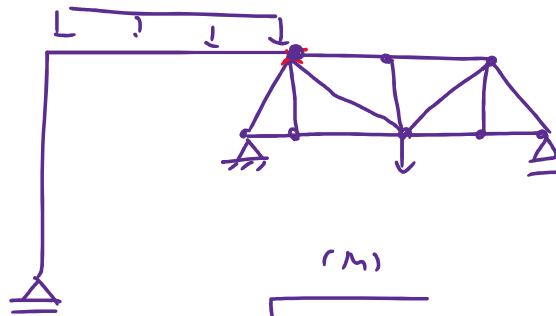
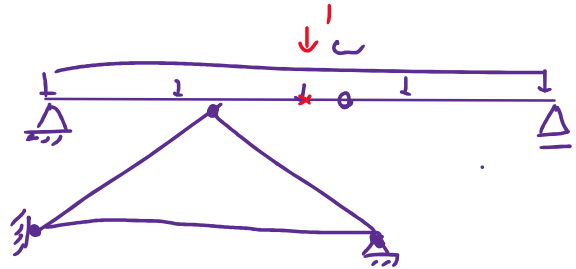
$$\delta = 0.021 \text{ m} = 2.1 \text{ cm}$$

x	h		
0.225	0.26875	0.58682	0.26407
0.675	0.30625	10.7074	4.81832
1.125	0.34375	35.0533	15.774
1.575	0.38125	70.5037	31.7267
2.025	0.41875	113.086	50.8889
2.475	0.45625	159.631	71.8339
2.925	0.49375	207.901	93.5554
3.375	0.53125	256.404	115.382
3.825	0.56875	304.18	136.881
4.275	0.60625	350.633	157.785
4.725	0.64375	395.416	177.937
5.175	0.68125	438.34	197.253
			1054.1
			0.02108

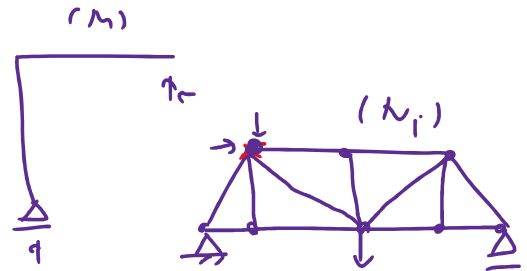




$$1 \times \delta = \int \frac{mM}{EI} dx + \sum \frac{n_i N_i l_i}{EA_i}$$



$$1 \times \delta = \int \frac{mM}{EI} dx + \sum \frac{n_i N_i l_i}{EA_i}$$



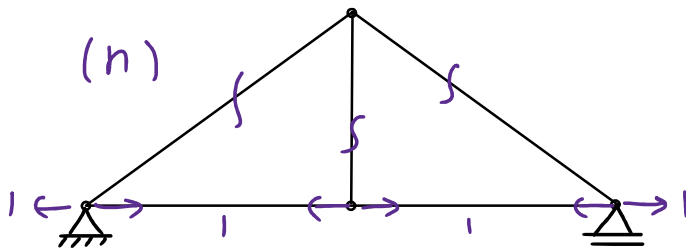
در اکثر سازه ها در مماندهی از جمله خرپاها، نیرو در محورها و EA در طول عضو ثابت هستند. بنابراین کار داخلی

ناشی از بار واحد را می توان به صورت زیر نوشت:

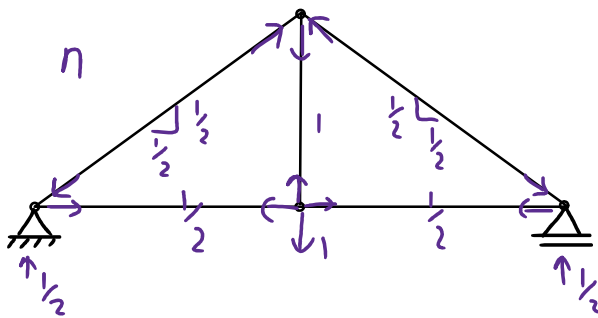
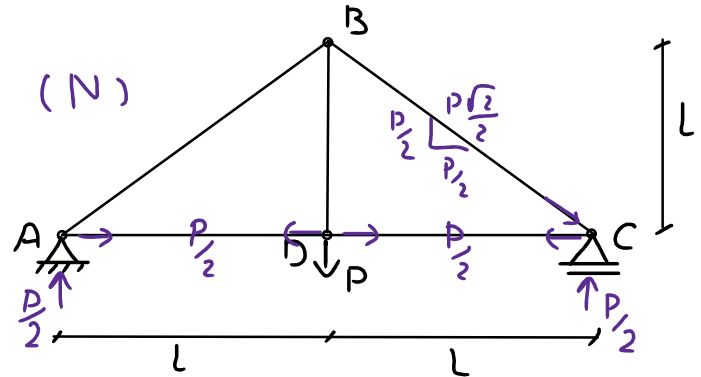
$$1 \times \delta = \sum \frac{n_i N_i L_i}{E_i A_i}$$

$$\int \frac{n N}{EA} dx$$

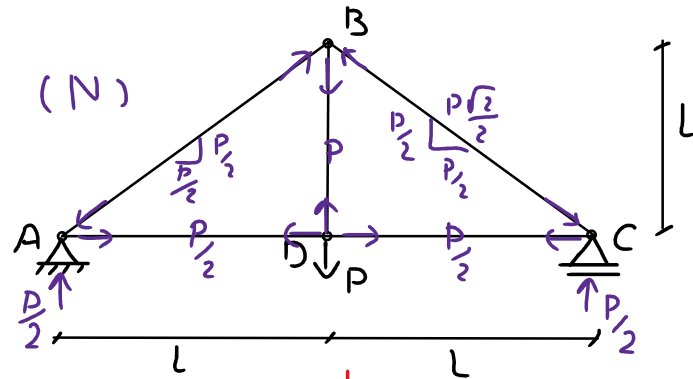
مثال: تغییر مکان افقی گره C را تغییر مکان ممان B و D را به دست آورید.



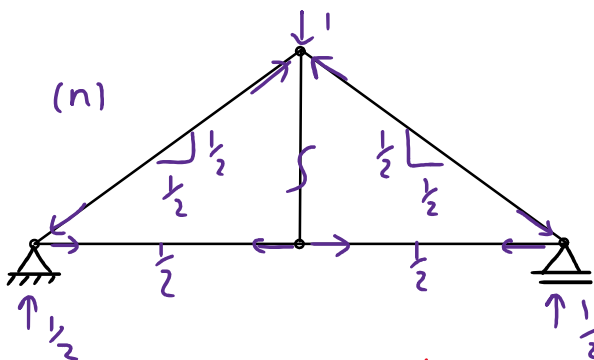
$$1 \times \delta_{Cx} = \frac{1}{EA} (1) \left(\frac{P}{2}\right) (L) \times 2 = \frac{PL}{EA}$$



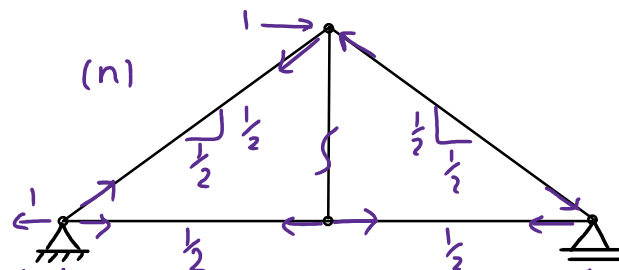
$$1 \times \delta_{By} = \frac{1}{EA} \left[\left(\frac{1}{2}\right) \left(\frac{P}{2}\right) (L) \times 2 + \left(-\frac{\sqrt{2}}{2}\right) \left(-\frac{\sqrt{2}}{2} P\right) (\sqrt{2} L) \times 2 + (1) (P) (L) \right] = \left(\frac{3}{2} + \sqrt{2}\right) \frac{PL}{EA}$$



$$\delta_{By} = \left(\frac{1}{2} + \sqrt{2}\right) \frac{PL}{EA}$$



$$1 \times \delta_{Bx} = \frac{1}{EA} \left[\left(\frac{\sqrt{2}}{2}\right) \left(-\frac{P\sqrt{2}}{2}\right) (\sqrt{2} L) + \left(-\frac{\sqrt{2}}{2}\right) \left(-\frac{P\sqrt{2}}{2}\right) (\sqrt{2} L) + 0 + \left(\frac{1}{2}\right) \left(\frac{P}{2}\right) (L) \times 2 \right] = \frac{PL}{2EA}$$

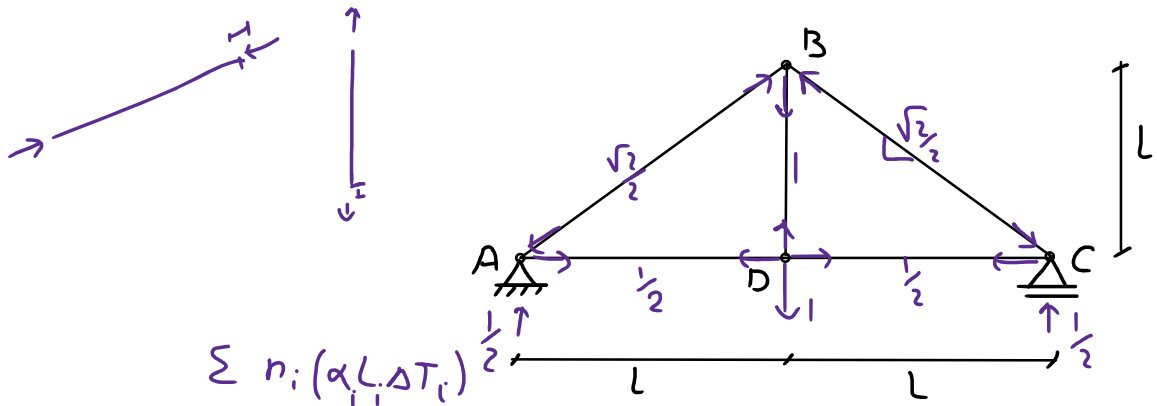


$$1 \times \delta_{Bx} = \frac{1}{EA} \left[\left(\frac{\sqrt{2}}{2} \right) \left(-\frac{P\sqrt{2}}{2} \right) (\sqrt{2}L) + \left(-\frac{\sqrt{2}}{2} \right) \left(-\frac{P\sqrt{2}}{2} \right) (\sqrt{2}L) + 0 + \left(\frac{1}{2} \right) \left(\frac{P}{2} \right) (L) \times 2 \right] = \frac{PL}{2EA}$$

اثر نشست تکیه‌گاه، تغییردها و نفوذ عضو در خرابی‌ها

مثال: مطلوب است تعیین مکان D در اثر نشست تکیه‌گاه C به اندازه 4 cm ، انحرافین دما در اعضا

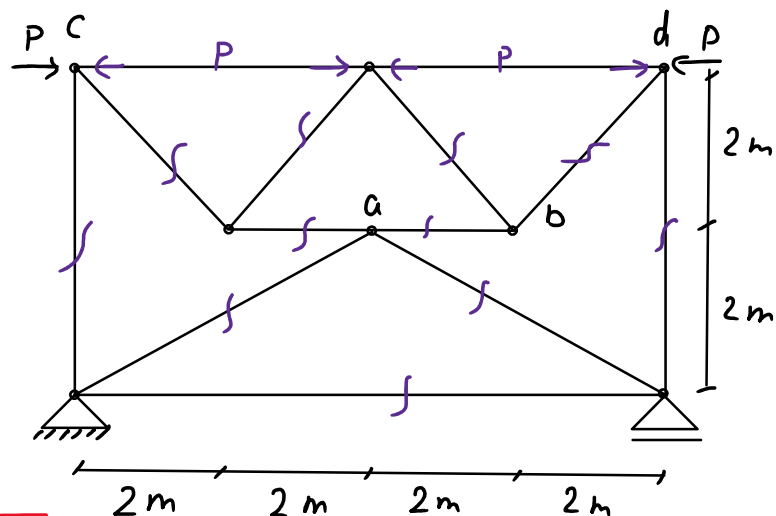
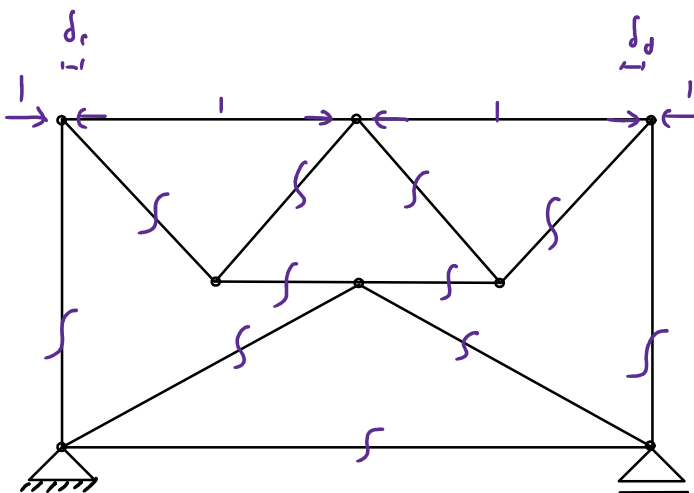
مایل به اندازه ΔT ، اضافه طول عضو BD به مقدار 3 و گویا بودن طول عضو CD به اندازه 2 cm .



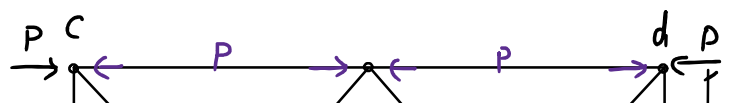
$$1 \times \delta - \frac{1}{2} (0.04) = \left(-\frac{\sqrt{2}}{2} \right) (\alpha \sqrt{2} L \Delta T) \times 2 + (1) (0.03) + \left(\frac{1}{2} \right) (-0.02)$$

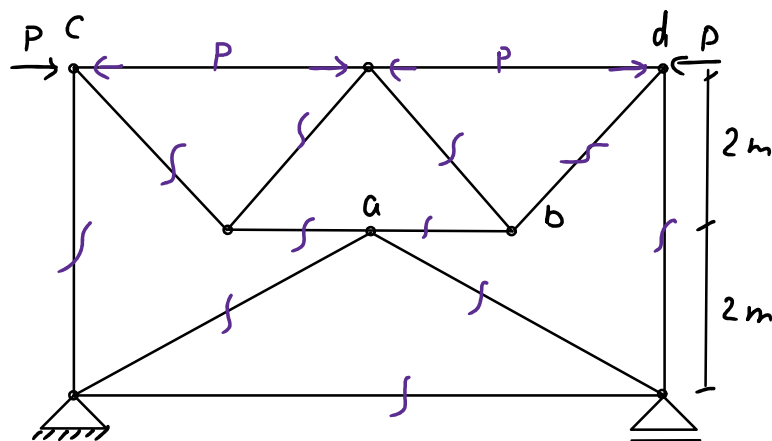
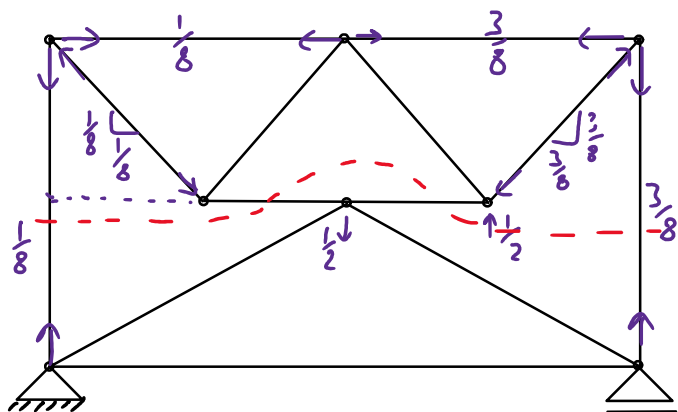
$$\delta_{Dy} = 0.94 + 2 \alpha L \Delta T$$

مثال: مطلوب است میزان نزدیک شدن یا دور شدن دو تیر C و d و دوران عضو a .



$$\delta_{C,d} = 1 \times \delta_c + 1 \times \delta_d = \frac{1}{EA} (-1) (-P) (4) \times 2 = \frac{8P}{EA}$$

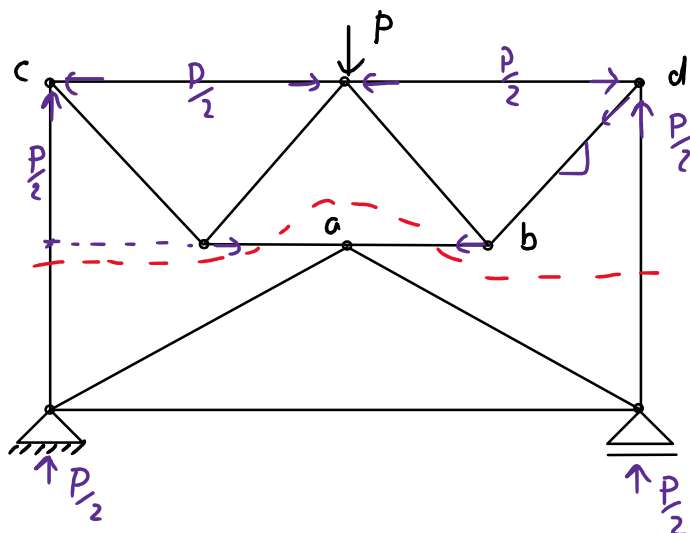
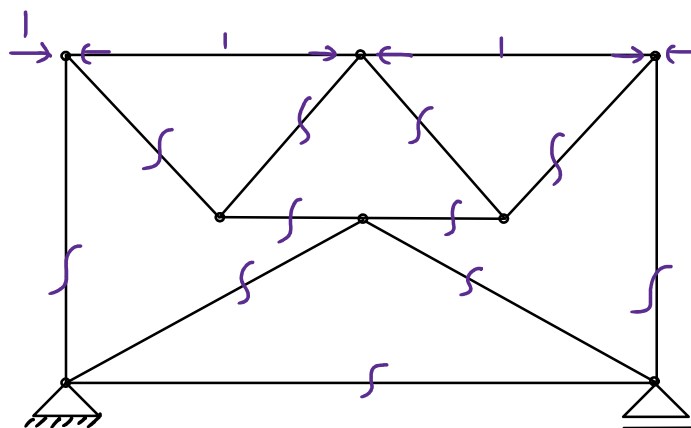




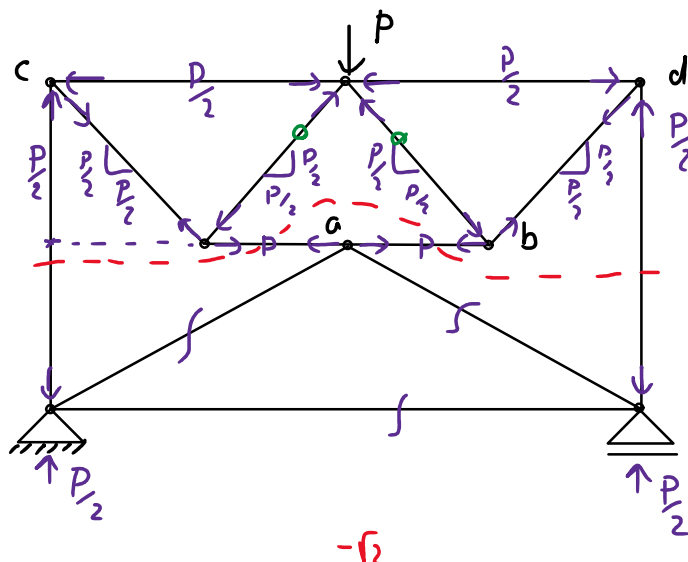
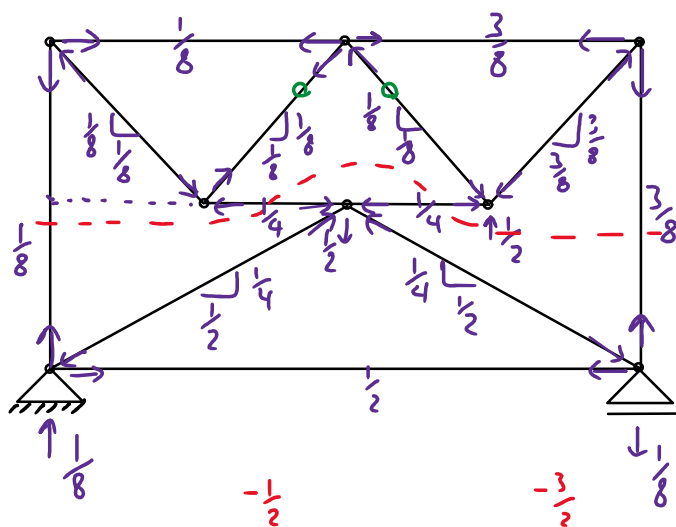
$$\theta_{ab} = \frac{1}{2} \delta_c + \frac{1}{2} \delta_b = \frac{1}{EA} \left[\left(\frac{1}{8} \right) (-P) (4) + \left(\frac{3}{8} \right) (-P) (4) \right] = \frac{-2P}{EA}$$

a ← b

مثال: معلوب است میزان نزدیک شدن یا دور شدن دگره c و d و دوران عضو a.

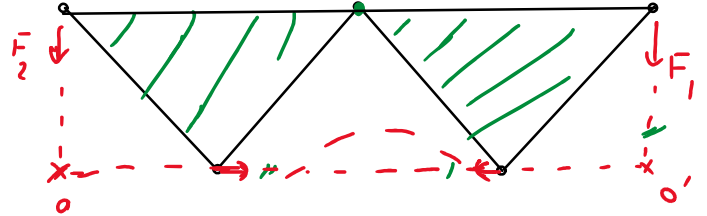


$$\delta_{c/d} = \frac{1}{EA} (-1) \left(-\frac{P}{2} \right) (4) \times 2 = \frac{4P}{EA}$$



$$FA\theta = \frac{1}{EA} \left[\left(-\frac{1}{8} \right) (-P) (4) + \left(\frac{3}{8} \right) (-P) (4) + \left(\frac{1}{2} \right) (-P) (4) \right] = \frac{-2P}{EA}$$

$$\theta_{ab} = (3 + \sqrt{2}) \frac{p}{EA} \Delta$$

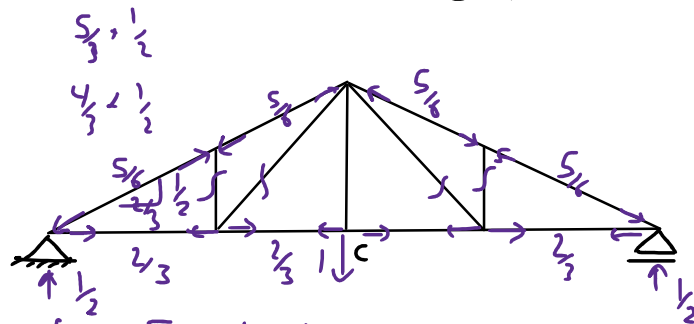
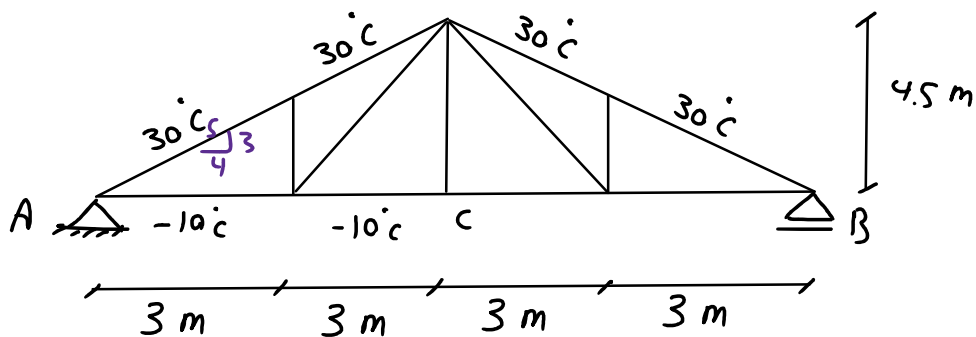


$$\sum M_o = 0 \rightarrow F_2$$

	n_i	N_i	l_i	$n_i N_i l_i$
a b	-	.	.	—
b c	.	.	.	
.				
.				
,				

 Σ

مثال: تغییر مکان قائم گره C را در اثر تغییر دما نشان داد. به دست آورید.

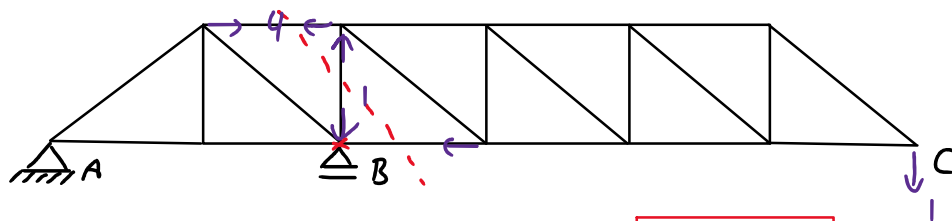
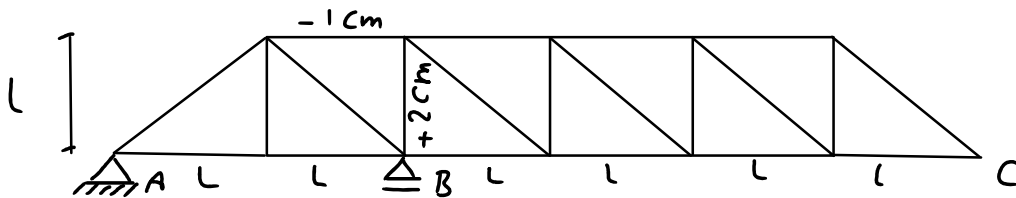


$$1 \times \delta_c = \sum n_i (\alpha_i L_i \Delta T_i)$$

$$1 \times \delta_c = \left(-\frac{5}{8}\right) (12 \times 10^{-6} \times 3.75 \times 30) \times 4 + \left(\frac{2}{3}\right) (12 \times 10^{-6} \times 3 \times (-10)) \times 2 = -0.005 \text{ m}$$

$$\delta_c = 5 \text{ mm} \uparrow$$

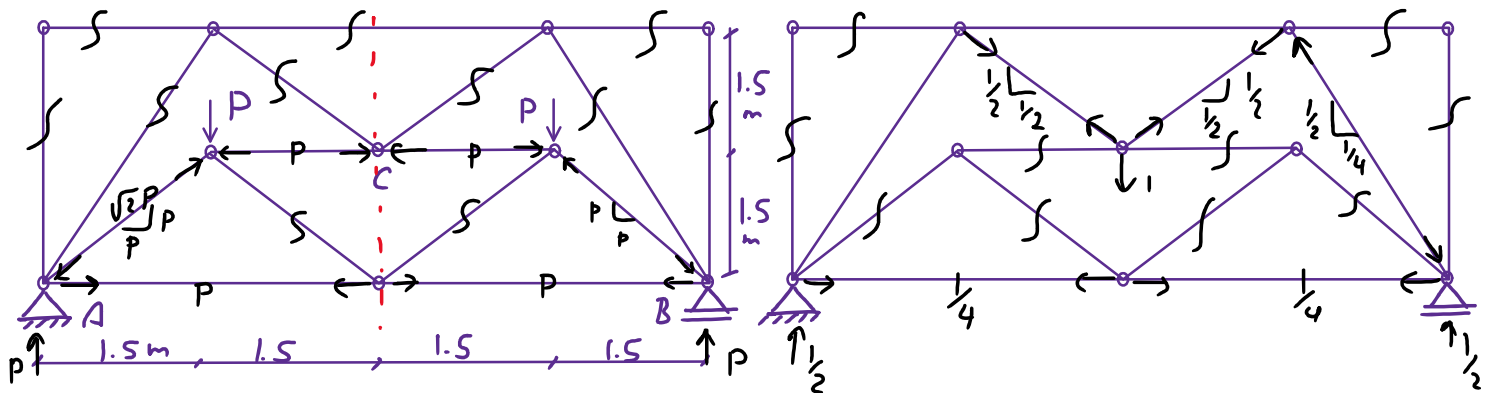
مثال: تغییر مکان قائم گره C در اثر کوتاه و بلند بودن اعضا مطابق شکل چندرات ؟



$$1 \times \delta_c = (-1)(0.02) + (4)(-0.01) = -0.06 \rightarrow \delta_c = 6 \text{ cm} \uparrow$$

مثال: جابجایی قائم گره C را تحت بار وارده به دست آورید.

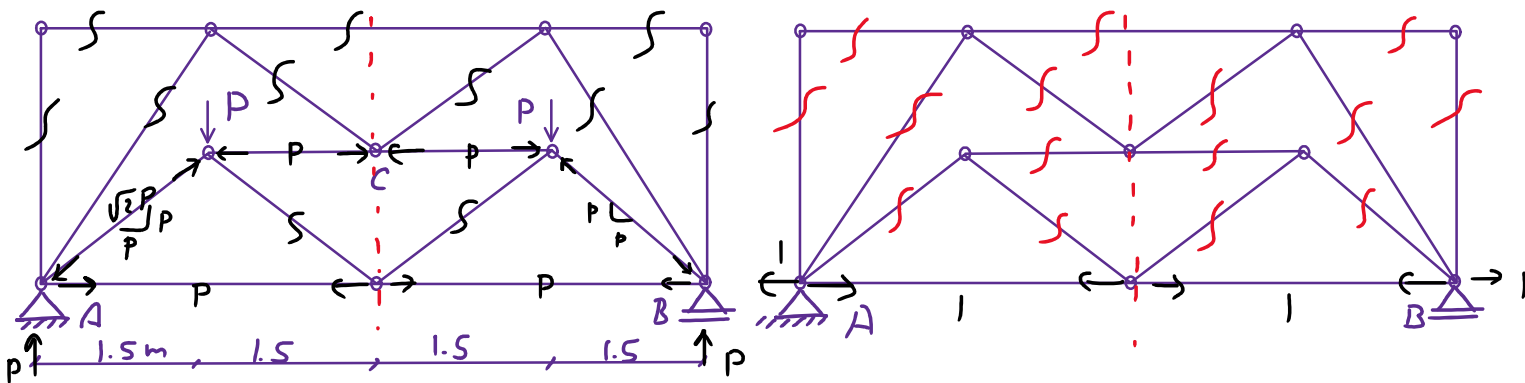




$$1 \times \delta_c = \sum \frac{n_i N_i L_i}{EA_i} = \left(\frac{1}{4}\right)(P)\left(\frac{3}{EA}\right) \times 2 = \frac{3}{2} \frac{P}{EA}$$

$$\delta_c = \frac{3}{2} \frac{P}{EA} \downarrow$$

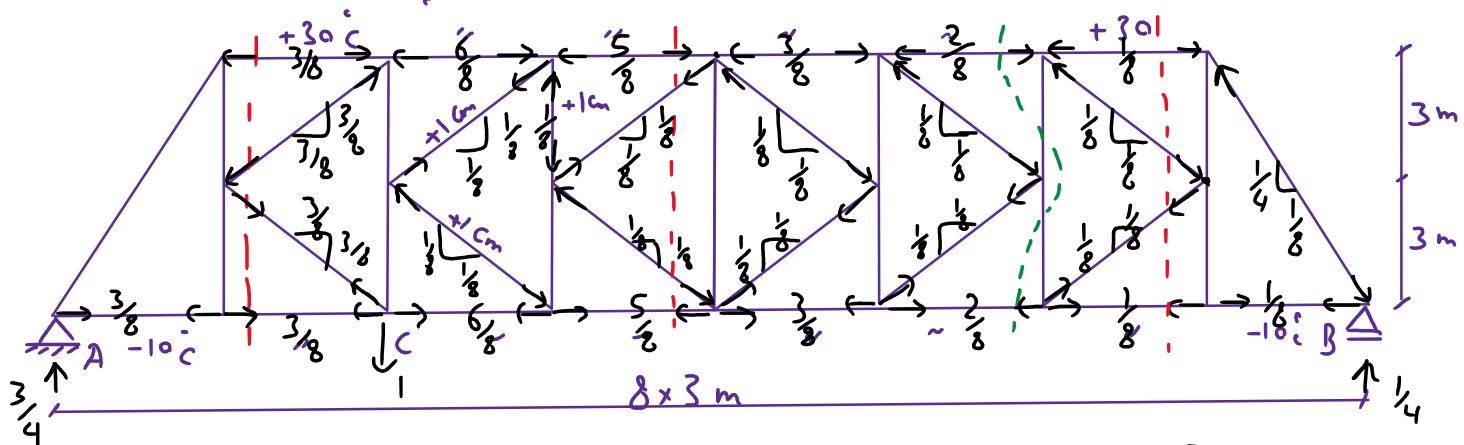
مثال: اگر در مثال فوق علاوه بر بارهای وارده، کلبه اعضا به قطر 30 و افزایش دما داشته باشد، جابجایی این کلبه گاه B را بدست آورید.



$$1 \times \delta_B = \sum \frac{n_i N_i L_i}{EA} + \sum n_i (\alpha L_i \Delta T_i) = \frac{(1)(P)(3)}{EA} \times 2 + (1)(\alpha)(3)(30) \times 2$$

$$\delta_B = \frac{6P}{EA} + 180\alpha \rightarrow$$

مثال: جابجایی قائم گره C تحت تغییر دما در تاربالای 30+ و تاربالایی 10- و نقص عضو را محاسبه کنید.



$$1 \times \delta = \sum n_i (\alpha L_i \Delta T_i) + (-\frac{\sqrt{2}}{2})(0.01) + (\frac{\sqrt{2}}{2})(0.01) + (-\frac{1}{2})(0.01) \frac{F_1}{EA} \leftarrow$$

4

$$1 \times \delta_c = \sum \eta_i (\alpha L \Delta T) + \left(-\frac{\sqrt{2}}{8}\right)(0.01) + \left(\frac{\sqrt{2}}{8}\right)(0.01) + \left(-\frac{1}{8}\right)(0.01)$$

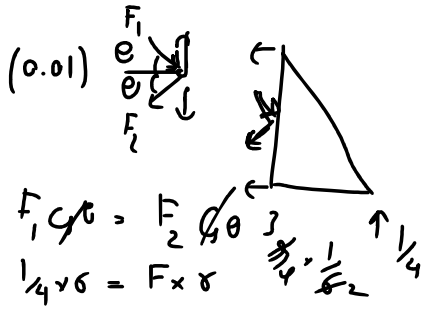
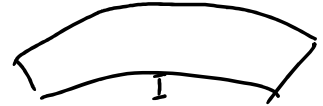
$$1 \times \delta_c = \alpha \times 3 \times 30 \sum \eta_i + \alpha \times 3 \times (-10) \sum \eta_i + \left(-\frac{1}{8}\right)(0.01)$$

$$20 \times \frac{24}{8} = 3$$

$$24 \times \frac{28}{8}$$

$$1 \times \delta_c = -375 \alpha - \frac{0.01}{8} = -375 \times 12 \times 10^{-6} - \frac{0.01}{8} = -4.5 \text{ mm} - 1.25 \text{ mm} = -5.75 \text{ mm}$$

$$\delta_c = 5.03 \text{ mm} \uparrow$$

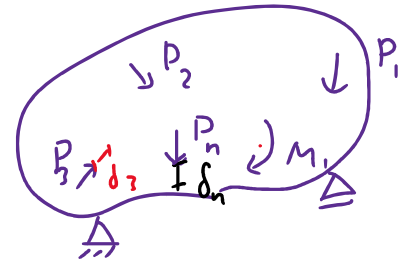


روش کاستیلیانو

این روش در سال ۱۸۷۹ میلادی توسط آلبرتو کاستیلیانو ارائه شد. در این روش فرض می‌شود که نشست کلیه گان، تغییر دما و نقص عضو نداریم.

در سازه‌های زیر فرض کنیم که تغییر مکان δ_n به اندازه $d\delta_n$ افزایش یابد. در این صورت داریم:

$$U = U(P_1, P_2, \dots, P_n, M_1, M_2, \dots, M_n, \delta_1, \delta_2, \dots, \underline{\delta_n}, \theta_1, \theta_2, \dots, \theta_n)$$



$$\begin{cases} dU = \frac{\partial U}{\partial \delta_n} d\delta_n \\ dU = P_n d\delta_n \end{cases}$$

$$\frac{\partial U}{\partial \delta_n} = P_n$$

تعیین اول
کاستیلیانو

$$f(x, y, z)$$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$$

اگر بار P_n به مقدار dP_n افزایش یابد، تغییر مکان ما به میزان $d\delta_n$ تا δ_n تغییر کند. با توجه به اینکه رفتار سازه خطی است، می‌توان فرض کرد که ابتدا بار dP_n و سپس بارهای P_1 تا P_n به سازه اعمال شود. بنابراین داریم:

$$\begin{cases} dU = \frac{\partial U}{\partial P_n} dP_n \\ dU = dP_n \delta_n + \frac{1}{2} dP_n d\delta_n \end{cases}$$

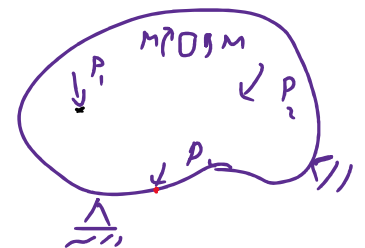
$$\frac{\partial U}{\partial P_n} = \delta_n$$

تعیین دوم
کاستیلیانو

$$U = \int \frac{M^2}{2EI} dx$$

$$M = P_1 m_1 + P_2 m_2 + \dots + P_n m_n$$

$$\frac{\partial U}{\partial P_n} = \int \frac{M (\frac{\partial M}{\partial P_n})}{EI} dx = \int \frac{M m_n}{EI} dx \stackrel{\text{بار واحد}}{=} \delta_n$$



مثال: تغییر مکان قائم سر آزاد تیر را محاسبه کنید.

$$\delta = \frac{\partial U}{\partial P} = \int \frac{M}{EI} \left(\frac{\partial M}{\partial P} \right) dx = \frac{1}{EI} \int_0^L (-Px)(-x) dx = \frac{Px^3}{3EI} \Big|_0^L = \frac{PL^3}{3EI}$$

$$M = -Px$$

$$\frac{\partial M}{\partial P} = -x$$

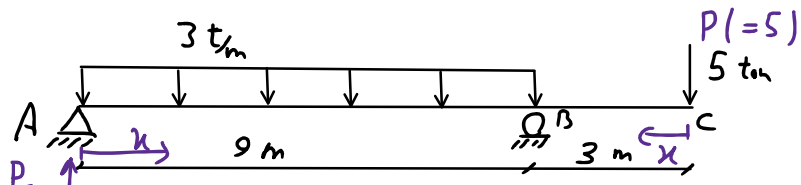
$$\frac{1}{EI} \int \frac{M}{EI} dx$$


$M = -Px - \bar{M}$

$\frac{\partial M}{\partial \bar{M}} = -1$

$$\theta = \frac{\partial U}{\partial \bar{M}} = \int \frac{M}{EI} \left(\frac{\partial M}{\partial \bar{M}} \right) dx = \frac{1}{EI} \int_0^L (-Px - \bar{M}) (-1) dx = \frac{Px^2}{2EI} \Big|_0^L = \frac{PL^2}{2EI}$$

مثال: تغییر مکان قائم C را به دست آورید. ($EI = 18000 \text{ t.m}^2$)



$P = 5 \text{ t}$

$AB: M = (13.5 - \frac{P}{3})x - \frac{3}{2}x^2$ $\frac{\partial M}{\partial P} = -\frac{x}{3}$

$BC: M = -Px$ $\frac{\partial M}{\partial P} = -x$

$$\delta_c = \frac{\partial U}{\partial P} = \int \frac{M}{EI} \left(\frac{\partial M}{\partial P} \right) dx = \frac{1}{EI} \int_0^9 (11.833x - 1.5x^2) \left(-\frac{x}{3} \right) dx + \frac{1}{EI} \int_0^3 (-5x)(-x) dx$$

$$\frac{1}{EI} \left(-\frac{11.833}{9} x^3 + \frac{1}{2} \frac{x^4}{4} \right) \Big|_0^9 + \frac{1}{EI} \left(\frac{5x^3}{3} \right) \Big|_0^3 = -\frac{138.375}{EI} + \frac{45}{EI} = \frac{-93.375}{EI} \uparrow$$

مثال: چرخش ست چپ مفصل C را به دست آورید.

$$\theta_{cl} = \frac{\partial U}{\partial \bar{M}} = \int \frac{M}{EI} \frac{\partial M}{\partial \bar{M}} dx$$

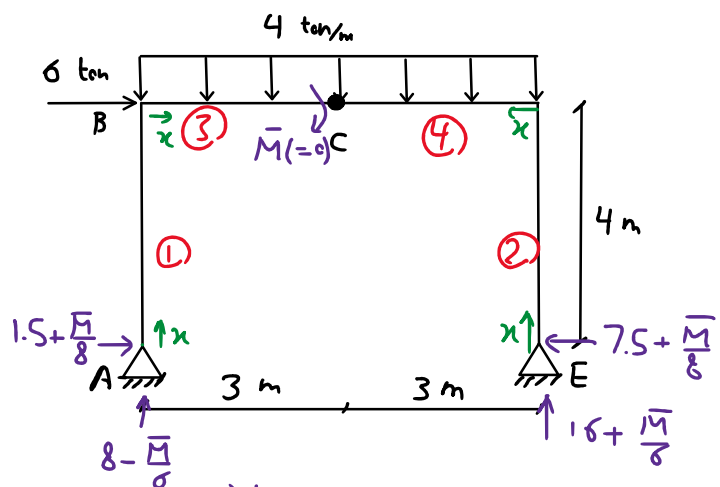
$$\sum M_c = 0$$

$$(6 - \frac{\bar{M}}{8})(3) - 4R_x - 12 \times 1.5 + \bar{M} = 0$$

$$6 + \frac{\bar{M}}{2} = 4R_x$$

① $M = -(1.5 + \frac{\bar{M}}{8})x$

② $M = -(7.5 + \frac{\bar{M}}{8})x$



$$\frac{\partial M}{\partial \bar{M}} = -\frac{x}{8}$$

$$\frac{\partial M}{\partial \bar{M}} = -\frac{x}{8}$$

$$\frac{\partial M}{\partial \bar{M}} = 1$$

$$(2) \quad M = -(7.5 + \frac{\bar{M}}{8})x$$

$$\frac{\partial M}{\partial \bar{M}} = -\frac{x}{8}$$

$$(3) \quad M = (8 - \frac{\bar{M}}{6})x - (1.5 + \frac{\bar{M}}{8})(4) - 2x^2$$

$$\frac{\partial M}{\partial \bar{M}} = -\frac{1}{2} - \frac{x}{8}$$

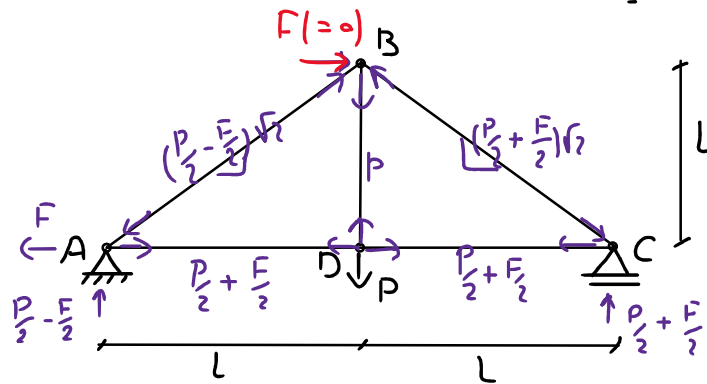
$$(4) \quad M = (16 + \frac{\bar{M}}{6})x - (7.5 + \frac{\bar{M}}{8})(4) - 2x^2$$

$$\frac{\partial M}{\partial \bar{M}} = -\frac{1}{2} + \frac{x}{6}$$

$$EI \theta_c = \int_0^4 (-1.5x)(-\frac{x}{8}) dx + \int_0^4 (-7.5x)(-\frac{x}{8}) dx + \int_0^3 (-2x^2 + 8x - 6)(-\frac{1}{2} - \frac{x}{6}) dx + \int_0^3 (-2x^2 + 16x - 30)(-\frac{1}{2} + \frac{x}{6}) dx = 4 + 20 - 0.75 + 12.75 \rightarrow \theta_c = \frac{36}{EI}$$

مثال: جابجایی انتگره B را بدست آورید.

$$U = \frac{1}{2} \sum \frac{N^2 L}{EA}$$

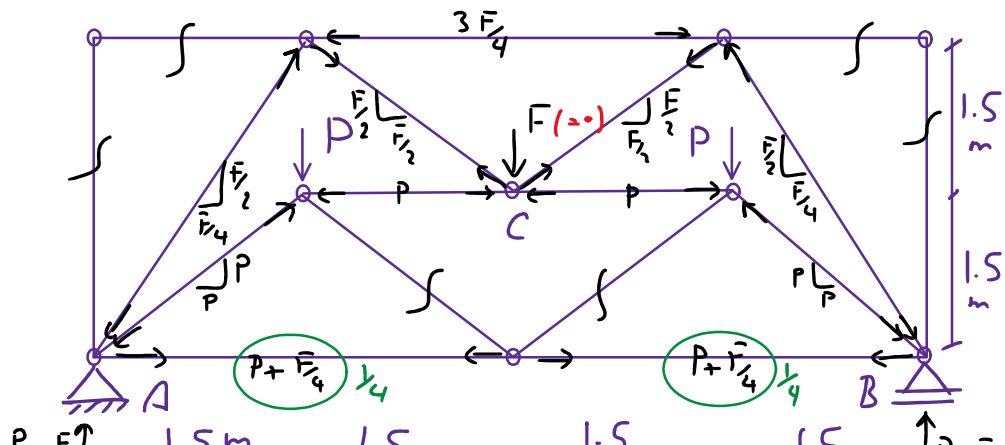


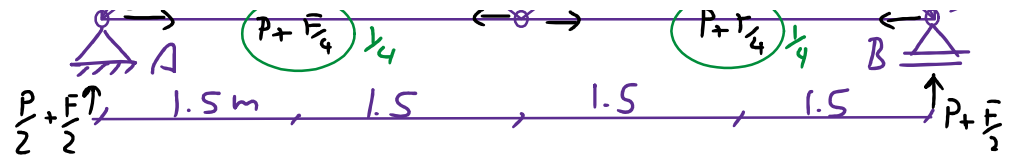
$$\delta_B = \frac{\partial U}{\partial F} = \sum N \left(\frac{\partial N}{\partial F} \right) \frac{L}{EA}$$

$$EA \delta_B = \left(\frac{P}{2} + \frac{F}{2} \right) \left(\frac{1}{2} \right) (L) \times 2 + \sqrt{2} \left(\frac{P}{2} + \frac{F}{2} \right) \left(\frac{\sqrt{2}}{2} \right) (\sqrt{2} L) + \sqrt{2} \left(\frac{P}{2} - \frac{F}{2} \right) \left(\frac{-\sqrt{2}}{2} \right) (\sqrt{2} L) + 0$$

$$\delta_B = \frac{PL}{2EA}$$

مثال: جابجایی قائم گره C را تحت بار وارده به دست آورید.





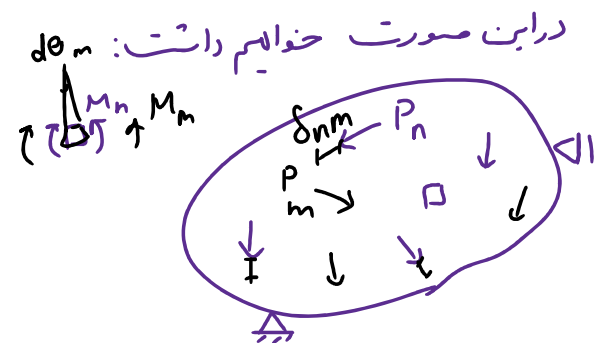
$$\delta_c = \frac{\partial U}{\partial F} = \sum N \left(\frac{\partial N}{\partial F} \right) \frac{L}{EA} = (P) \left(\frac{1}{4} \right) \frac{3}{EA} \times 2 = \frac{3}{2} \frac{P}{EA}$$

قانون بیتی در سال ۱۸۷۲ میلادی توسط انریکو بیتی ارائه شد.

برای بدست آوردن این قضیه، سازه شکل زیر را تحت انرژیتیم نیروها P_n در نظر بگیرید. حال اگر سیستم نیروها P_m به سازه وارد شود، تغییر مکان ایجاد شده توسط P_m در رابطه انرژیتیم P_n با δ_{nm} نشان داده می شود.

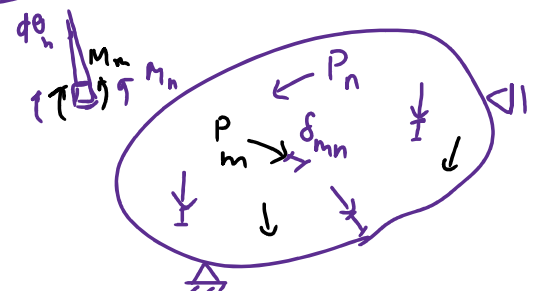
$$W_{ext} = U$$

$$\textcircled{1} \sum P_n \delta_{nm} = \int M_n \frac{M_m}{EI} dx$$



حال اگر ابتدا سیستم نیروها P_m در سازه موجود باشد و سپس سیستم نیروها P_n را به سازه وارد کنیم، داریم:

$$\textcircled{2} \sum P_m \delta_{mn} = \int M_m \frac{M_n}{EI} dx$$



$$\sum P_n \delta_{nm} = \sum P_m \delta_{mn}$$

قانون بیتی

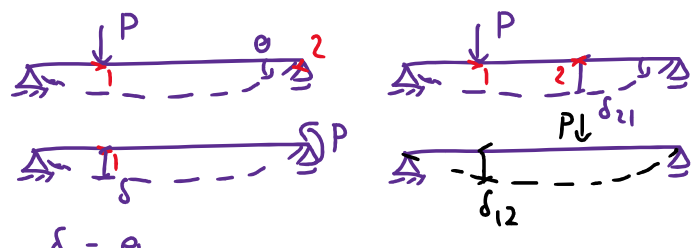
بنابراین از $\textcircled{1}$ و $\textcircled{2}$ داریم:

کار انجام شده توسط سیستم نیروها P_m به علت تغییر شکل سازه در انرژیتیم نیروها P_n برابر است با کار انجام شده توسط سیستم نیروها P_n به علت تغییر شکل سازه در انرژیتیم نیروها P_m .

قانون ماکسول حالت خاصی از قانون بیتی است که در آن نقطه یک بار P در دو حالت به سازه وارد می شود.

$$\delta_{21} = \delta_{12}$$

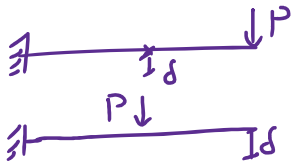
قانون ماکسول



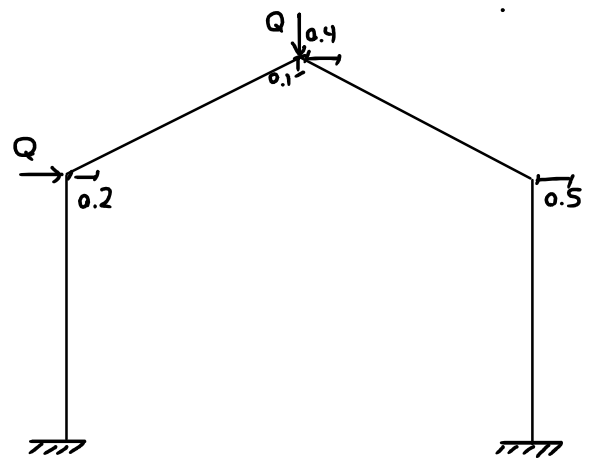
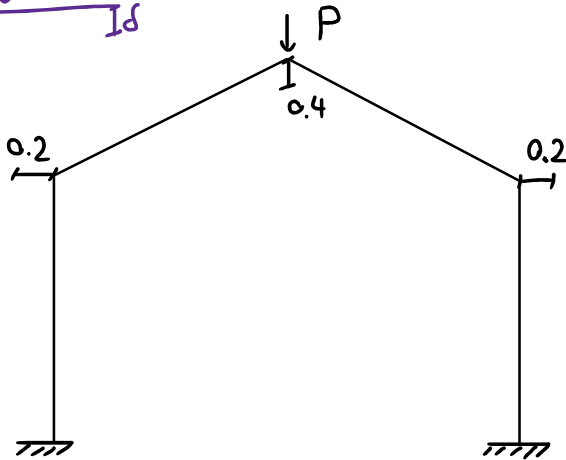
$$\Delta \delta_1 \quad \Delta \delta_{12}$$

$$\delta = \theta$$

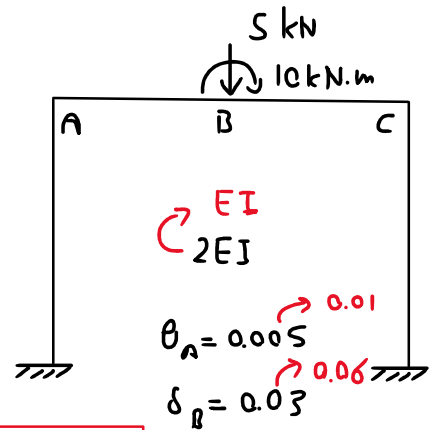
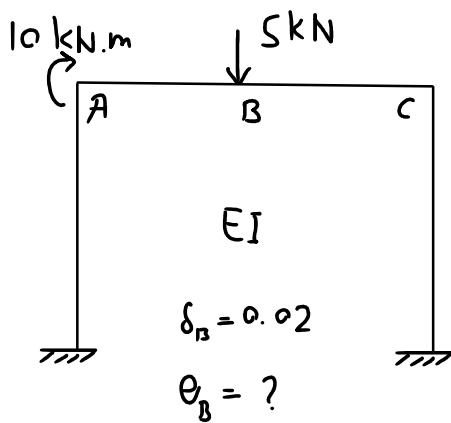
یعنی تغییر مکان در نقطه ۱ وقتی بار در نقطه ۱ باشد برابر است با تغییر مکان در نقطه ۲ وقتی همان بار در نقطه ۱ اعمال شود.



مثال: رابطه P و Q را پیدا کنید.



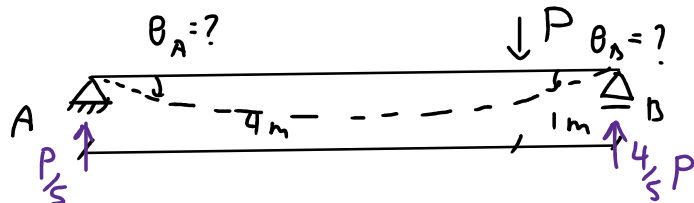
$$P(0.1) = -Q(0.2) + Q(0.4) \rightarrow \boxed{P = 2Q}$$



مثال:

$$10 \times 0.01 + 5 \times 0.06 = 5 \times 0.02 + 10 \theta_B \rightarrow \boxed{\theta_B = 0.02 \text{ rad}}$$

$$\delta_1 = \frac{1 \times 4^3}{3EI} + \frac{1 \times 4^2}{2EI} = \frac{29.333}{EI}$$



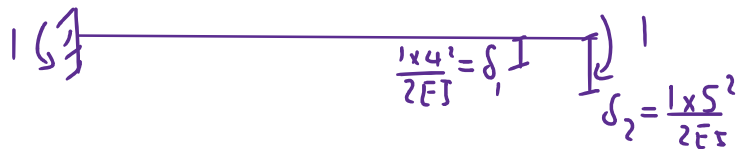
مثال:

$$\delta_2 = \frac{1 \times 5^3}{3EI} = \frac{41.667}{EI}$$

$$P \times \delta_1 = 0 \quad 4 \times 1 = 1 \times 5 \times \delta_2$$

$$\left(\frac{P}{5}\right)(0) + P\delta_1 - \left(\frac{4}{5}P\right)\delta_2 = 1 \times 0 - 5\theta_A + 0$$

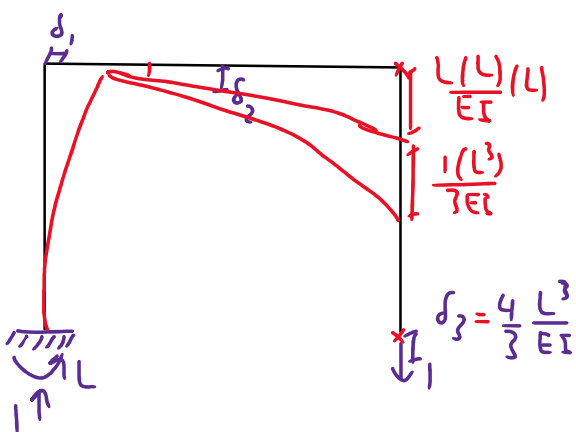
$$P\left(\frac{29.333}{EI}\right) - \left(\frac{4}{5}P\right)\left(\frac{41.667}{EI}\right) = -5\theta_A \rightarrow \theta_A = \frac{4}{5} \frac{P}{EI}$$



$$P\delta_1 - \left(\frac{4}{5}P\right)\delta_2 = -1\theta_A - 1\theta_B$$

$$P\left(\frac{8}{EI}\right) - \left(\frac{4}{5}P\right)\left(\frac{12.5}{EI}\right) = -\frac{4}{5} \frac{P}{EI} - \theta_B \rightarrow \theta_B = \frac{6}{5} \frac{P}{EI}$$

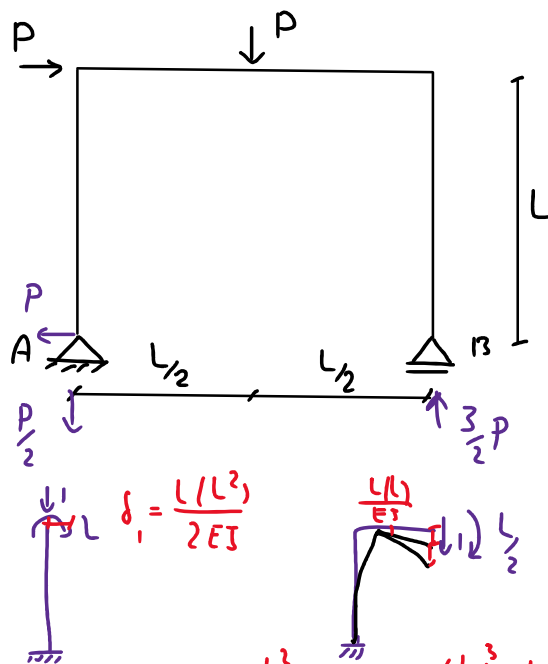
مثال: درغاب شکل زیر θ_A را به کمک قانون بیتی ج دت آورید.



$$L\theta_A = P\delta_1 + P\delta_2 - \frac{3}{2}P\delta_3$$

$$L\theta_A = P\left(\frac{L^3}{2EI}\right) + P\left(\frac{29}{48} \frac{L^3}{EI}\right) - \frac{3}{2}P\left(\frac{2}{3} \frac{L^3}{EI}\right)$$

$$\theta_A = \frac{43}{48} \frac{PL^2}{EI}$$

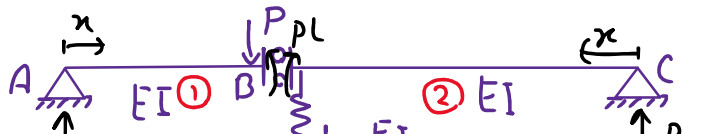


$$\delta_1 = \frac{L^2}{EI} \left(\frac{L}{2}\right) + \frac{1 \times \left(\frac{L}{2}\right)^3}{3EI} + \frac{L \times \left(\frac{L}{2}\right)^2}{2EI} = \frac{1}{2} + \frac{1}{24} + \frac{1}{16} = \frac{24+2+3}{48} = \frac{29}{48} \frac{L^3}{EI}$$

مثال: δ_{BL} و θ_A را به روش کاستیلیانو به دست آورید.

$$\delta_{BL} = \frac{\partial U}{\partial P} \quad U = \int \frac{M^2}{2EI} dx + \frac{F^2}{2k}$$

$$\delta = \int \frac{M}{EI} \frac{\partial M}{\partial P} dx + \frac{F}{k} \frac{\partial F}{\partial P}$$



$$\delta_{B_L} = \int \frac{M}{EI} \left(\frac{\partial M}{\partial P} \right) dx + \frac{F}{k} \left(\frac{\partial F}{\partial P} \right)$$

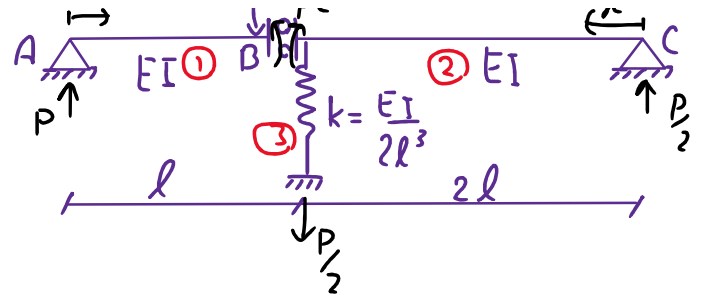
$$\textcircled{1} M = Px \quad \frac{\partial M}{\partial P} = x$$

$$\textcircled{2} M = \frac{P}{2}x \quad \frac{\partial M}{\partial P} = \frac{x}{2}$$

$$\textcircled{3} F = \frac{P}{2} \quad \frac{\partial F}{\partial P} = \frac{1}{2}$$

$$\begin{aligned} \delta_{B_L} &= \frac{1}{EI} \int_0^l (Px)(x) dx + \frac{1}{EI} \int_0^{2l} \left(\frac{P}{2}x \right) \left(\frac{x}{2} \right) dx + \frac{\frac{P}{2}}{\frac{EI}{2l^3}} \left(\frac{1}{2} \right) \\ &= \frac{1}{EI} \left(\frac{PL^3}{3} + \frac{P}{4} \frac{(2L)^3}{3} \right) + \frac{PL^3}{2EI} = \frac{3}{2} \frac{PL^3}{EI} \end{aligned}$$

$$\delta_{B_L} = \frac{3}{2} \frac{PL^3}{EI}$$



$$\theta_A = \frac{\partial U}{\partial \bar{M}} = \int \frac{M}{EI} \frac{\partial M}{\partial \bar{M}} dx + \frac{F}{k} \left(\frac{\partial F}{\partial \bar{M}} \right)$$

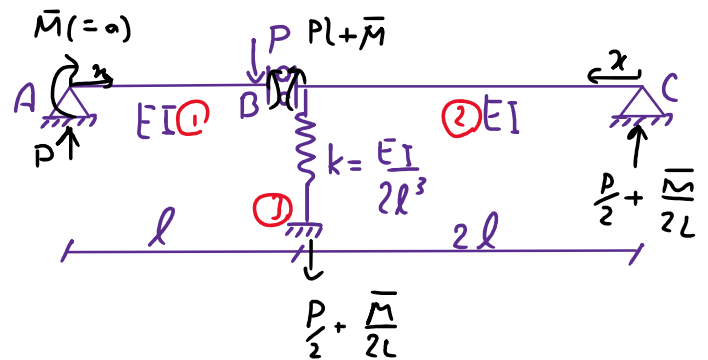
$$\textcircled{1} M = Px + \bar{M} \quad \frac{\partial M}{\partial \bar{M}} = 1$$

$$\textcircled{2} M = \left(\frac{P}{2} + \frac{\bar{M}}{2l} \right) x \quad \frac{\partial M}{\partial \bar{M}} = \frac{x}{2l}$$

$$\textcircled{3} F = \frac{P}{2} + \frac{\bar{M}}{2l} \quad \frac{\partial F}{\partial \bar{M}} = \frac{1}{2l}$$

$$\begin{aligned} \theta_A &= \frac{1}{EI} \int_0^l (Px)(1) dx + \frac{1}{EI} \int_0^{2l} \left(\frac{P}{2}x \right) \left(\frac{x}{2l} \right) dx + \frac{\frac{P}{2}}{\frac{EI}{2l^3}} \left(\frac{1}{2l} \right) \\ &= \frac{1}{EI} \left[\frac{PL^2}{2} + \frac{P}{4l} \frac{(2L)^3}{3} \right] + \frac{PL^2}{2EI} = \frac{5}{3} \frac{PL^2}{EI} \end{aligned}$$

$$\theta_A = \frac{5}{3} \frac{PL^2}{EI}$$



فصل پنجم : تحلیل سازه‌ها نامعین به روش نیروی

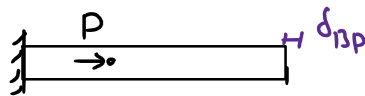
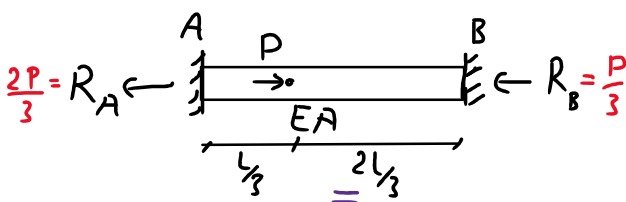
روش تحلیل سازه‌ها نامعین

① روش ما نیروی (نر) } سازه‌ها تغییر شکل ما
 کاستی‌ها (تغییر از حد حالت) }
 نیرو مزدوج }
 سه لنگر }
 ② روش ما تغییر مکان (نر) }
 تیب-انت }
 توزیع لنگر }
 کان }

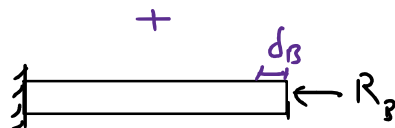
روش نیروی	مجموعات	معادلات
روش تغییر مکان	تغییر مکان (نر) (θ, δ)	نیرو (M, F)
روش نیروی	نیرو (M, F)	تغییر مکان (نر) (θ, δ)

مجهول R_B

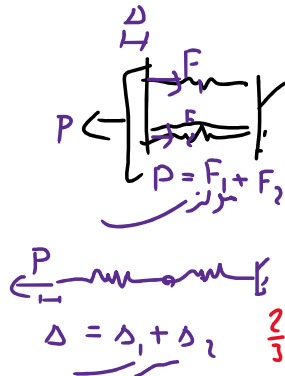
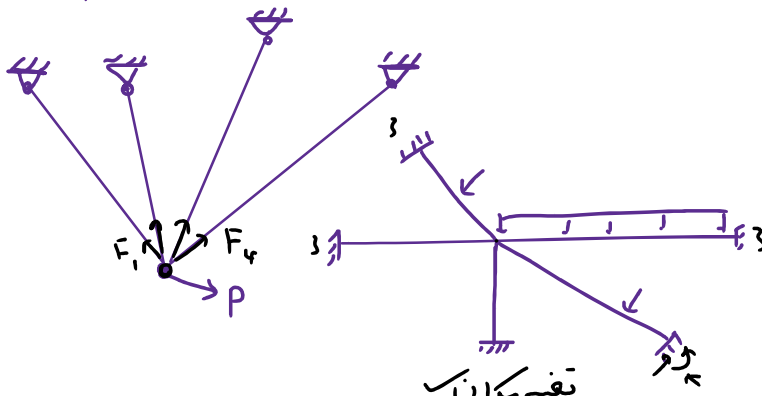
معادله $\delta_B =$



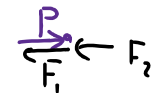
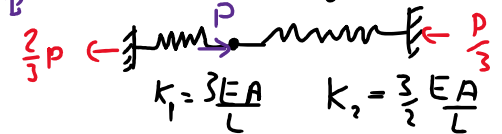
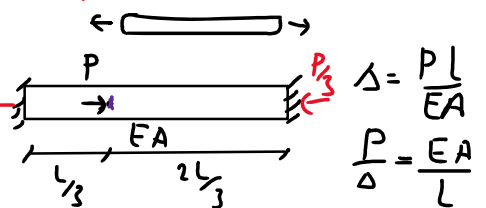
سوپرپوزیشن



$$\delta_B = \frac{P(L/3)}{EA} - \frac{R_B L}{EA} = 0 \rightarrow \frac{P(L/3)}{EA} = \frac{R_B L}{EA} \rightarrow R_B = \frac{P}{3}$$



روش تغییر مکان (نر)

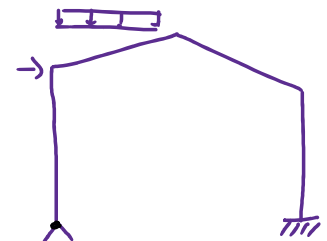


معادله نیروی $F_1 + F_2 = P$

مجهول $k_1 \Delta + k_2 \Delta = P$

$\Delta = \frac{P}{k_1 + k_2} = \frac{2}{9} \frac{PL}{EA}$

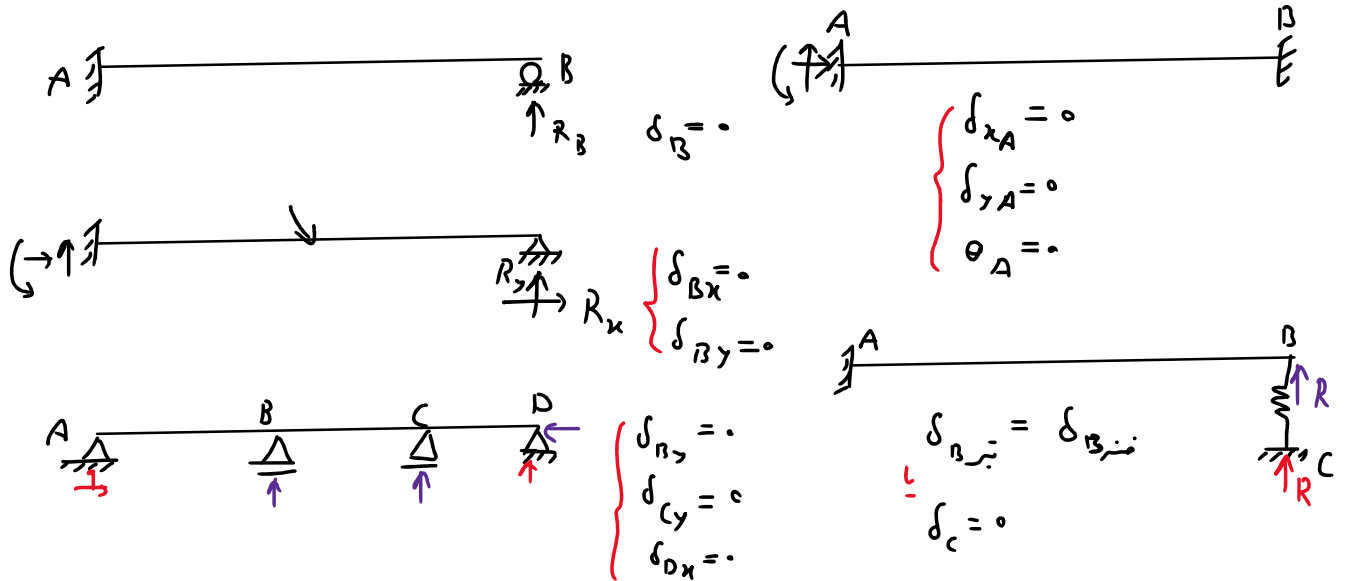
$F_1 = k_1 \Delta = \left(\frac{3EA}{L} \right) \left(\frac{2}{9} \frac{PL}{EA} \right) = \frac{2}{3} P$





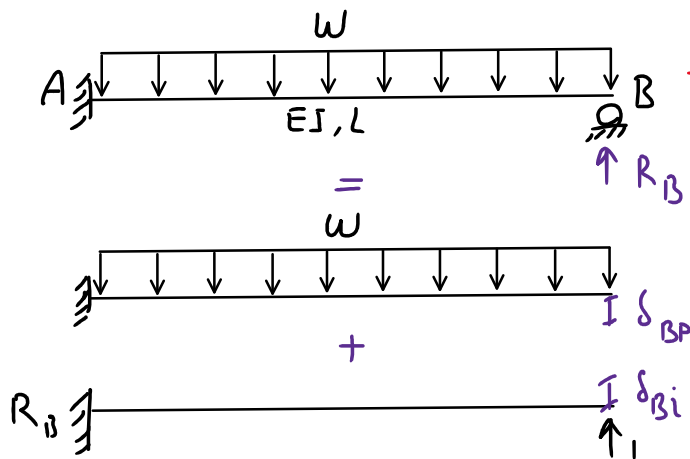
ردست سازان تغییر شکل ها

در یک سازه n درجه نامعین، n محمول نبردنی اضافه وجود دارد که با معادلات تعادل به دست می آید. اما هر محمول اضافه یک معادله تغییر مکانی اضافه نیز به همراه دارد. بنابراین با نوشتن n معادله n محمول کلیه محمول ها نیز با اضافی به دست می آید. در انتهای سایر محمولات از معادلات تعادل استاتیکی تعیین شده و سازه کاملاً تحلیل می شود.



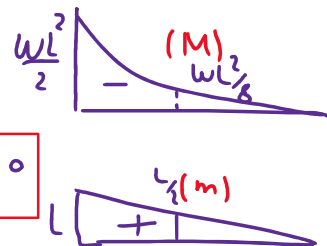
مثال: نیروی تکیه زبر را تحلیل کنید.

راه حل اول



$$\delta_B = 0$$

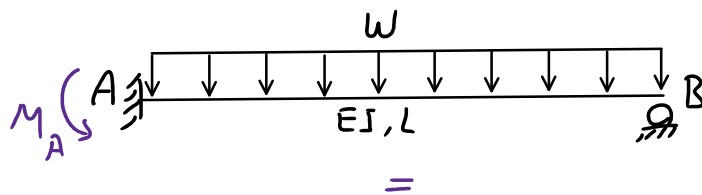
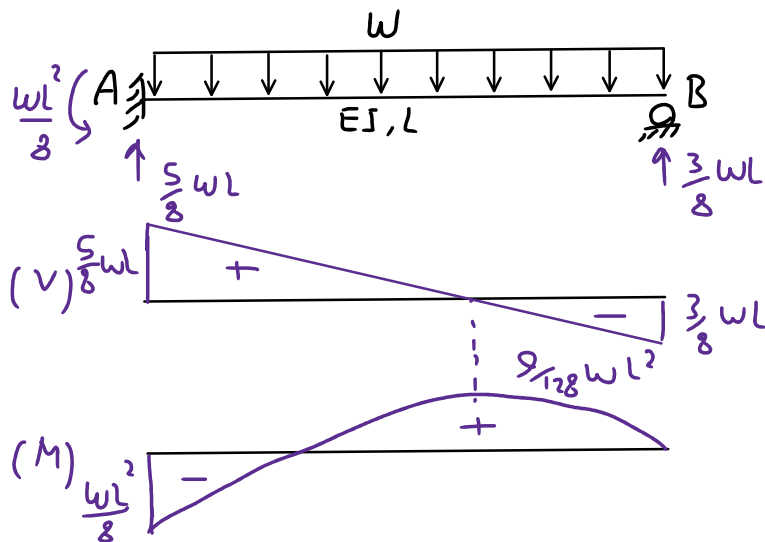
$$\delta_{Bp} + R_B \delta_{Bi} = 0$$



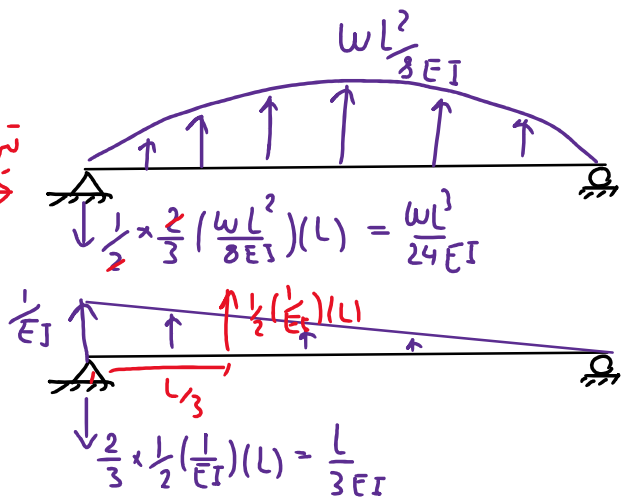
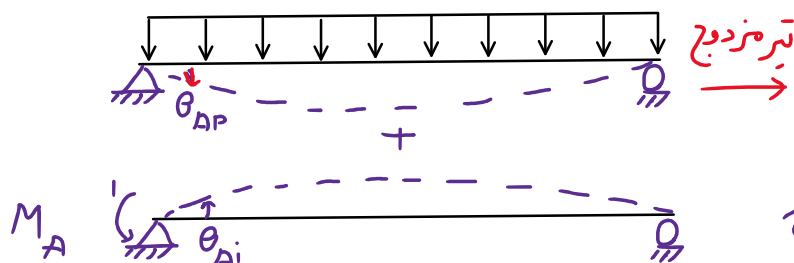
$$\delta_{Bp} = \int \frac{m M}{EI} dx = \frac{L}{6} \left[\left(-\frac{wL^2}{2} \right) (L) + 4 \left(-\frac{wL^2}{8} \right) \left(\frac{L}{2} \right) + 0 \right] = -\frac{wL^4}{8EI}$$

$$\delta_{Bi} = \int \frac{m^2}{EI} dx = \frac{L}{3} (L)(L) = \frac{L^3}{3EI}$$

$$-\frac{WL^4}{8EI} + R_B \frac{L^3}{3EI} = 0 \rightarrow \boxed{R_B = \frac{3}{8} WL}$$



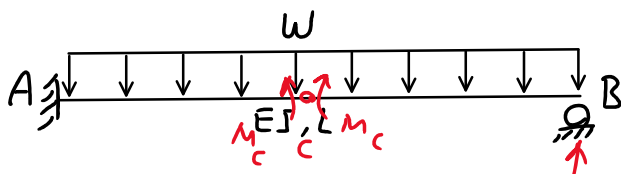
راه حل دوم



$$\theta_A = 0$$

$$\theta_{Ap} + M_A \theta_{Ai} = 0$$

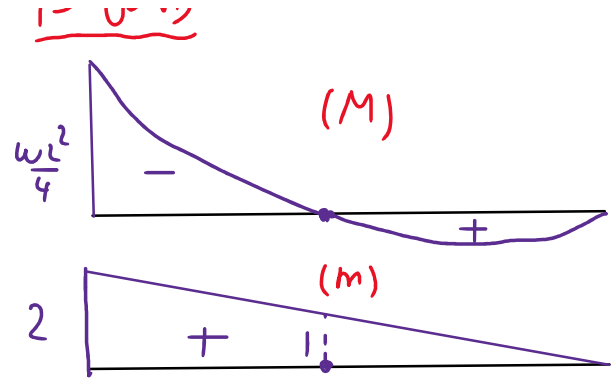
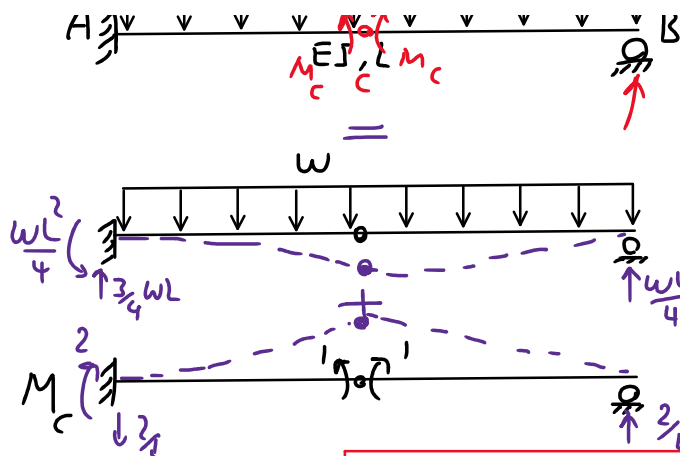
$$-\frac{WL^3}{24EI} + M_A \left(\frac{L}{3EI} \right) = 0 \rightarrow \boxed{M_A = \frac{WL^2}{8}}$$



راه حل سوم

Λ

رابطه

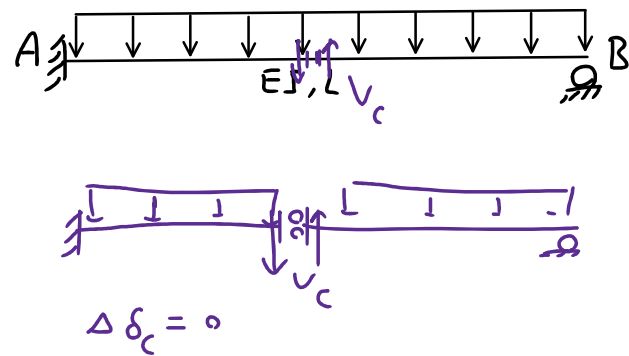
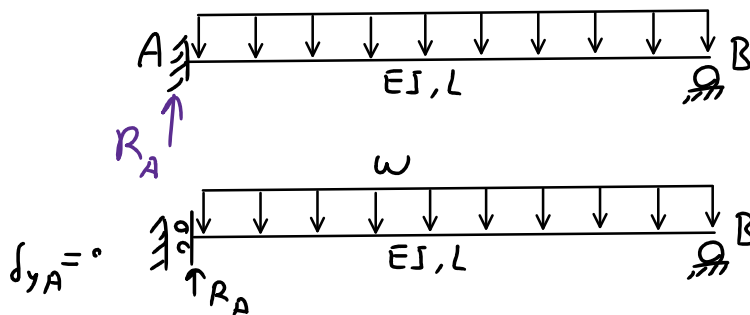


$$\Delta \theta_c = 0 \rightarrow \Delta \theta_{cp} + M_c \Delta \theta_i = 0$$

$$\Delta \theta_{cp} = \int \frac{mM}{EI} dx = \frac{L}{6} \left[\left(-\frac{wL^2}{4} \right) (2) + 0 + 0 \right] = -\frac{wL^3}{12EI}$$

$$\Delta \theta_{ci} = \int \frac{m^2}{EI} dx = \frac{L}{3} (2)(2) = \frac{4}{3} \frac{L}{EI}$$

$$-\frac{wL^3}{12EI} + M_c \left(\frac{4}{3} \frac{L}{EI} \right) = 0 \rightarrow M_c = \frac{wL^2}{16}$$



نوشتن معادلات سازگاری برای سازه ۲ درجه نامعین

$\delta_A = 0$
 $\delta_B = 0$

$\delta_{AP} + R_A \delta_{AA} + R_B \delta_{AB} = 0$
 $\delta_{BP} + R_A \delta_{BA} + R_B \delta_{BB} = \delta_B$

$(\delta_{BB} + \frac{1}{k}) R_B$

تغییر مکان ناشی از بار خارجی، تغییر دما...
 تغییرات سازه + تغییرات در سازه (مقاومت)

$\delta_A = 0$
 $\delta_B = 0$

$\delta_{B, beam} = \delta_{spring}$

$\delta_{beam} + \delta_{spring}$
 $\delta_{beam} + \frac{1}{k} R_B$

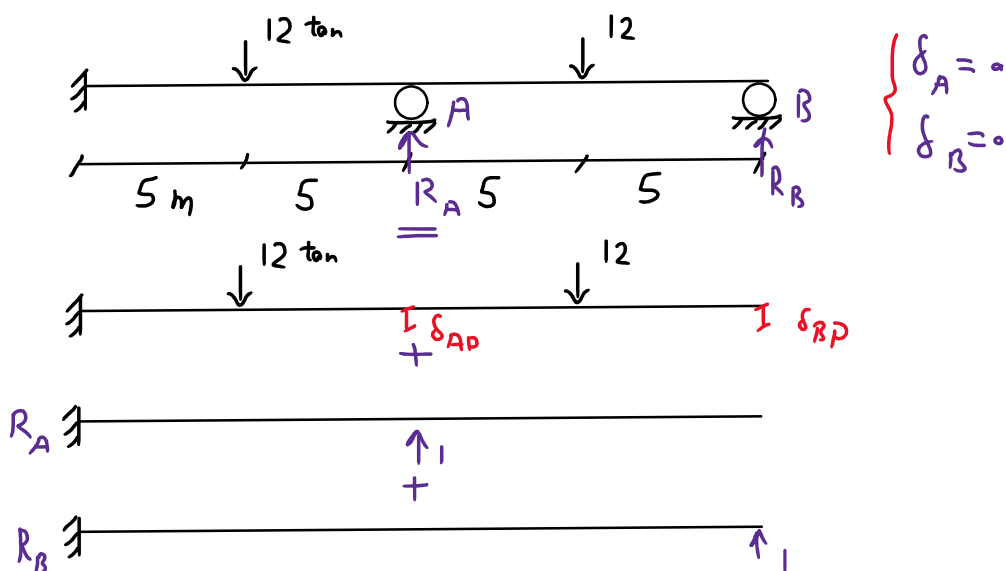
نوشتن معادلات سازگاری برای سازه n درجه نامعین

با توجه به اینکه سازه n درجه نامعین، ابتدا n محمول اضافه را در سازه مشخص می‌کنیم. سپس معادلات سازگاری را برای سازه n درجه نامعین، معادله صورت زیر می‌نویسیم:

با طوری برای سازه " درجه ها جفت، ابتدا n محمول اضافه راد سازه منتخبی کنیم. پس معادلات سازگاری را بر اساس جمع آثار قوا به صورت زیر تقسیم:

$$\begin{cases} \delta_{ap} + \delta_{aa} R_a + \delta_{ab} R_b + \delta_{ac} R_c + \dots + \delta_{an} R_n = 0 \\ \delta_{bp} + \delta_{ba} R_a + \delta_{bb} R_b + \delta_{bc} R_c + \dots + \delta_{bn} R_n = 0 \\ \vdots \\ \delta_{np} + \delta_{na} R_a + \delta_{nb} R_b + \delta_{nc} R_c + \dots + \delta_{nn} R_n = 0 \end{cases} \quad \begin{cases} \delta_{ap} \\ \delta_{bp} \\ \vdots \\ \delta_{np} \end{cases} + \begin{bmatrix} \delta_{aa} & \delta_{ab} & \dots & \delta_{an} \\ \delta_{ba} & \delta_{bb} & \dots & \delta_{bn} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{na} & \delta_{nb} & \dots & \delta_{nn} \end{bmatrix} \begin{Bmatrix} R_a \\ R_b \\ \vdots \\ R_n \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix}$$

مثال: تیر شکل زیر را تحلیل کنید.



$$\delta_{Ap} = \left[\frac{12 \times 5^3}{3EI} + \frac{12 \times 5^2 \times 5}{2EI} \right] + \left[\frac{12 \times 10^3}{3EI} + \frac{60 \times 10^2}{2EI} \right] = \frac{1250}{EI} + \frac{7000}{EI} = -\frac{8250}{EI}$$

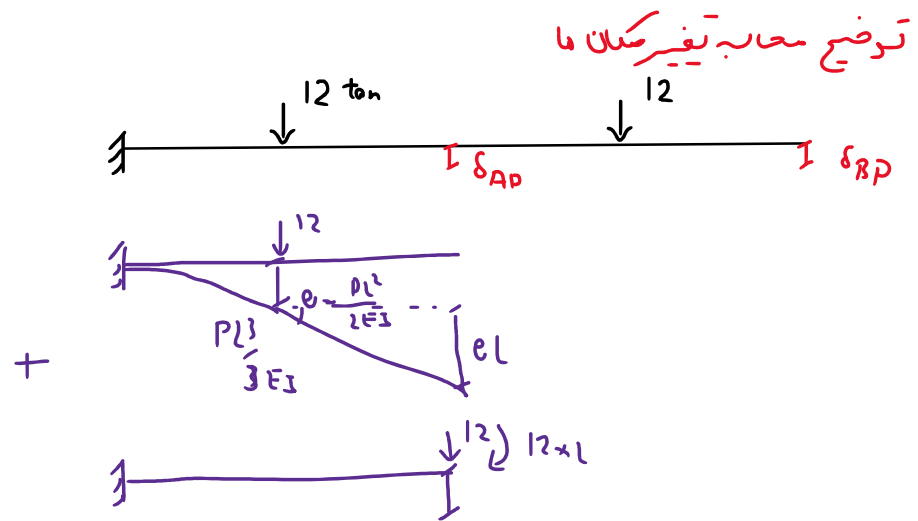
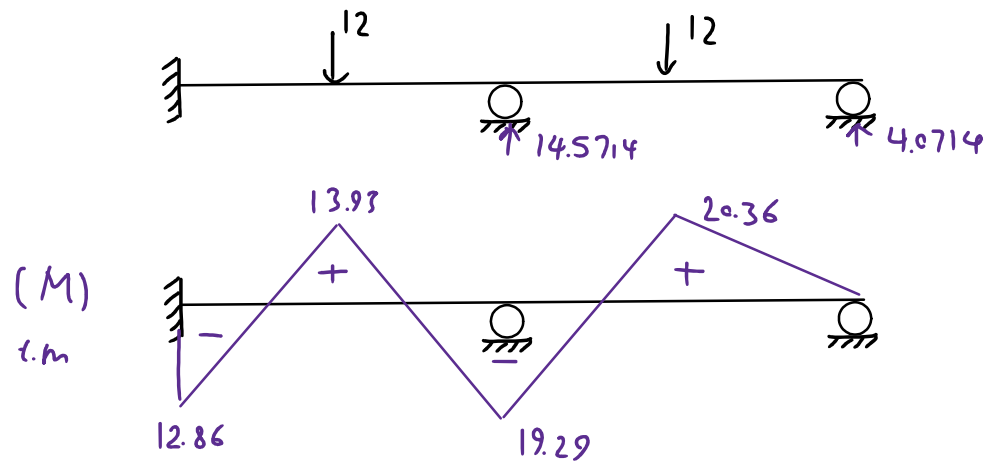
$$\delta_{AA} = \frac{1 \times 10^3}{3EI} = \frac{333.33}{EI} \quad \delta_{AB} = \frac{1 \times 10^3}{3EI} + \frac{10 \times 10^2}{2EI} = \frac{833.33}{EI}$$

$$\delta_{Bp} = \left[\frac{12 \times 5^3}{3EI} + \frac{12 \times 5^2 \times 15}{2EI} \right] + \left[\frac{12 \times 15^3}{3EI} + \frac{12 \times 15^2 \times 5}{2EI} \right] = \frac{2750}{EI} + \frac{20250}{EI} = -\frac{23000}{EI}$$

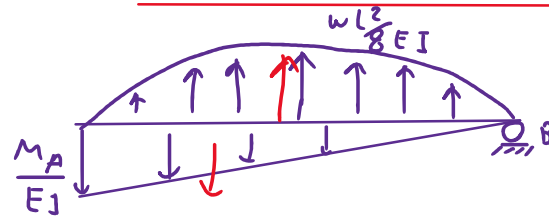
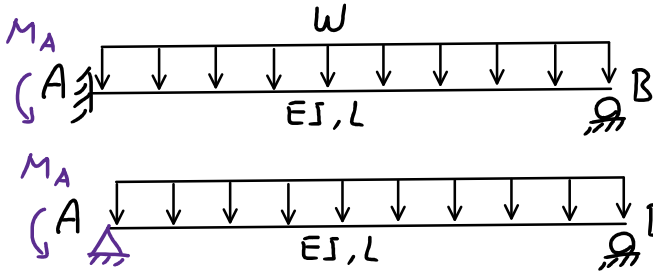
$$\delta_{BA} = \delta_{AB} = \frac{833.33}{EI} \quad \delta_{BB} = \frac{1 \times 20^3}{3EI} = \frac{2666.67}{EI}$$

$$\begin{cases} -8250 + 333.33 R_A + 833.33 R_B = 0 \\ -23000 + 833.33 R_A + 2666.67 R_B = 0 \end{cases} \rightarrow$$

$$\begin{cases} R_A = 14.5714 \text{ ton} \\ R_B = 4.0714 \text{ ton} \end{cases}$$



تحليل تيرنا معين با معادلات تعادل تير مزدوج

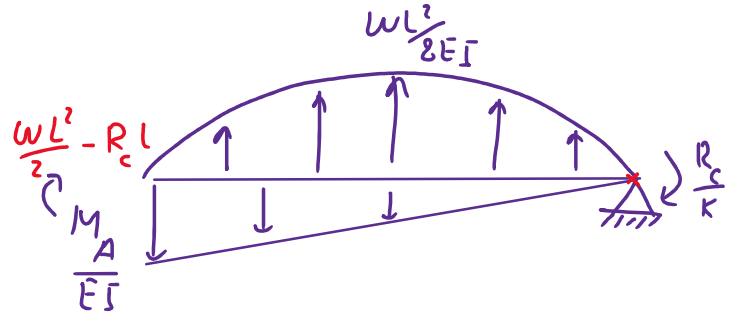
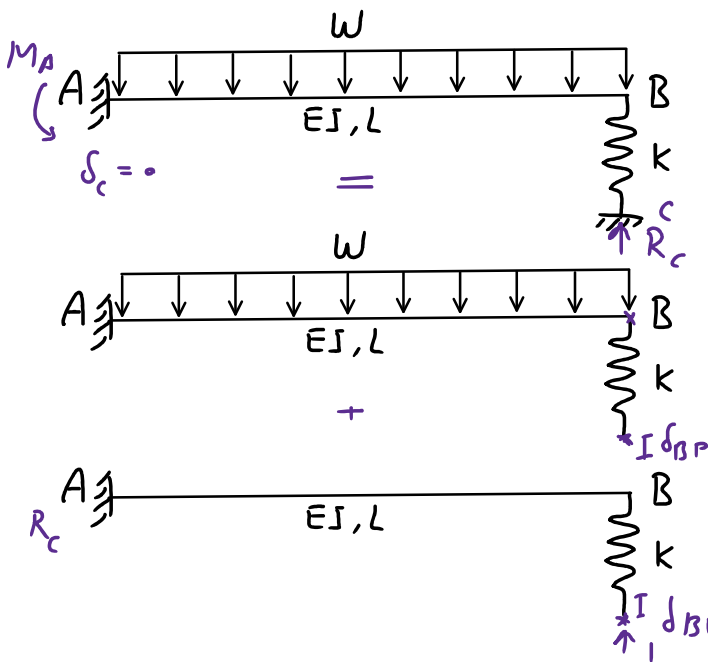


$\theta_A = 0 \rightarrow \sum M_B = 0 \rightarrow \frac{2}{3} \left(\frac{wL^2}{8EI} \right) (L) \left(\frac{L}{2} \right) - \frac{1}{2} \left(\frac{M_A}{EI} \right) (L) \left(\frac{2}{3} L \right) = 0$

$\frac{wL^4}{24EI} = \frac{1}{3} \frac{M_A L^2}{EI} \rightarrow M_A = \frac{wL^2}{8}$

* در واقع به جای نوشتن معادلات گزینا در تیر اصل می توان معادلات تعادل در تیر مزدوج را نوشت.

مثال: عکس العمل تیر را بر روش گزینا تغییر شکل و معادلات تعادل تیر مزدوج بدست آورید.



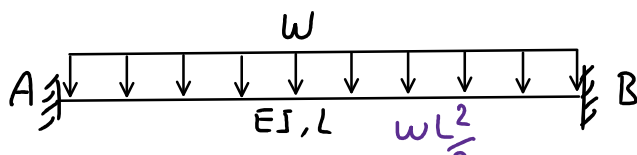
$\frac{2}{3} \left(\frac{wL^2}{8EI} \right) (L) \left(\frac{L}{2} \right) - \frac{1}{2} \left(\frac{wL^2}{2EI} - \frac{R_c L}{EI} \right) (L) \left(\frac{2}{3} L \right) + \frac{R_c}{k} = 0$

$\frac{1-4}{24} \frac{wL^4}{EI} = R_c \left(\frac{L^3}{3EI} + \frac{1}{k} \right)$

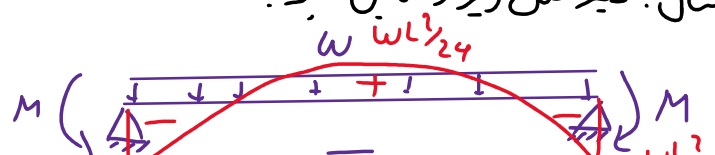
$R_c = \frac{3}{8} wL \frac{1}{\left(1 + \frac{3EI}{kL^3} \right)}$

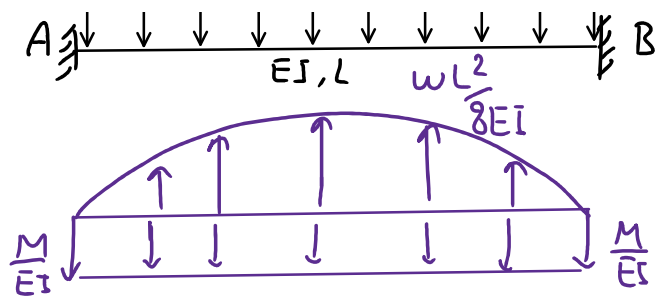
$\delta_{BP} + R_c \delta_{Bi} = 0$

$\frac{wL^4}{8EI} - \left(\frac{R_c L^3}{3EI} + \frac{R_c}{k} \right) = 0 \rightarrow R_c = \frac{\frac{3}{8} wL}{1 + \frac{3EI}{kL^3}}$



مثال: تیر شکل زیر را تحلیل کنید.

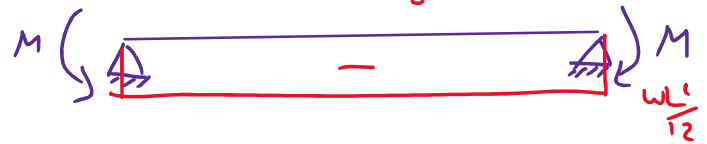
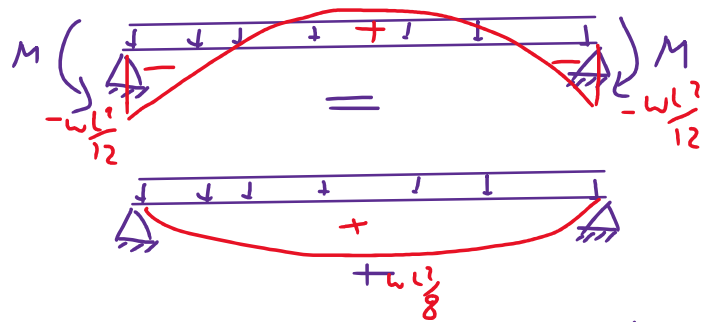




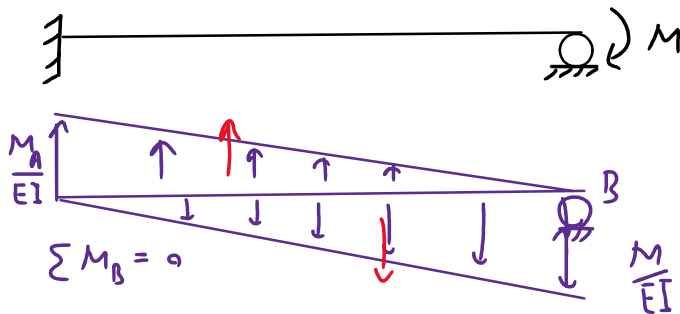
$$\sum F_y = 0$$

$$\frac{2}{3} \left(\frac{wL}{4} \right) (L) - \left(\frac{M}{EI} \right) (L) = 0$$

$$M_A = M_B = -\frac{wL^2}{12}$$



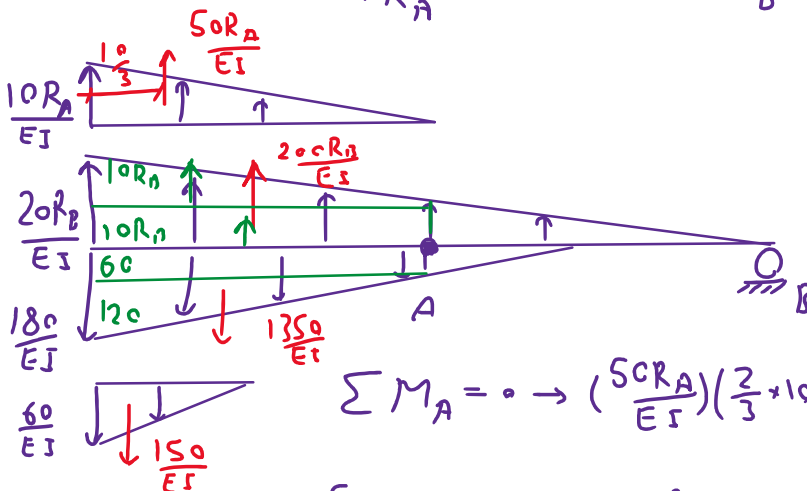
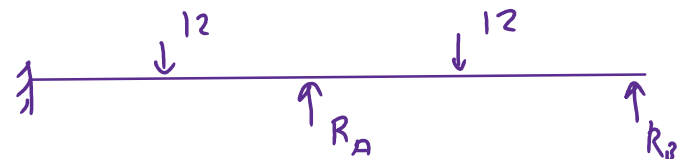
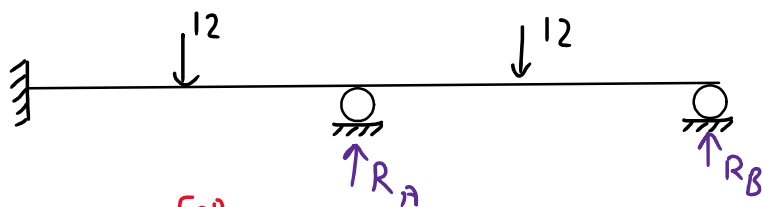
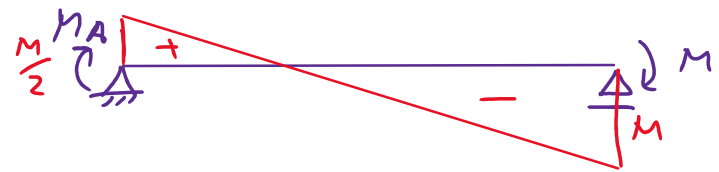
مثال :



$$\sum M_B = 0$$

$$\frac{1}{2} \left(\frac{M_A}{EI} \right) (L) \left(\frac{2}{3} L \right) - \frac{1}{2} \left(\frac{M}{EI} \right) (L) \left(\frac{L}{3} \right) = 0$$

$$M_A = \frac{M}{2}$$



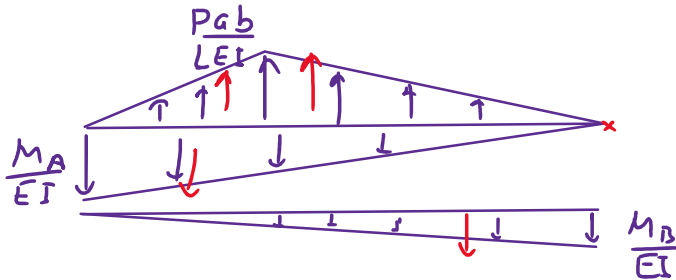
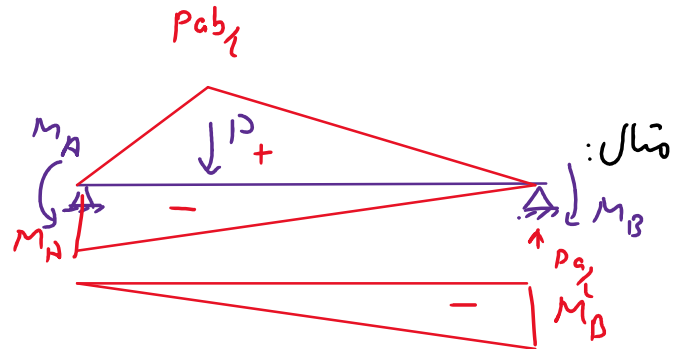
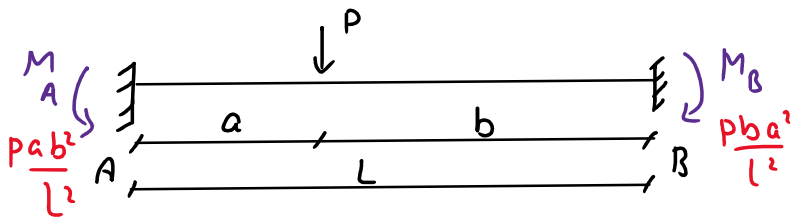
$$\sum M_A = 0 \rightarrow \left(\frac{50R_A}{EI} \right) \left(\frac{2}{3} \times 10 \right) + \left(\frac{10R_B}{EI} \right) (10) (5) + \frac{1}{2} \left(\frac{10R_B}{EI} \right) (10) \left(\frac{2}{3} \times 10 \right)$$

$$- \left[\left(\frac{60}{EI} \right) (10) (5) + \frac{1}{2} \left(\frac{120}{EI} \right) (10) \left(\frac{2}{3} \times 10 \right) \right] - \left(\frac{150}{EI} \right) \left(\frac{25}{3} \right) = 0$$

$$* 333.33 R_A + 833.33 R_B - 8250 = 0$$

$$\Sigma M_D = 0 \rightarrow \left(\frac{50 R_A}{EI}\right)(16.67) + \left(\frac{200 R_B}{EI}\right)\left(\frac{2}{3} \times 20\right) - \left(\frac{1350}{EI}\right)(15) - \left(\frac{150}{EI}\right)(18.33) = 0$$

$$* 833.33 R_A + 2666.67 R_B - 23000 = 0$$



$$\Sigma F_y = 0 \rightarrow \frac{1}{2} \left(\frac{Pab}{LEI} \right) (L) - \frac{1}{2} \left(\frac{M_A}{EI} \right) (L) - \frac{1}{2} \left(\frac{M_B}{EI} \right) (L) = 0$$

$$\Sigma M_D = 0 \rightarrow \frac{1}{2} \left(\frac{Pab}{LEI} \right) (b) \left(\frac{2}{3} b \right) + \frac{1}{2} \left(\frac{Pab}{LEI} \right) (a) \left(b + \frac{a}{3} \right) - \left(\frac{M_A L}{2EI} \right) \left(\frac{2L}{3} \right) - \left(\frac{M_B L}{2EI} \right) \left(\frac{1}{3} L \right) = 0$$

$$\begin{cases} M_A + M_B = \frac{Pab}{L} \\ 2M_A + M_B = \frac{Pab}{L} + \frac{Pab^2}{L^2} \end{cases}$$

$$M_A = \frac{Pab^2}{L^2}$$

$$M_B = \frac{Pba^2}{L^2}$$

جبر نریس

$$6 \times \frac{Pab}{L} \left(\frac{b^2}{3} + \frac{ab}{2} + \frac{a^2}{6} \right) = \frac{L^2}{3} (M_A + M_B)$$

$$\frac{Pab}{L^3} (2b^2 + 3ab + a^2) = 2(M_A + M_B)$$

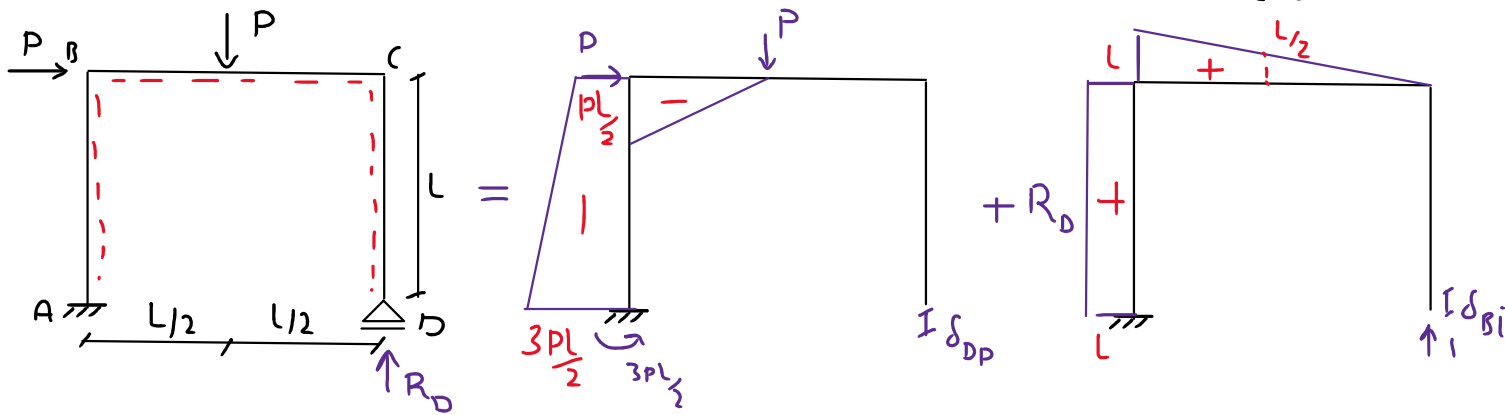
$$(a+b)^2 + b^2 + a^2 = L^2 + bL$$

$$\frac{Pab}{L^3} (L+b) = 2(M_A + \frac{M_B}{2})$$

$$Pab^2 \quad Pab \quad Pab \quad L$$

$$-\frac{Pcb^2}{L^2} + \frac{Pcb}{L} = \frac{Pcb}{L} \left(\frac{1 - \frac{b}{L}}{\frac{L-b}{L}} \right) a$$

مثال: یکسازه‌ی کلیه‌گاه D را به دست آورید.



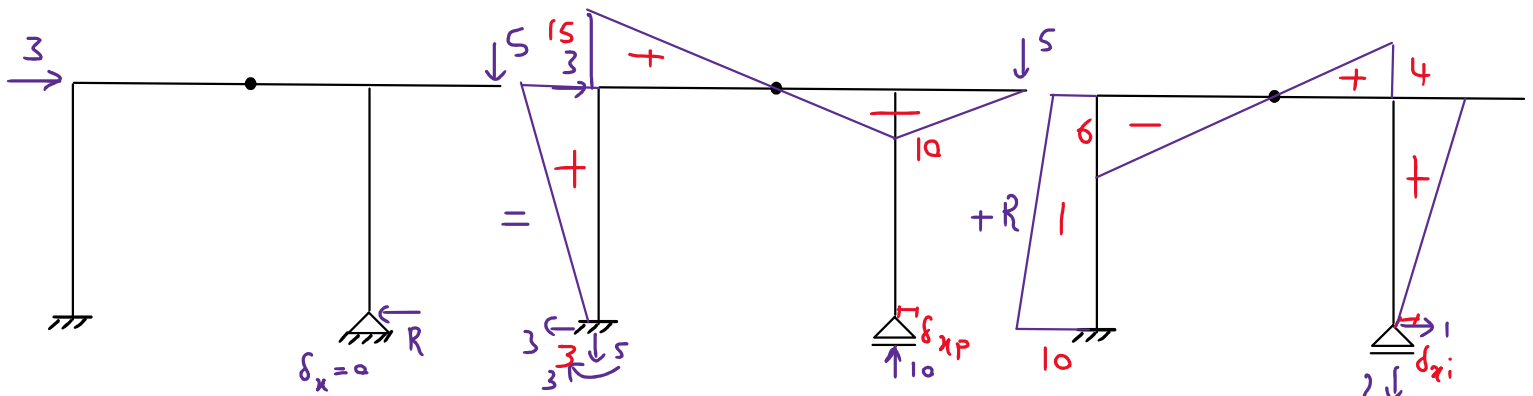
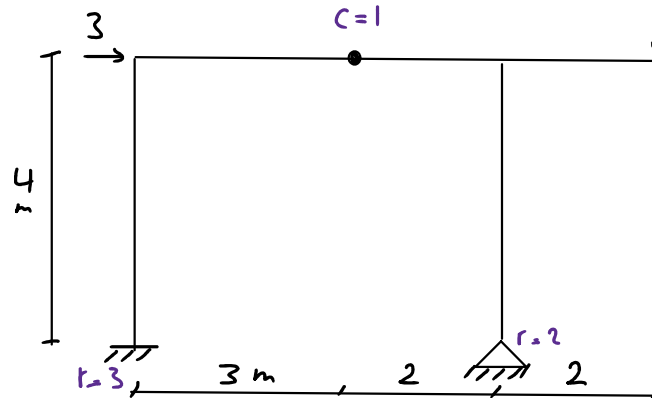
$$\delta_D = 0 \rightarrow \delta_{Dp} + R_D \delta_{Di} = 0$$

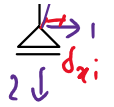
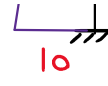
$$1 \times \delta_{Dp} = \int \frac{mM}{EI} dx = \left(\frac{L}{6}\right) \left[2 \left(-\frac{PL}{2}\right)(L) + \left(-\frac{PL}{2}\right)\left(\frac{L}{2}\right) \right] + (L)(-PL)(L) = -\frac{53}{48} \frac{PL^3}{EI}$$

$$1 \times \delta_{Di} = \int \frac{m^2}{EI} dx = \left(\frac{L}{3}\right)(L^2) + (L^2)(L) = \frac{4}{3} \frac{L^3}{EI}$$

$$-\frac{53}{48} \frac{PL^3}{EI} + R_D \left(\frac{4}{3} \frac{L^3}{EI}\right) = 0 \rightarrow R_D = \frac{53}{64} P$$

مثال: دیاگرام لنگر قاف شکل زیر را رسم نمایید.



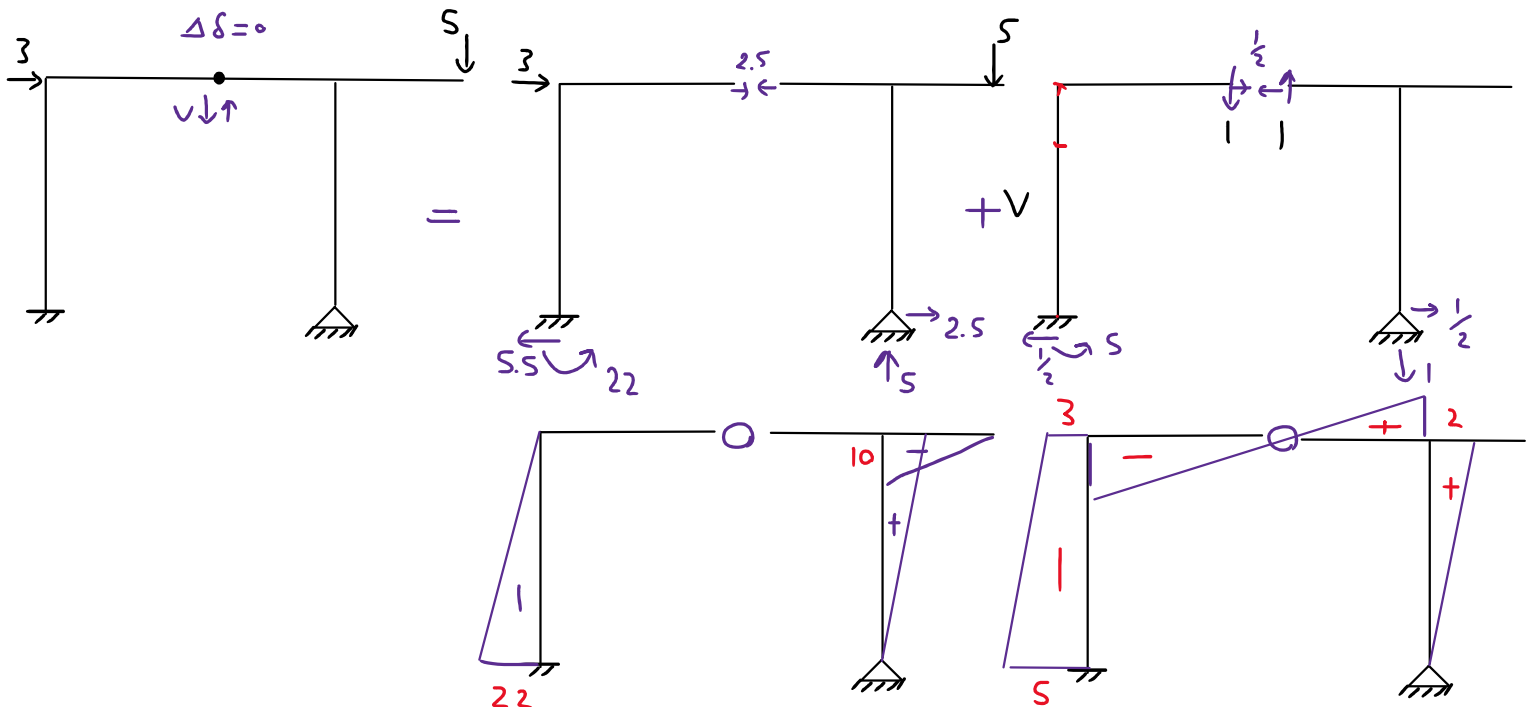
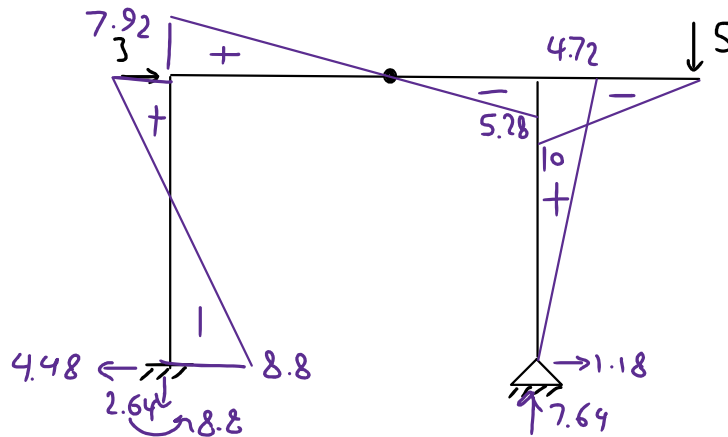


$$1 \times \delta_{xp} = \int \frac{mM}{EI} dz = \left(\frac{4}{6}\right) [(3)(-10) + 4(9)(-8) + (15)(-6)] + \left(\frac{3}{3}\right)(15)(-6) + \left(\frac{2}{3}\right)(-10)(4) =$$

$$\delta_{xp} = \frac{-388.67}{EI}$$

$$1 \times \delta_{xi} = \int \frac{m^2}{EI} dx = \left(\frac{4}{6}\right) [(-10)^2 + 4(-8)^2 + (-6)^2] + \left(\frac{3}{3}\right)(-6)^2 + \left(\frac{2}{3}\right)(4)^2 + \left(\frac{4}{3}\right)(4)^2 = \frac{329.33}{EI}$$

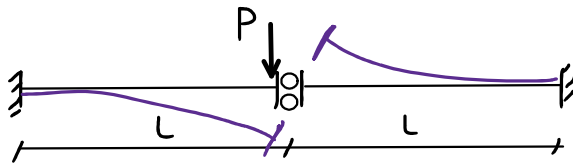
$$\delta_{xp} + R \delta_{xi} = 0 \rightarrow \frac{-388.67}{EI} + R \frac{329.33}{EI} = 0 \rightarrow \boxed{R = 1.18} \rightarrow$$



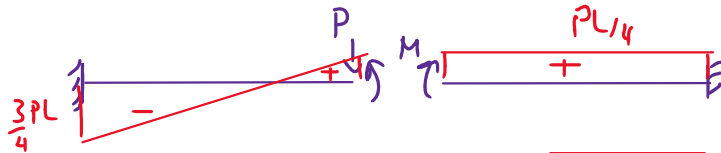
$$1 \times \Delta \delta_p = \int \frac{mM}{EI} dx = \left(\frac{4}{3}\right) [2(-22)(-5) + (-22)(-3)] + \left(\frac{4}{3}\right)(10)(2) = \frac{217.3}{EI}$$

$$I \times \Delta \delta_i = \int \frac{m^2}{EI} dx = \left(\frac{4}{6}\right) [(-5)^2 + 4(-4)^2 + (-3)^2] + \left(\frac{3}{3}\right)(-3)^2 + \left(\frac{2}{3}\right)(2)^2 + \left(\frac{4}{3}\right)(2^2) = \frac{82.3}{EI}$$

$$\frac{217.3}{EI} + v \frac{82.3}{EI} = 0 \rightarrow v = -2.64$$



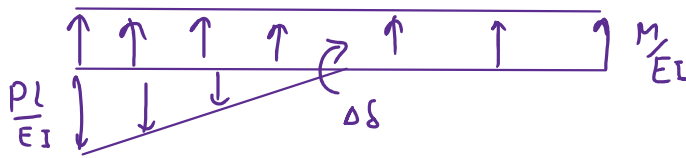
مثال: مطلوب است تحلیل نیروی زبر.



$$\theta_L = \theta_R$$

سازگار تغییر شکل ها

$$\frac{PL^2}{2EI} - \frac{ML}{EI} = \frac{ML}{EI} \rightarrow M = \frac{PL}{4}$$



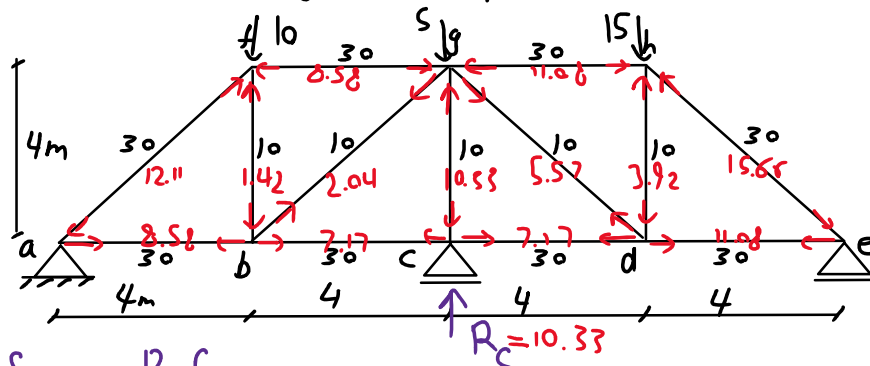
تعداد نیروی زبر

$$\theta \quad v$$

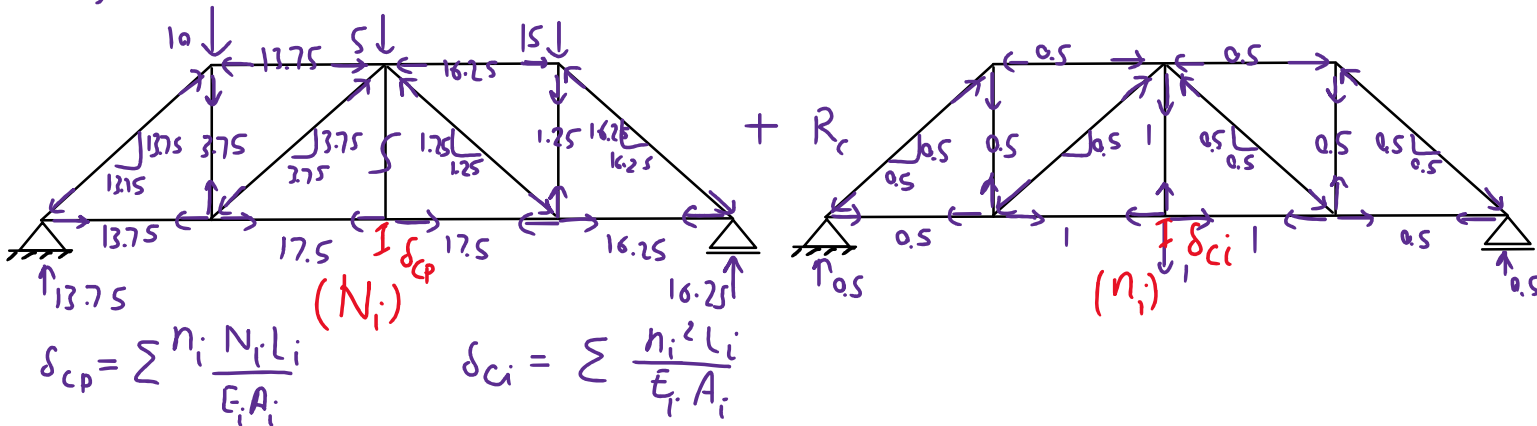
$$\delta_1, \delta_2 \quad M_1, M_2$$

$$\sum F_y = 0 \rightarrow \frac{M}{EI}(2L) - \frac{1}{2} \left(\frac{PL}{EI} \right) (L) = 0 \rightarrow M = \frac{PL}{4}$$

مثال : خرابی شکل زیر را تحلیل کنید. (مساحت اعضا بر حسب cm^2 درون آن‌ها نوشته شده است)



$$\delta_{cy} = 0 \rightarrow \delta_{cp} + R_c \delta_{ci} = 0$$



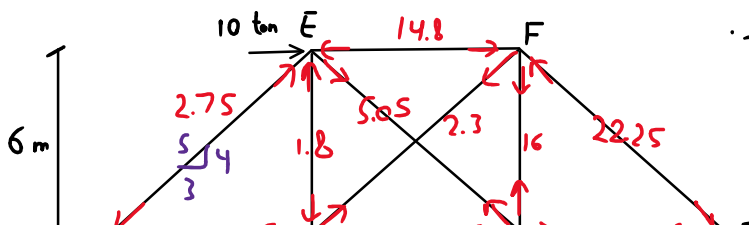
$$\delta_{cp} = \frac{1819.41}{E} \quad \delta_{ci} = \frac{176.09}{E}$$

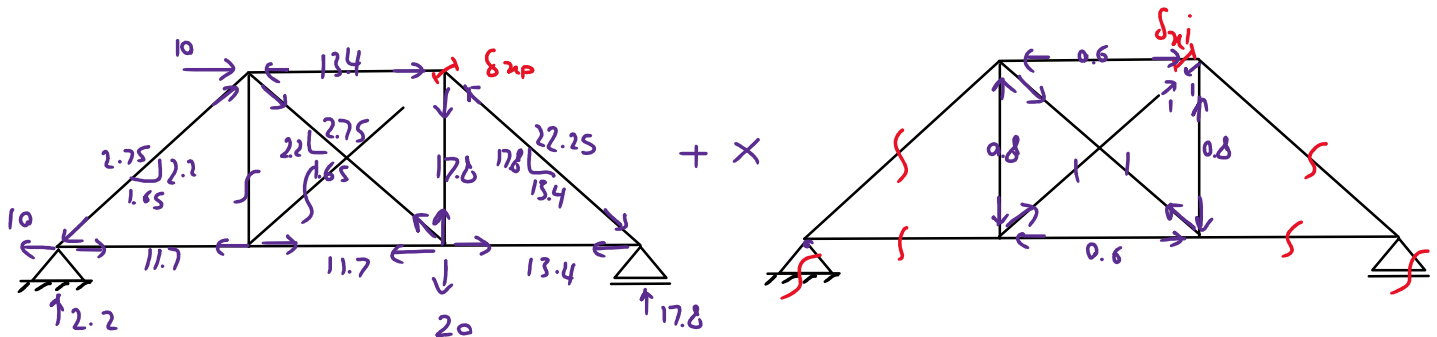
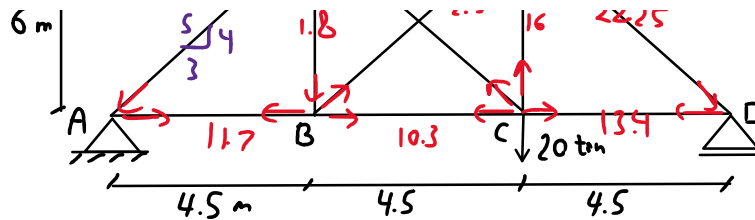
$$\frac{1819.41}{E} + R_c \frac{176.09}{E} = 0$$

$$R_c = -10.33$$

Element	Ai	Li	ni	Ni	$n_i \cdot N_i \cdot L_i / A_i$	$n_i^2 \cdot L_i / A_i$	$N_i + R_c \cdot n_i$
ab	30	400	0.5	13.75	91.67	3.33	8.58
bc	30	400	1	17.5	233.33	13.33	7.17
cd	30	400	1	17.5	233.33	13.33	7.17
de	30	400	0.5	16.25	108.33	3.33	11.08
fg	30	400	-0.5	-13.75	91.67	3.33	-8.58
gh	30	400	-0.5	-16.25	108.33	3.33	-11.08
af	30	566	-0.71	-19.45	260.54	9.51	-12.11
bf	10	400	0.5	3.75	75.00	10.00	-1.42
bg	10	566	-0.71	-5.3	212.99	28.53	2.04
cg	10	400	1	0	0.00	40.00	-10.33
dg	10	566	-0.71	-1.77	71.13	28.53	5.57
dh	10	400	0.5	1.25	25.00	10.00	-3.92
he	30	566	-0.71	-23	308.09	9.51	-15.66
					1819.41	176.09	-10.33

مثال : خرابی شکل زیر را تحلیل کنید.





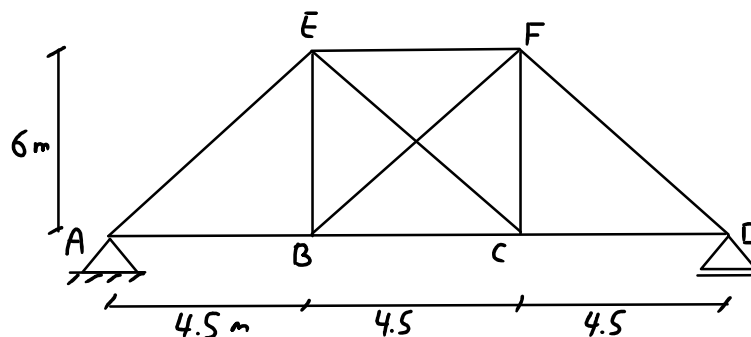
$$\delta x_p + X \delta x_i = 0$$

$$1 \times \delta x_p = \sum \frac{n_i N_i L_i}{EA} = \frac{1}{EA} \left[0 + (-13.4)(-0.6)(4.5) + (17.8)(-0.8)(6) + (11.7)(-0.6)(4.5) + (2.75)(1)(7.5) + 0 \right] = \frac{-59.85}{EA}$$

$$1 \times \delta x_i = \sum \frac{n_i^2 L_i}{EA} = \frac{1}{EA} \left[2 \times (-0.6)^2 (4.5) + 2 \times (-0.8)^2 (6) + 2 \times (1)^2 (7.5) \right] = \frac{25.92}{EA}$$

$$\frac{-59.85}{EA} + X \frac{25.92}{EA} = 0 \rightarrow X = 2.3 \text{ ton}$$

مثال (الف) آلریال خارجی خپا (AE, EF, FD) به میزان 33 انرژیش دسایابد، بزرگترین خپا را به دست آورید.
(ب) در صورتی که علاوه بر تغییر دما، عضو BF در اثر خطای مونتاژ 2m کوتاه تر اجرا شده باشد، بزرگترین انحراف را در نقطه E به اندازه 3



$$\alpha = 1.2 \times 10^{-5} \text{ } ^\circ \text{C}^{-1}$$

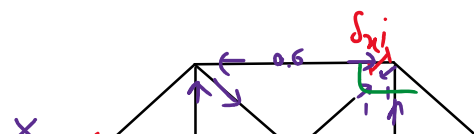
$$E = 2 \times 10^6 \text{ kg/cm}^2$$

$$A = 50 \text{ cm}^2$$

$$EA = 100 \times 10^6 \text{ kg} = 10^5 \text{ ton}$$

$$\delta x_p + X \delta x_i = 0$$

$$+ 0.003$$



$$\delta_{iP} + X \delta_{xi} = 0$$

$$1 \times \delta_{xP} + 0 \times 0.03 = \sum \eta_i (\alpha L_i \Delta T_i)$$

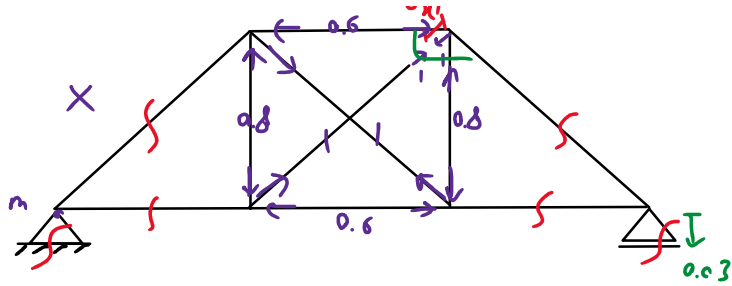
$$\delta_{xP} = 0 + 0 + (-0.6)(1.2 \times 10^{-5} \times 4.5 \times 33) = -0.001 \text{ m} = -1 \text{ mm}$$

$$1 \times \delta_{xi} = \frac{2925 \text{ ton} \cdot \text{m}}{EA} = \frac{2592}{10^5} = 0.26 \times 10^{-3} \text{ m} = 0.26 \text{ mm}$$

$$-1 + X(0.26) = 0 \rightarrow X = 3.85 \text{ ton}$$

$$1 \times \delta_{xP} = (-0.6)(1.2 \times 10^{-5} \times 4.5 \times 33) + 1 \times (-0.002) = -0.001 - 0.002 = -0.003 \text{ m}$$

$$-3 + X(0.26) = 0 \rightarrow X = 11.5 \text{ ton}$$



طبق قضیه دوم کاستیلیانو $\delta \frac{\partial U}{\partial P_n} = \delta_n$ است.

در تحلیل سازه های نامعین قیدهای اضافی، بزرگسای محمول اضافی و تغییر مکان معلوم (معمولاً برابر صفر) به وجود می آید. بنابراین می توان محمولات اضافی را از معادله $\frac{\partial U}{\partial P_n} = 0$ به دست آورد.

یعنی یکنس محمول اضافی باید مقدار صفر را از زیر یکنس داخلی در سازه حد اول شود.

* در قضیه کاستیلیانو، نسبت تکیه گاه، نقص صفر و تغییر مکان داریم. (محدودیت روش)

* این روش نسبت به روش سازگاری تغییر شکل ما، تعداد دانتهای بیشتری دارد. (مزیت)

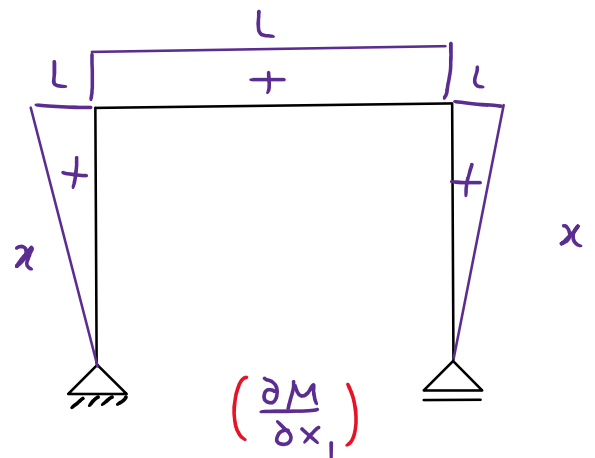
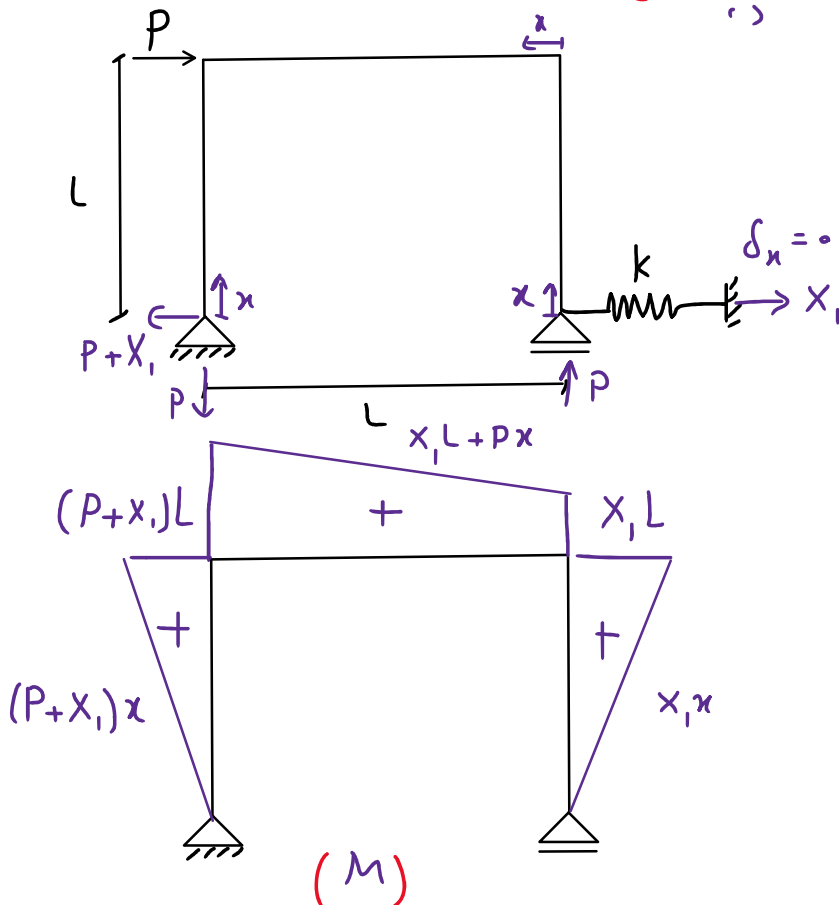
$$\begin{aligned} \delta_x = 0 &\rightarrow \frac{\partial U}{\partial R_x} = 0 \\ \delta_y = 0 &\rightarrow \frac{\partial U}{\partial R_y} = 0 \\ \delta_\theta = 0 &\rightarrow \frac{\partial U}{\partial M} = 0 \end{aligned}$$

مثال: مطلوب است عکس العمل تکیه گاه میانه.

$$\frac{\partial U}{\partial x_1} = 0$$

$$U = \frac{1}{2} \int \frac{M^2}{EI} dx + \frac{1}{2} \frac{F^2}{k}$$

$$\frac{\partial U}{\partial x_1} = \int \frac{M}{EI} \left(\frac{\partial M}{\partial x_1} \right) dx + \frac{F}{k} \frac{\partial F}{\partial x_1} = 0$$



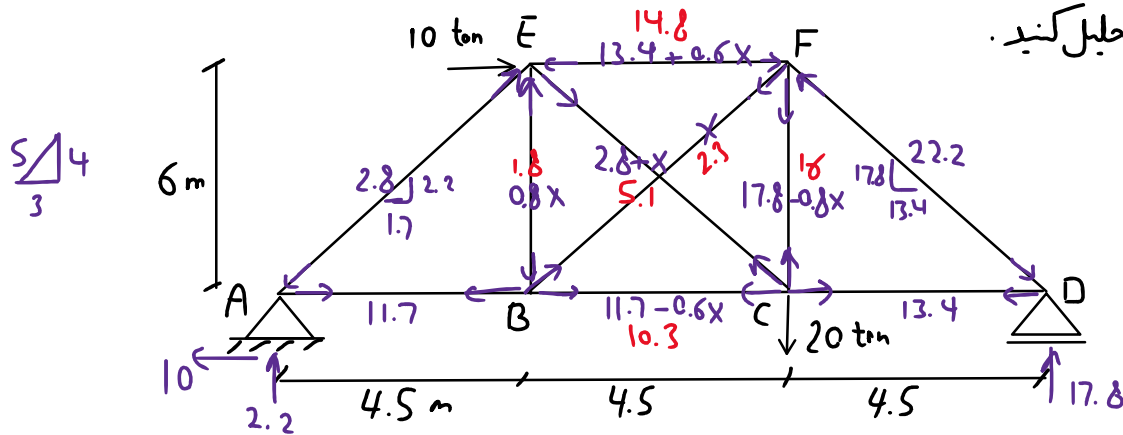
$$\frac{\partial U}{\partial x_1} = \int \frac{M}{EI} \left(\frac{\partial M}{\partial x_1} \right) dx + \frac{F}{k} \frac{\partial F}{\partial x_1} = 0$$

$$\frac{1}{EI} \left[\left(\frac{L}{3} \right) (P + X_1) L \left(\frac{L}{3} \right) + \left(\frac{L}{3} \right) (X_1) L \left(\frac{L}{3} \right) + (L) \left(X_1 L + \frac{PL}{2} \right) \left(\frac{L}{3} \right) \right] + \left(\frac{X_1}{K} \right) (1) = 0$$

$$\frac{1}{EI} \left[\frac{PL^3}{3} + \frac{X_1 L^3}{3} + \frac{X_1 L^3}{3} + X_1 L^3 + \frac{PL^3}{2} \right] + \frac{X_1}{K} = 0$$

$$\frac{1}{EI} \left[\frac{5}{6} PL^3 + \frac{5}{3} X_1 L^3 \right] + \frac{X_1}{K} = 0 \rightarrow X_1 = \frac{-P}{2 + \frac{6}{5} \left(\frac{EI}{L^3 K} \right)}$$

مثال : خرابی‌ها را تحلیل کنید.



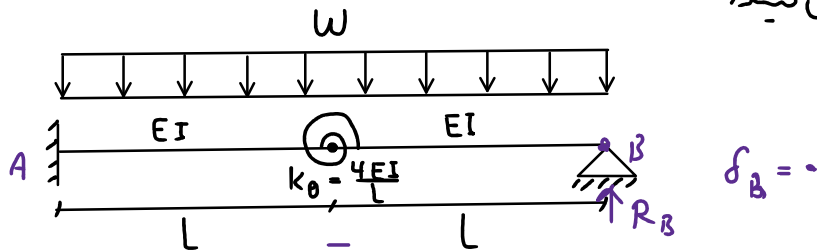
$$\frac{\partial U}{\partial X} = 0 \rightarrow U = \frac{1}{2} \sum \frac{N^2 L}{EA} \rightarrow \frac{\partial U}{\partial X} = \sum \frac{N}{EA} \left(\frac{\partial N}{\partial X} \right) L = 0$$

$$\frac{1}{EA} \left[(-0.8X)(-0.8)(6) + (11.7 - 0.6X)(-0.6)(4.5) + (17.8 - 0.8X)(-0.8)(6) + \right. \\ \left. - (13.4 + 0.6X)(-0.6)(4.5) + (X)(1)(7.5) + (2.8 + X)(1)(7.5) \right] = 0$$

$$-58.86 + 25.92 X = 0 \rightarrow X = 2.3 \text{ ton}$$

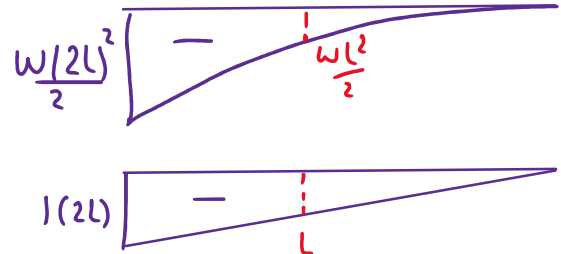
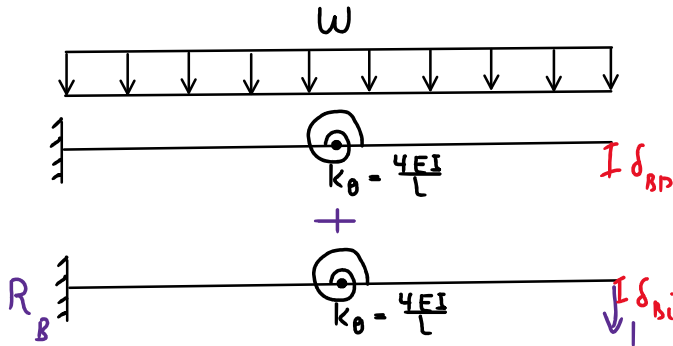
مثال: تیر شکل زیر را به روش های مختلف تحلیل کنید.

① سازه را تغییر شکل ما



$$\delta_B = 0$$

(الف) بار واحد



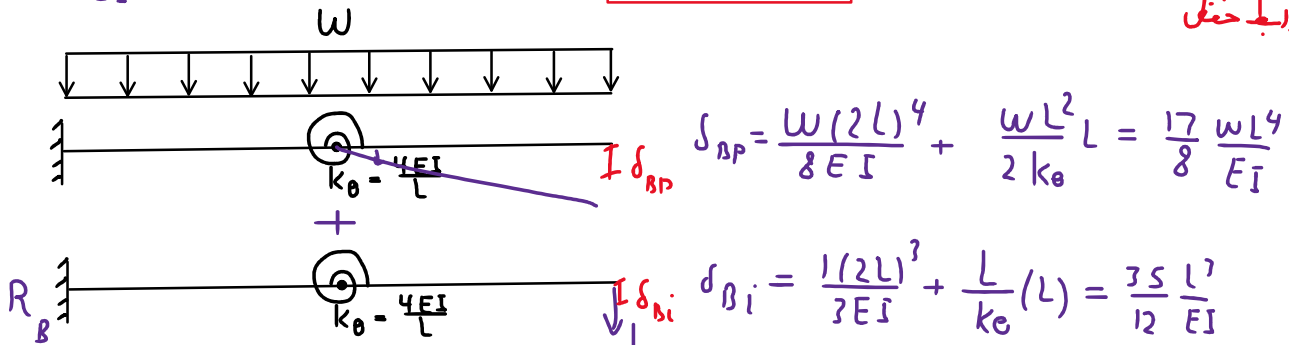
$$\delta_{Bp} + R_B \delta_{Bi} = 0 \quad 1 \times \delta_{Bp} = \int \frac{m M}{EI} dx + m \frac{M}{k_0}$$

$$1 \times \delta_{Bp} = \frac{1}{EI} \left(\frac{2L}{2} \right) \left[(2wL^2)(-2L) + 4 \left(\frac{wL^2}{2} \right) (-L) + 0 \right] + (L) \left(\frac{wL^2}{2k_0} \right) = \frac{17}{8} \frac{wL^4}{EI}$$

$$1 \times \delta_{Bi} = \frac{1}{EI} \left(\frac{2L}{2} \right) (2L)^2 + \frac{L^2}{k_0} = \left(\frac{8}{3} + \frac{1}{4} \right) \frac{L^3}{EI} = \frac{35}{12} \frac{L^3}{EI}$$

$$\frac{17}{8} \frac{wL^4}{EI} + R_B \left(\frac{35}{12} \frac{L^3}{EI} \right) = 0 \rightarrow R_B = \frac{51}{70} wL$$

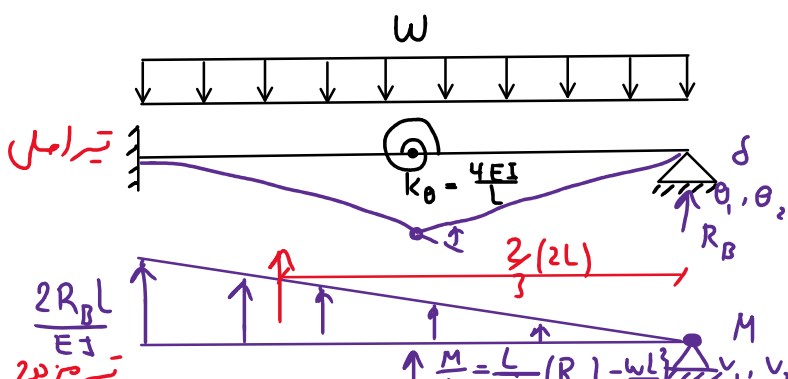
(ب) ردیاب حفظ



$$\delta_{Bp} = \frac{w(2L)^4}{8EI} + \frac{wL^2}{2k_0} L = \frac{17}{8} \frac{wL^4}{EI}$$

$$\delta_{Bi} = \frac{1}{3EI} (2L)^3 + \frac{L}{k_0} (L) = \frac{35}{12} \frac{L^3}{EI}$$

② معادلات تعادل تیر مزدوج



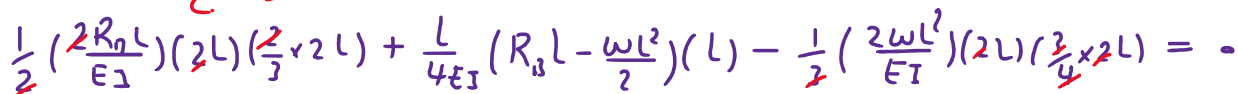


Diagram of a beam of total length $2L$. The beam is supported by a fixed support at the left end and a roller support at the right end. A uniformly distributed load W acts downwards over the entire length of the beam. A clockwise moment $k_0 = \frac{4EI}{L}$ is applied at the center of the beam. The beam is divided into two segments of length L each. The reaction at the roller support is labeled R_R .

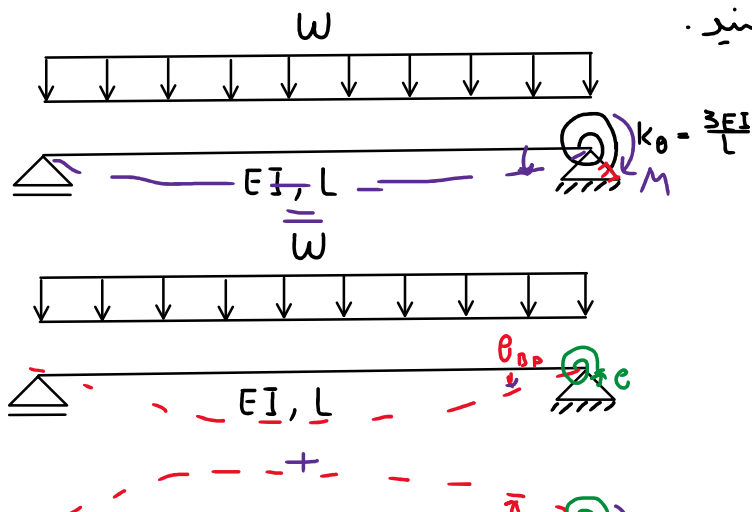
$$\frac{\partial U}{\partial R_B} = \int \frac{M}{EI} \left(\frac{\partial M}{\partial R_B} \right) dx + \frac{M}{k_\theta} \frac{\partial M}{\partial R_B} = 0$$

$$\text{نیر } M = R_0 x - \frac{\omega x^2}{2} \quad \frac{\partial M}{\partial R_0} = x$$

$$\frac{1}{EI} \int_0^{2L} (R_B x - \frac{w x^2}{2}) dx + \frac{L}{4EI} (R_B L - \frac{w L^2}{2}) = 0$$

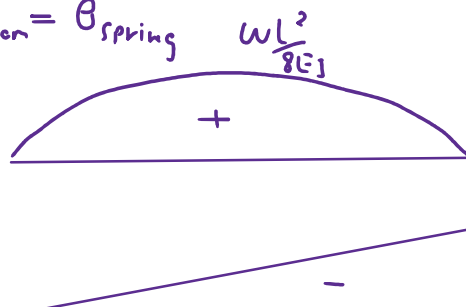
$$R_B = \frac{51}{70} \omega L$$

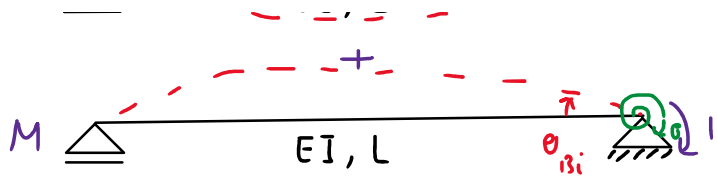
مثال: تر شکل زیر را به روش های مختلف تحلیل کنید.




 $\theta = 0$
 $\theta_{\text{beam}} = \theta_{\text{spring}}$

① سازشوں بغیر شکل ما





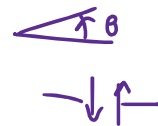
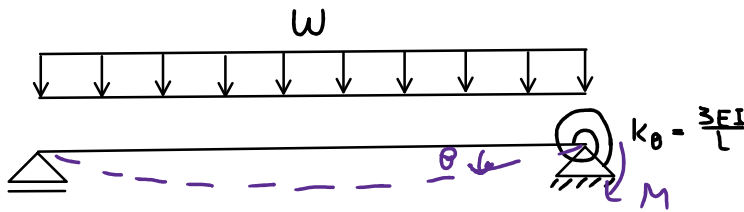
$$1 \times \theta_{Bp} = \int \frac{M}{EI} dx = \left(\frac{L}{3}\right) \left(\frac{wL^2}{8EI}\right) (-1) = -\frac{wL^3}{24EI}$$

$$1 \times \theta_{Bi} = \int \frac{M}{EI} dx = \left(\frac{L}{3}\right) (-1)^2 = \frac{L}{3EI}$$

$$-\frac{wL^3}{24EI} + \frac{ML}{3EI} = -\frac{ML}{3EI} \rightarrow \boxed{M = \frac{wL^2}{16}}$$

$$-\frac{wL^2}{24EI} + M\left(\frac{L}{3EI} + \frac{1}{3EI}\right) = 0$$

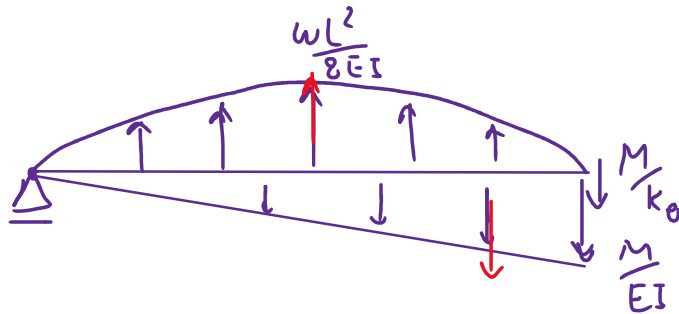
$$M = \frac{wL^2}{16}$$



تیر مزدوج

$$\delta = 0$$

$$\theta = \frac{M}{k_0}$$



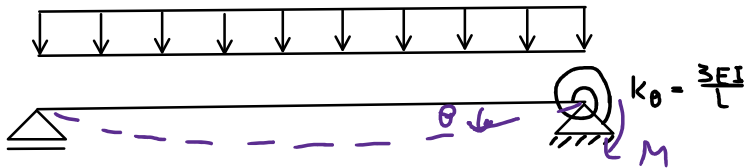
$$M = 0$$

$$V = \frac{M}{k_0}$$

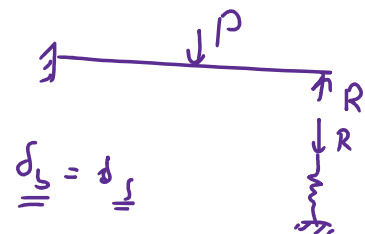
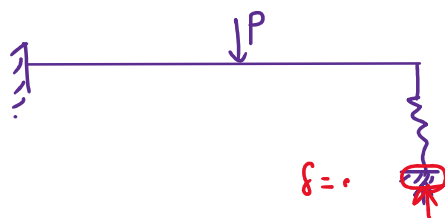
$$\frac{2}{3} \left(\frac{wL^2}{8EI}\right) \left(\frac{L}{2}\right) - \frac{M}{k_0} (L) - \frac{1}{2} \left(\frac{M}{EI}\right) \left(\frac{2L}{3}\right) = 0$$

$$\frac{wL^4}{24} - \frac{ML^2}{3} - \frac{ML^2}{3} = 0 \rightarrow \boxed{M = \frac{wL^2}{16}}$$

روش توزیع لنگر کراس



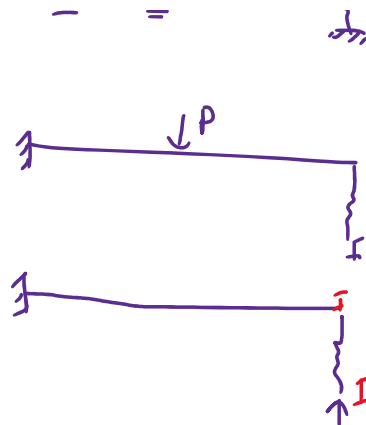
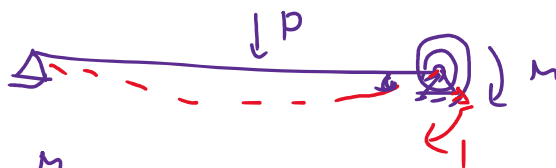
$$M = \frac{wL^2}{8} \times \frac{1}{2} = \frac{wL^2}{16}$$



$$- \frac{PL^3}{6EI} + \frac{RL^3}{6EI} - \frac{RL^3}{6EI} = 0$$

$$-\circ \frac{PL^3}{3EI} + \frac{RL^3}{3EI} = -\frac{R}{k} \quad R\left(\frac{1}{k_b} + \frac{1}{k_r}\right)$$

$$\circ \frac{DL^3}{EI} - \circ \frac{ML}{EI} = \frac{M}{k_\theta}$$



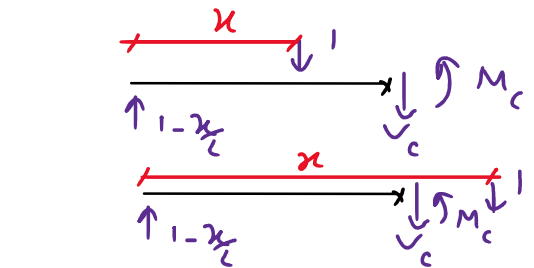
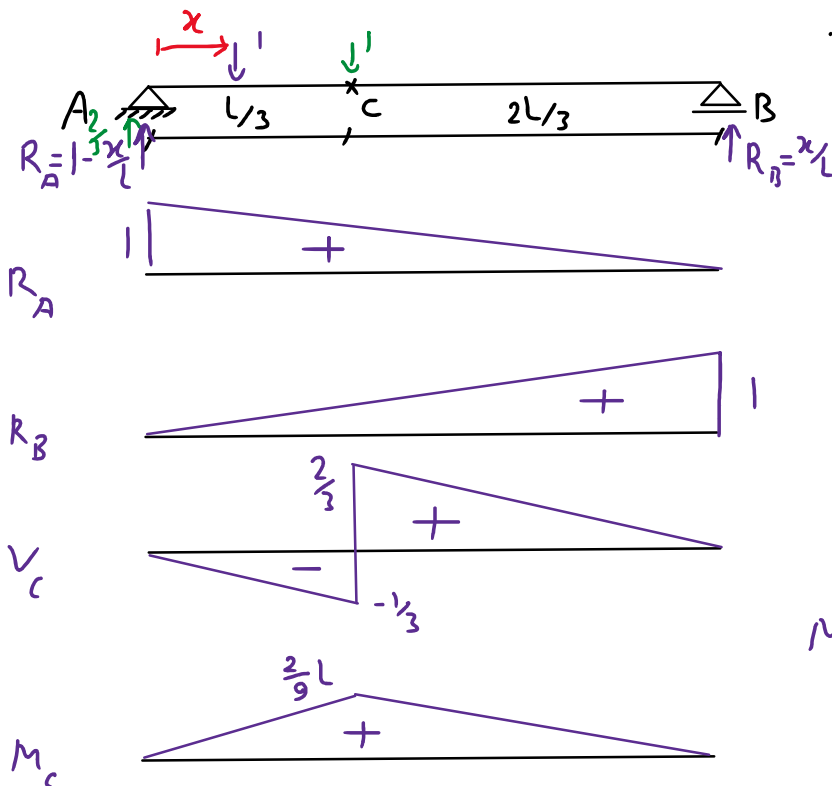
فصل ششم: خط تأثیر سازه های معین

خط تأثیر تابعی است که مقدار یک نیرو (عکس العمل یک گام R ، یزوس برین V ، گند خشی M) را در متن بار واحد در نقاط مختلف قرار گرفته است، نشان می دهد.

برای رسم خط تأثیر سازه های معین در روش وجود دارد:

- ۱- نوشتن معادلات تعادل
- ۲- روش کار مجازی (اصل مولر-بریلو)

مثال: خط تأثیر R_A ، R_B ، V_c ، M_c داریم بنابراین.



$$V_c = \begin{cases} 1 - \frac{x}{L} & 0 \leq x \leq \frac{L}{3} \\ -\frac{x}{L} & \frac{L}{3} < x \leq L \end{cases}$$

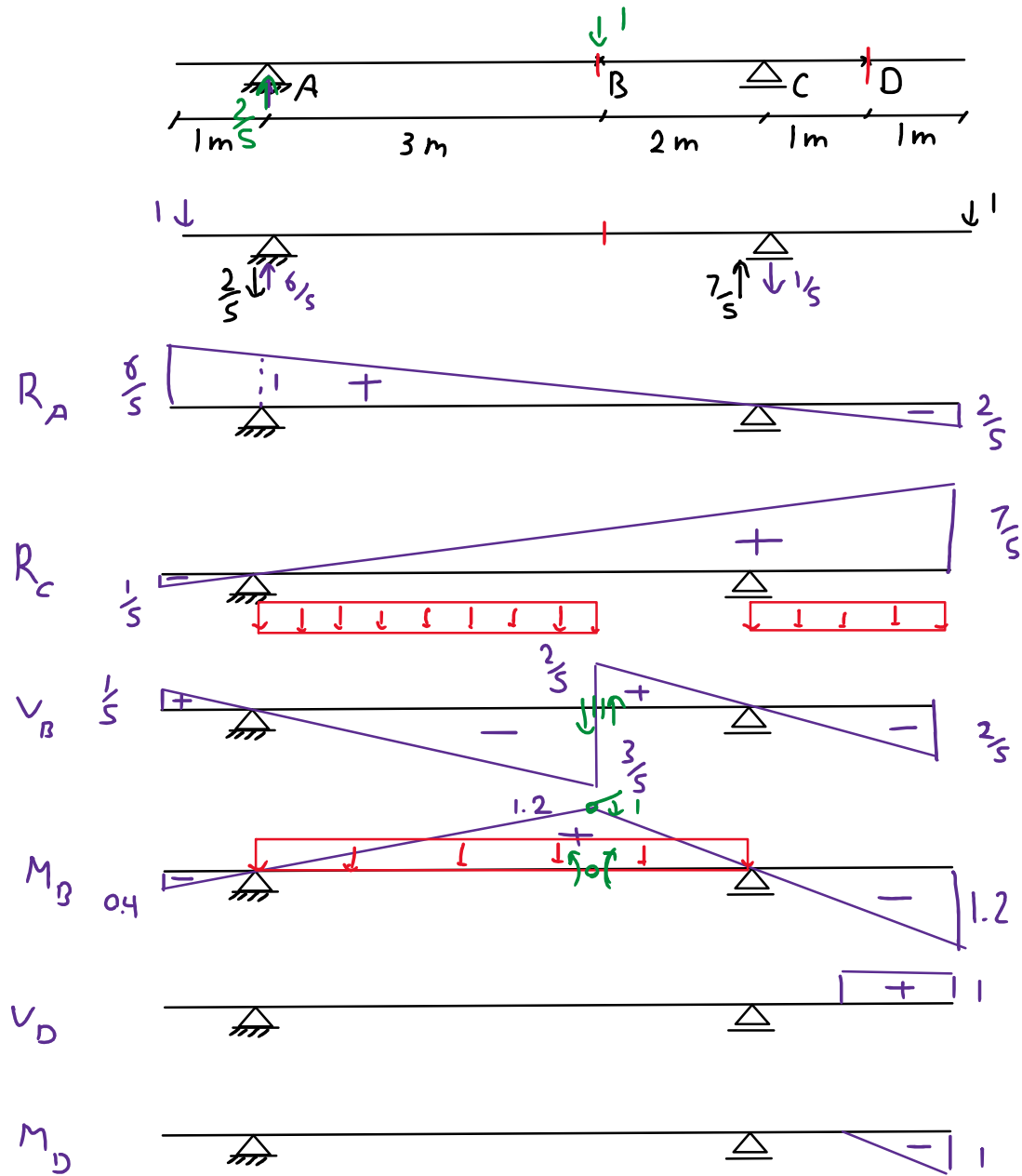
$$M_c = \begin{cases} (1 - \frac{x}{L})(\frac{L}{3}) = \frac{2}{3}x & 0 \leq x \leq \frac{L}{3} \\ (\frac{x}{L})(\frac{L}{3}) = \frac{1}{3}(L - x) & \frac{L}{3} < x \leq L \end{cases}$$

* مقدار خط تأثیر R_A ، V_c ، M_c در هر نقطه دلخواه x ، نشان دهنده عکس العمل یک گام، نیرو برش و گند خشی نقطه ثابت C روی تیر است، متناهی که بار واحد در نقطه x قرار داده شده است.

* خط تأثیر سازه های معین به صورت چند خط است.

* برای رسم خط تأثیر به روش معادلات تعادل، لازم نیست به صورت بالا تابع نوشته شود، بلکه تعیین نقاط کلیدی در حل کردن آنها کافی است.

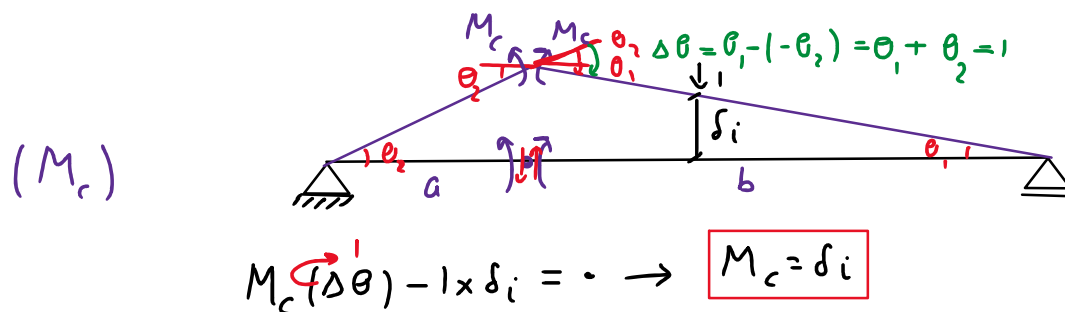
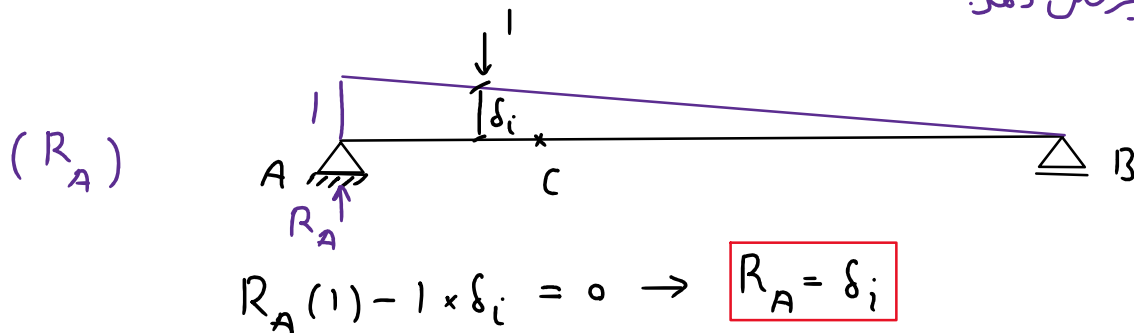
مثال: مطلوب است اثریم خط تأثیر R_c ، R_D ، V_D ، M_D ، V_B ، M_B .



در سال ۱۸۸۶ میلادی، مولر-برسلر یک تکنیک قدیمی برای رسم خط تأثیر ارائه داد.

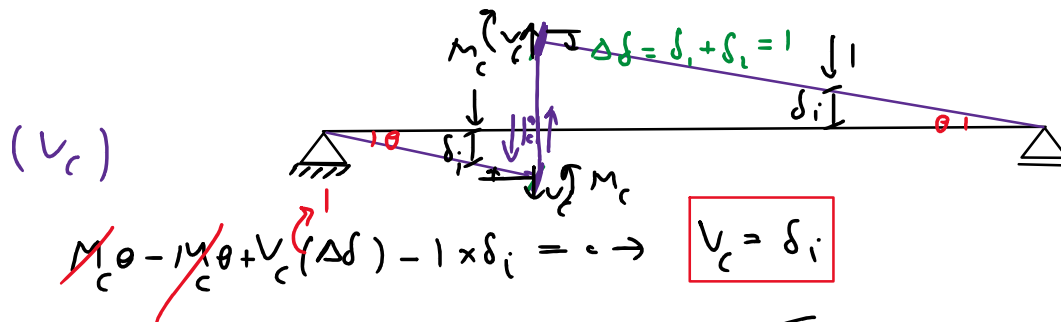
اصل مولر-برسلر:

خط تأثیر یک نیرو برابر است با تابع تغییر شکل سازه وقتی که نیروی مورد نظر از سازه حذف شده و سازه آزاد شده تحت اثر همان نیرو تغییر شکل دهد.

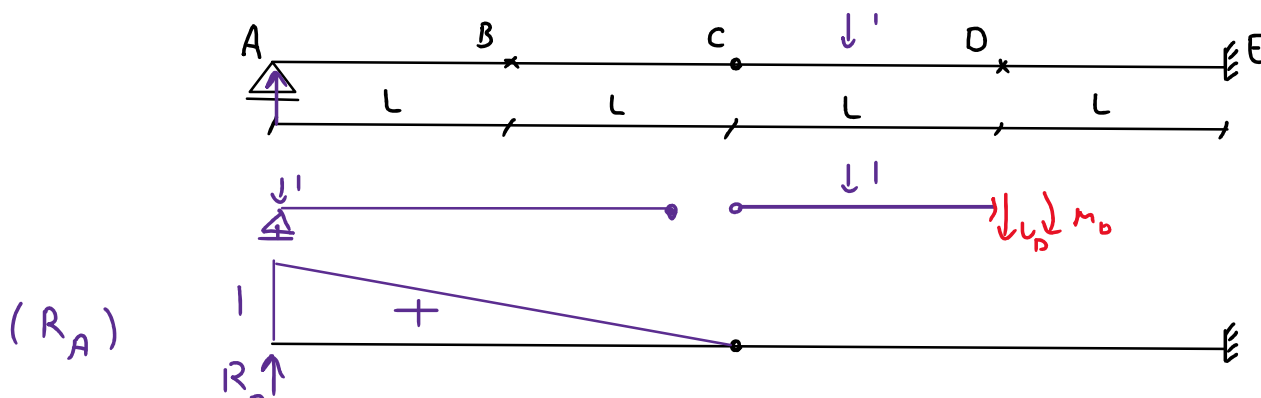


$$\frac{y}{a} + \frac{y}{b} = 1$$

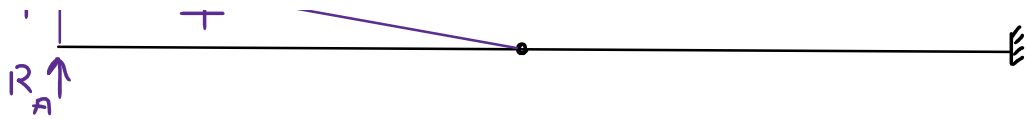
$$y = \frac{ab}{a+b}$$



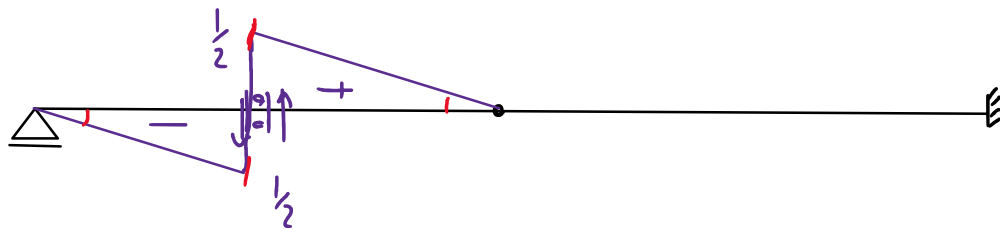
مثال: خط تأثیر R_A ، M_B ، V_B ، M_D را رسم کنید.



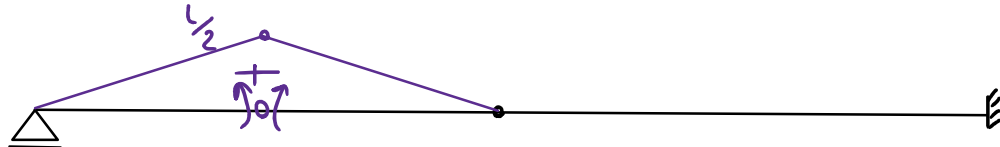
(R_A)



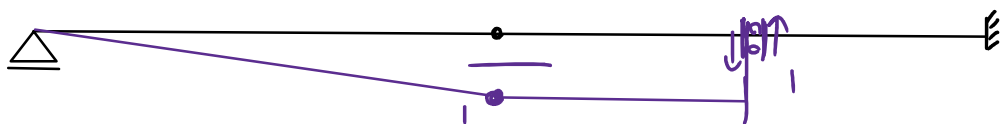
(V_B)



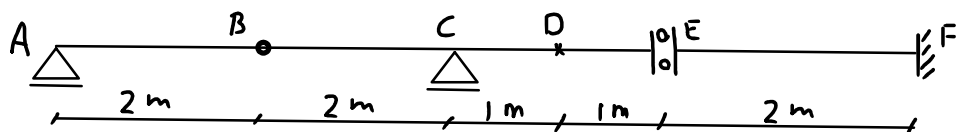
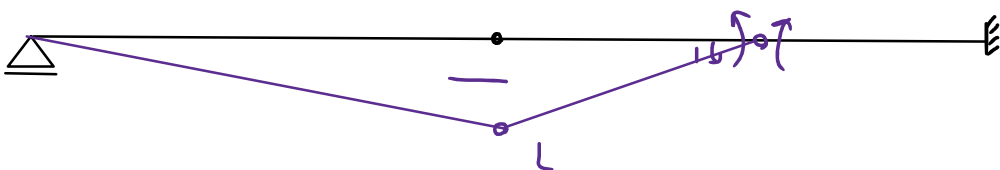
(M_B)



(V_D)

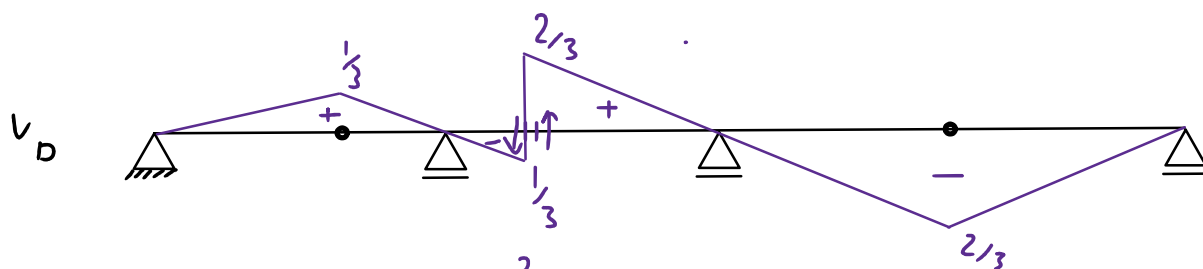
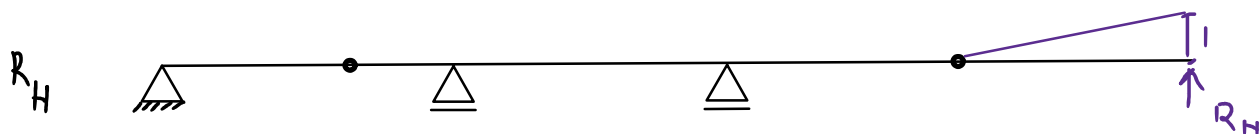
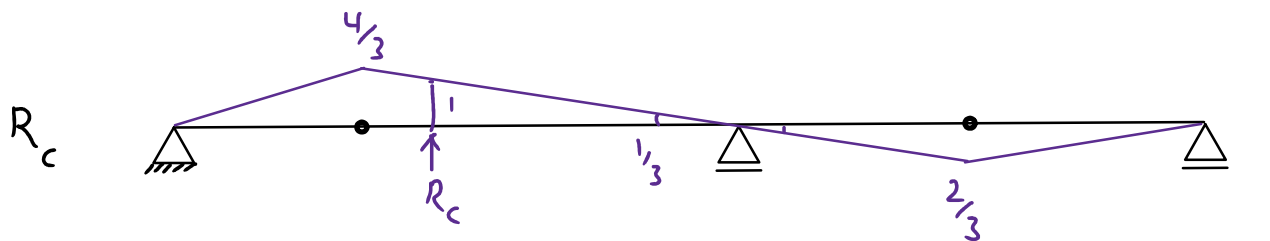
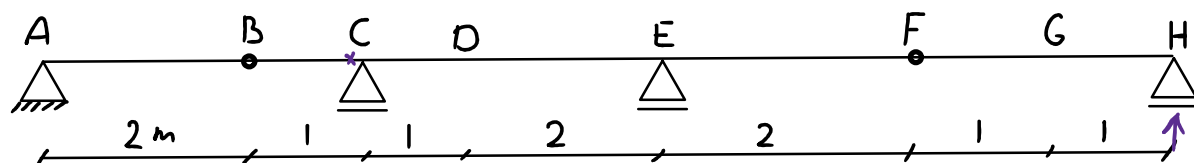


(M_D)



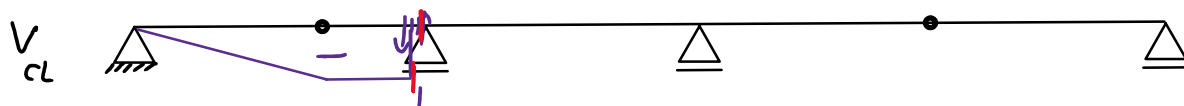
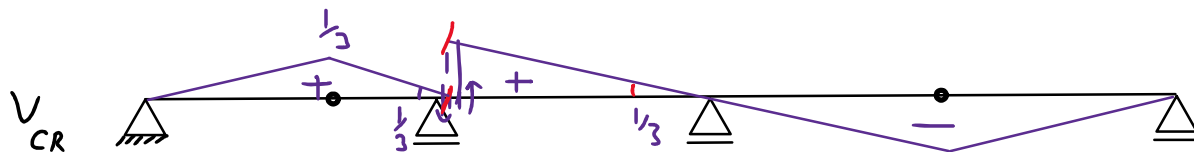
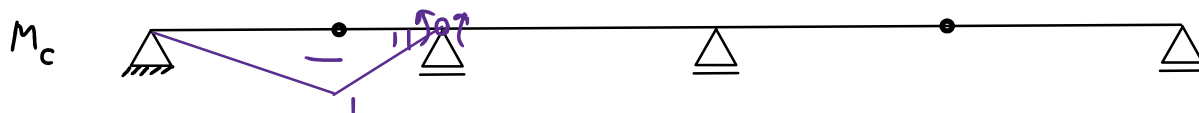
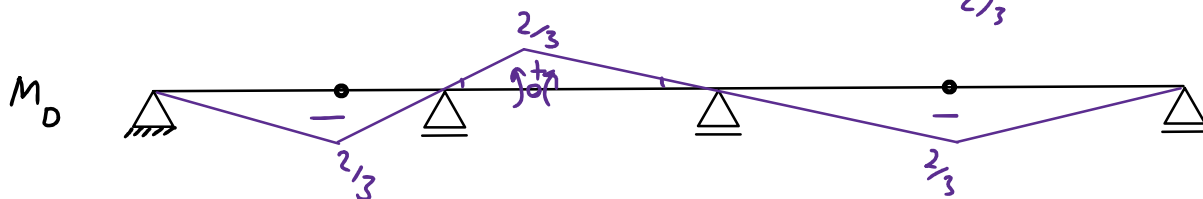
مثال: در تیر شکل زیر، خطوط تأثیر V_{CR} , M_c , M_D , V_D , R_H , R_c

V_F , V_{cl} را رسم نماید.

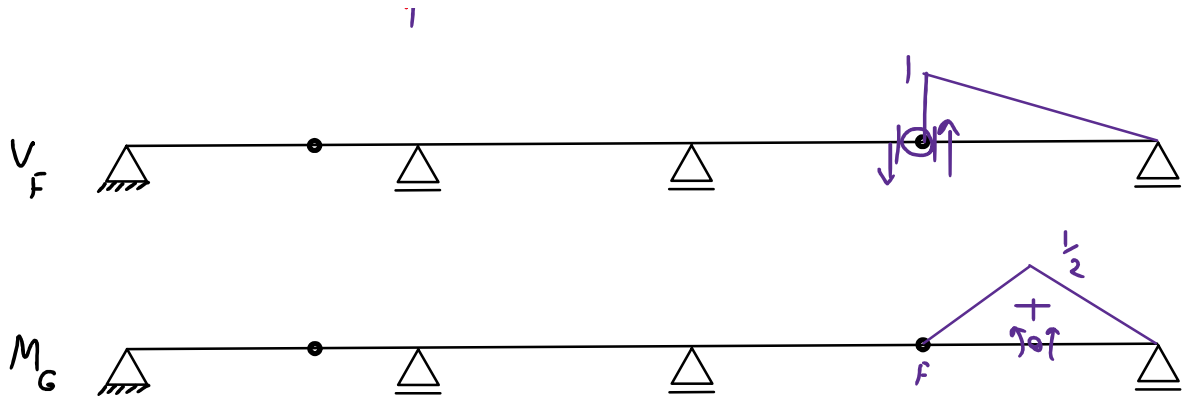


~~0.1 0.2 0.1~~

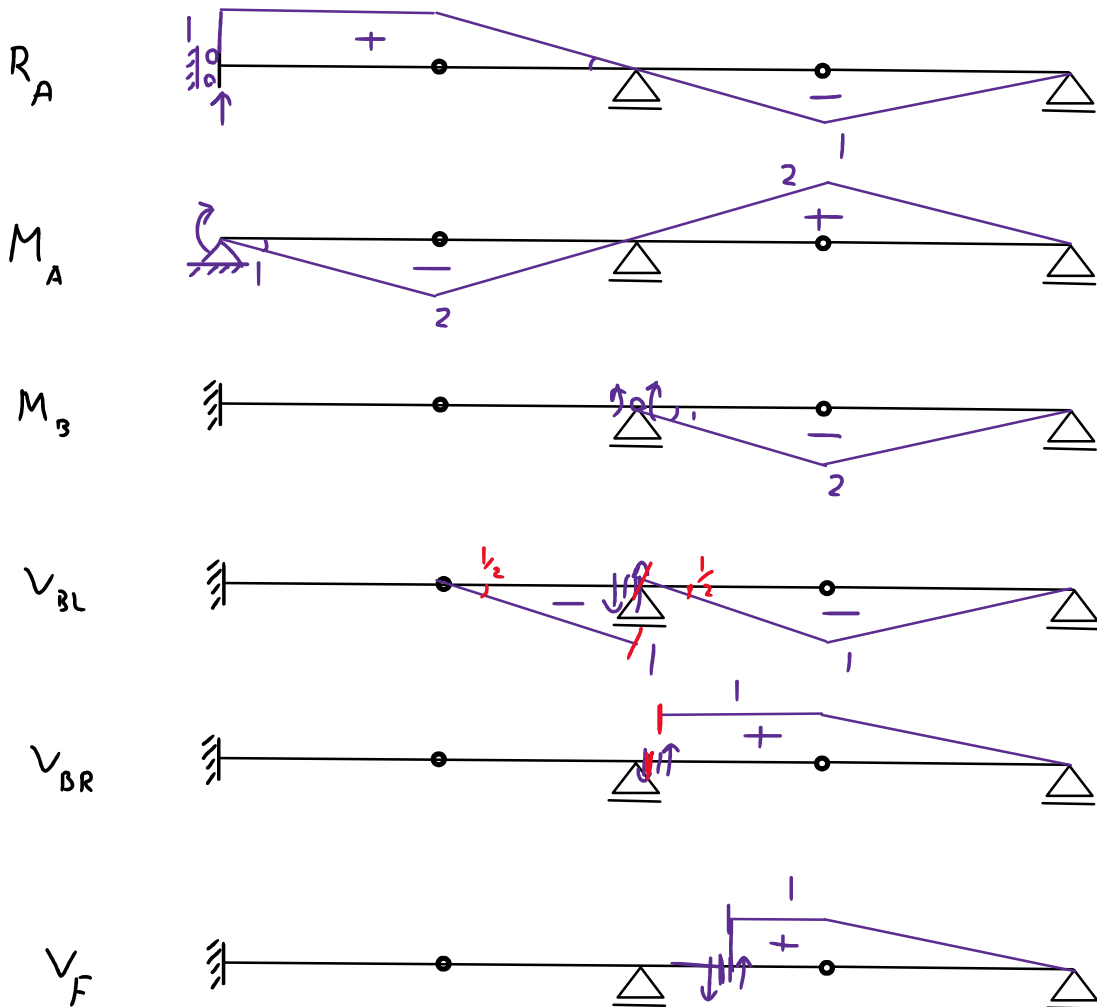
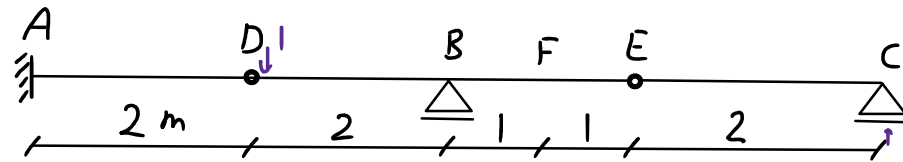
$$\frac{1 \times 2}{1+2}$$



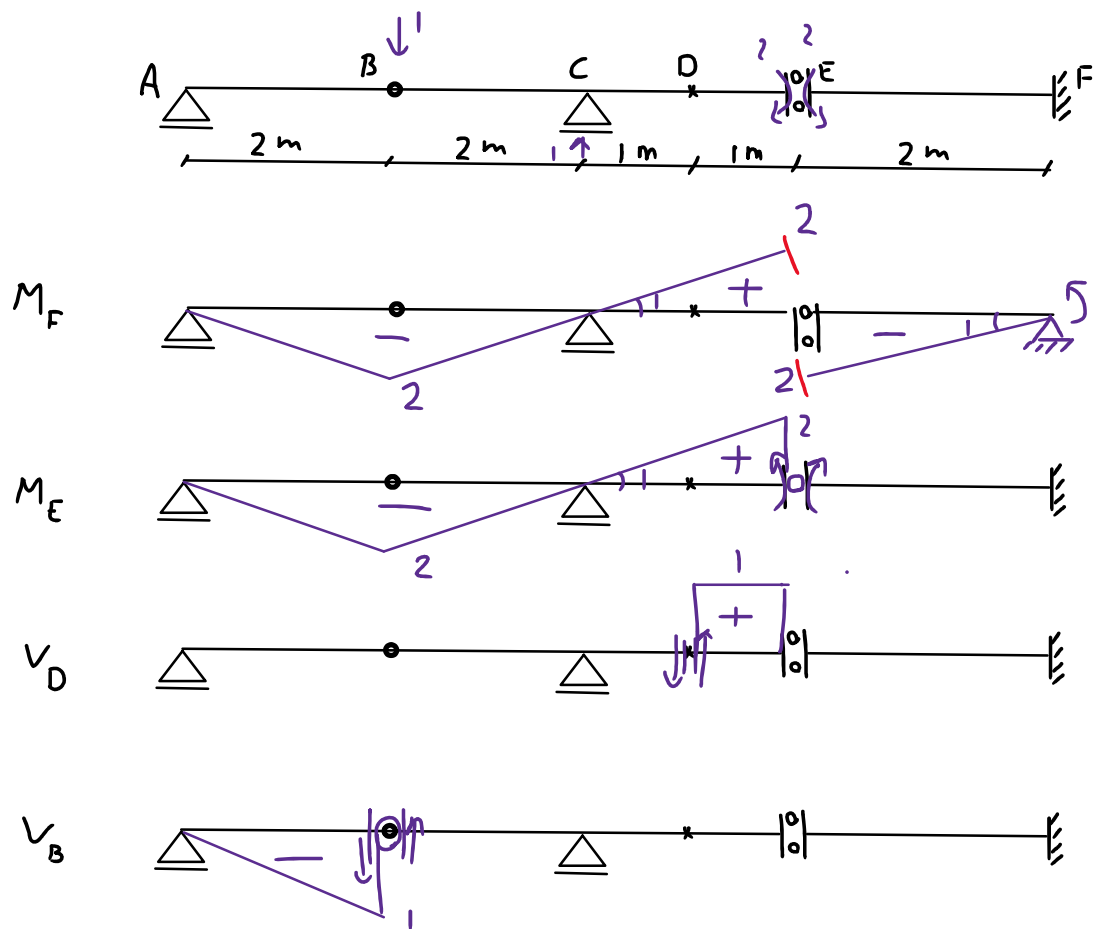
1/



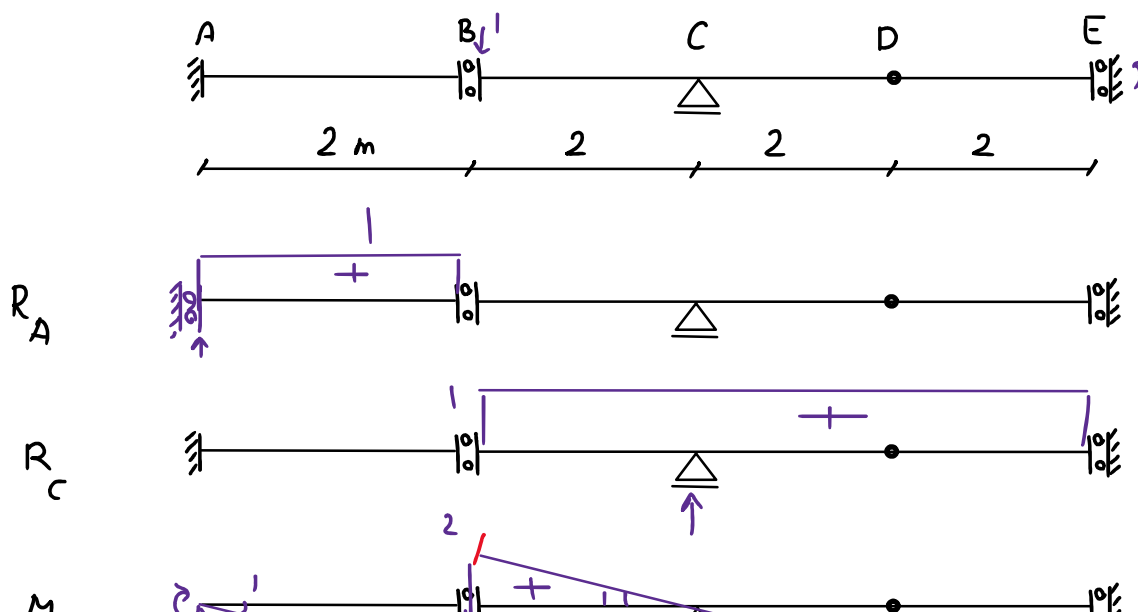
مثال: خطوط تأثیر R_A ، M_A ، M_B ، V_{BL} ، V_{BR} و V_F را رسم کنید.

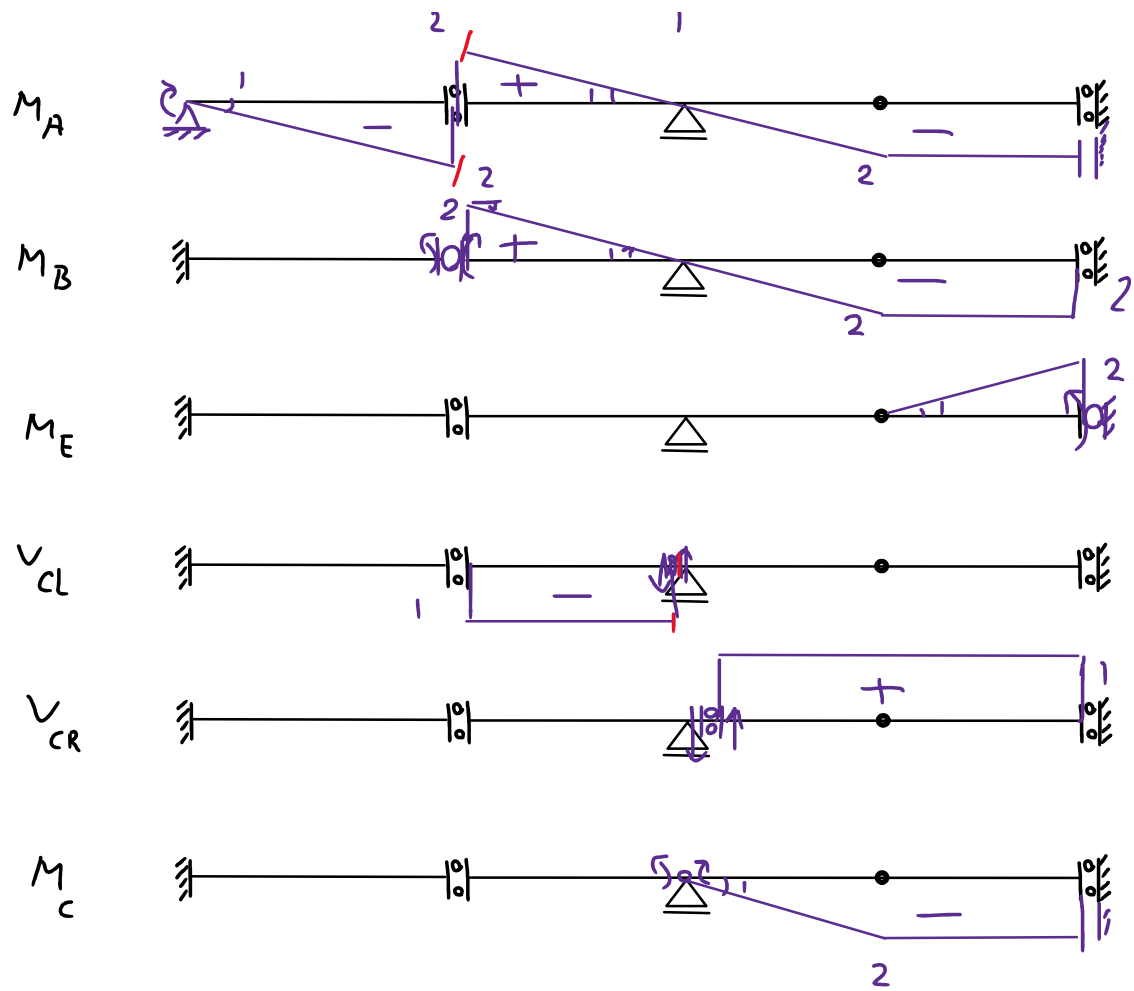


مثال: خطوط تأثیر V_D و V_B را رسم کنید.

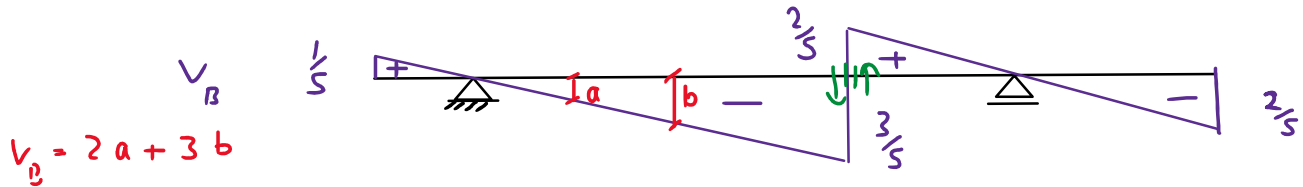
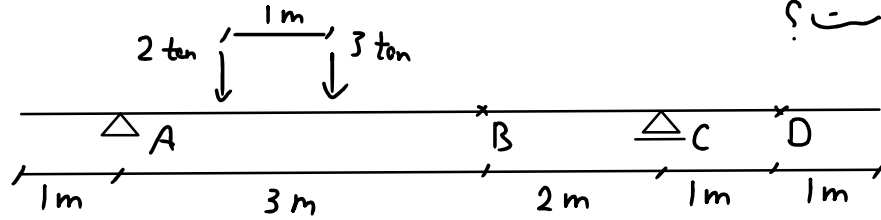


مثال: خطوط تأثیر M_A , M_B , M_C , M_E , V_{CL} , V_{CR} را رسم کنید.

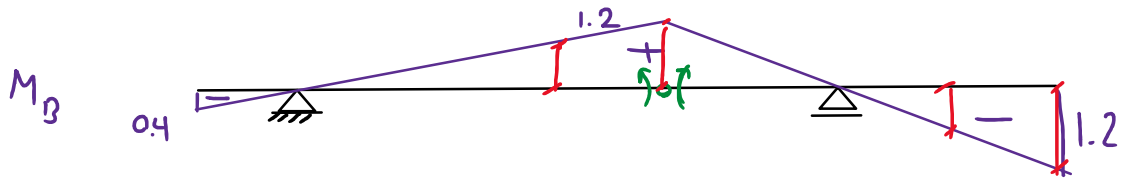




مثال: اگر دو بار مطابق شکل زیر روی تیر حرکت کند، ماکزیمم لنگر و برش در نقطه B چقدر است؟

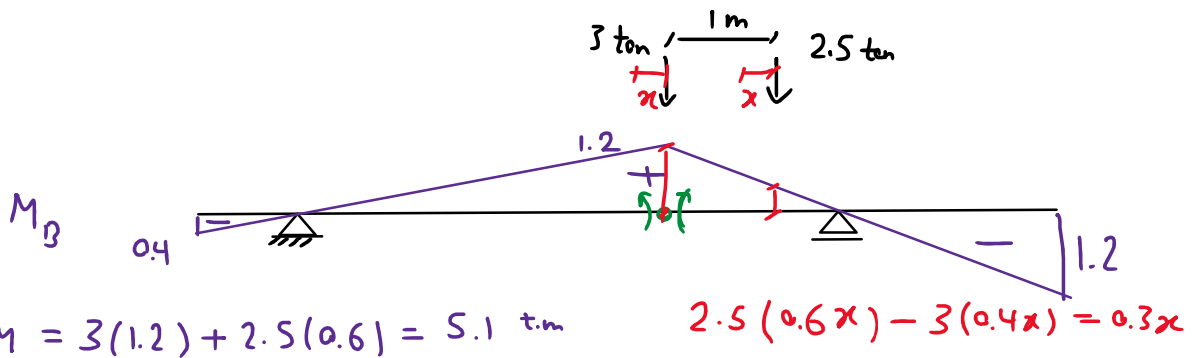


در B $V_{max} = 3(-0.6) + 2(-0.4) = \underline{\underline{-2.6 \text{ ton}}}$



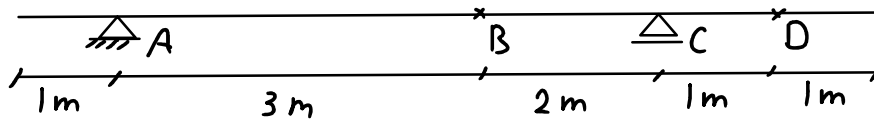
در B $M_{max}^+ = 2(0.8) + 3(1.2) = \underline{\underline{5.2 \text{ t.m}}}$

در انتهای تیر $M_{max}^- = 2(-0.6) + 3(-1.2) = \underline{\underline{-4.8 \text{ t.m}}}$



در B $M_{max}^+ = 3(0.8) + 2.5(1.2) = \underline{\underline{5.4 \text{ t.m}}}$

مثال: اگر بار گسترده 2 با طول متغیر روی تیر وارد شود، حداکثر لنگر و برش نقطه B را بدست آورید.



Shear Force Diagram (V) and Load Diagram (w):

The load diagram shows a uniformly distributed load (UDL) of w acting downwards over a length of 5m. The shear force diagram shows a linear variation of shear force from $V_B = \frac{1}{5}$ at support B to $V_C = \frac{2}{5}$ at support C. The area under the load diagram is divided into two triangles with heights $\frac{3}{5}$ and $\frac{2}{5}$ over lengths of 3m and 2m respectively.

$$V = \int (w dx) = w \int dx = w A$$

$$V_{max} = 2 \left[\frac{1}{2} \left(\frac{3}{5} \right) (3) + \frac{1}{2} \left(\frac{2}{5} \right) (2) \right] = \underline{\underline{2.6 \text{ ton}}}$$

Bending Moment Diagram (M) and Load Diagram (w):

The load diagram shows a UDL of w acting downwards over a length of 5m. The bending moment diagram shows a parabolic variation of moment. The maximum positive moment is $M_B^+ = 6 \text{ t.m}$ at support B. The diagram shows a peak of 1.2 at support B and a value of 0.4 at support C.

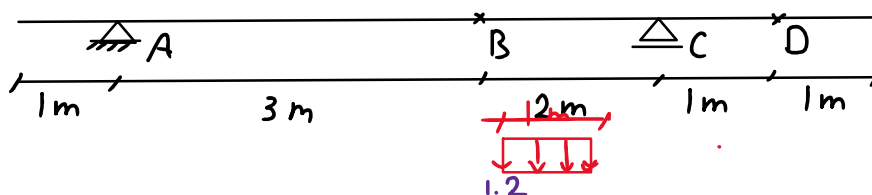
$$M_{max}^+ = 2 \left[\frac{1}{2} (1.2) (5) \right] = \underline{\underline{6 \text{ t.m}}}$$

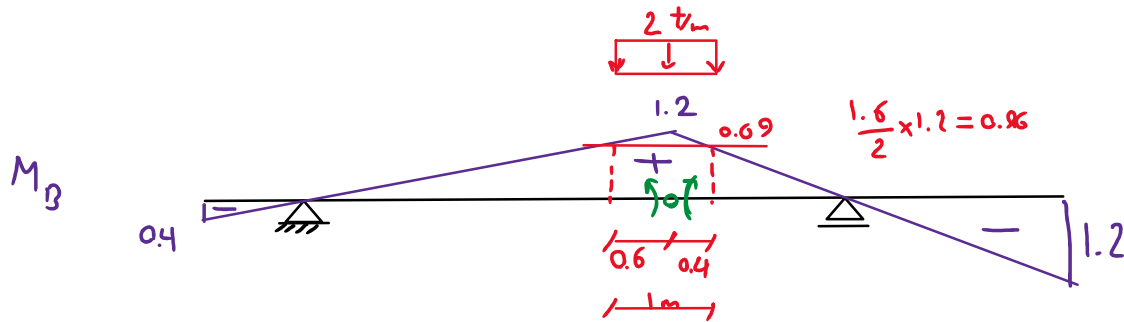
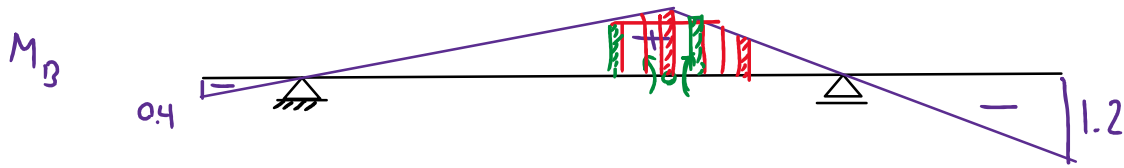
Bending Moment Diagram (M) and Load Diagram (w):

The load diagram shows a UDL of w acting downwards over a length of 5m. The bending moment diagram shows a parabolic variation of moment. The maximum negative moment is $M_{max}^- = 2.8 \text{ t.m}$ at support C. The diagram shows a peak of 1.2 at support B and a value of 0.4 at support C.

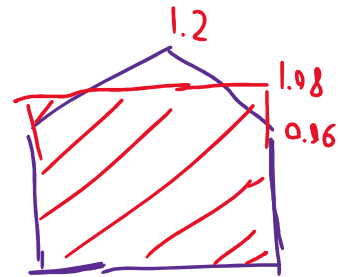
$$M_{max}^- = 2 \left[\frac{1}{2} (0.4) (1) + \frac{1}{2} (1.2) (2) \right] = \underline{\underline{2.8 \text{ t.m}}}$$

مثال: اگر بار گسترده 2 t/m به طول 1 m روی تیر وارد نشود، حداکثر لنگر و برش نقطه B را بدست آورید.

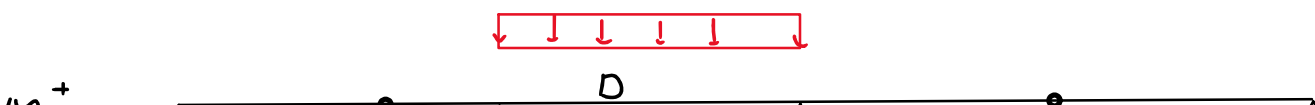
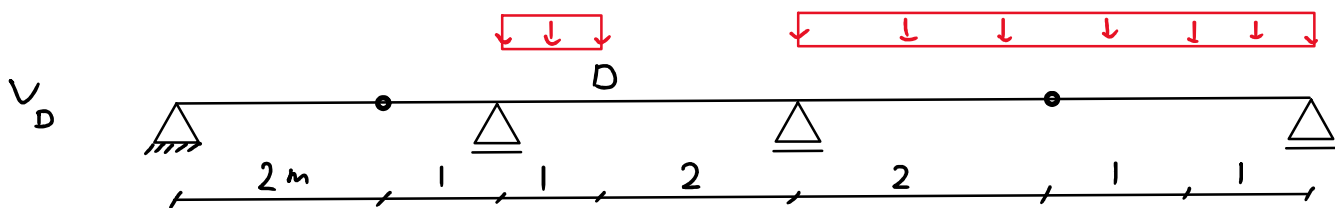
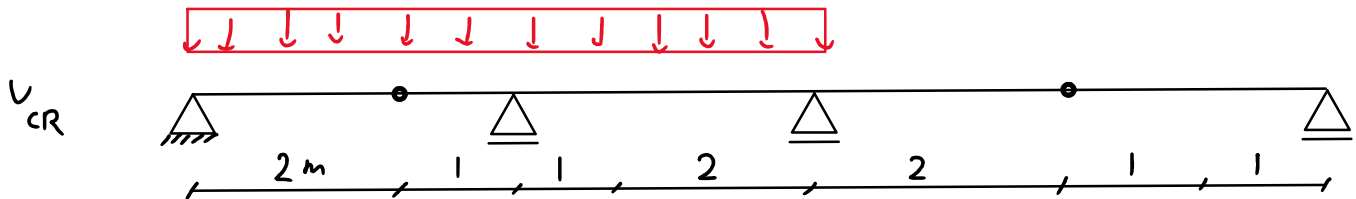
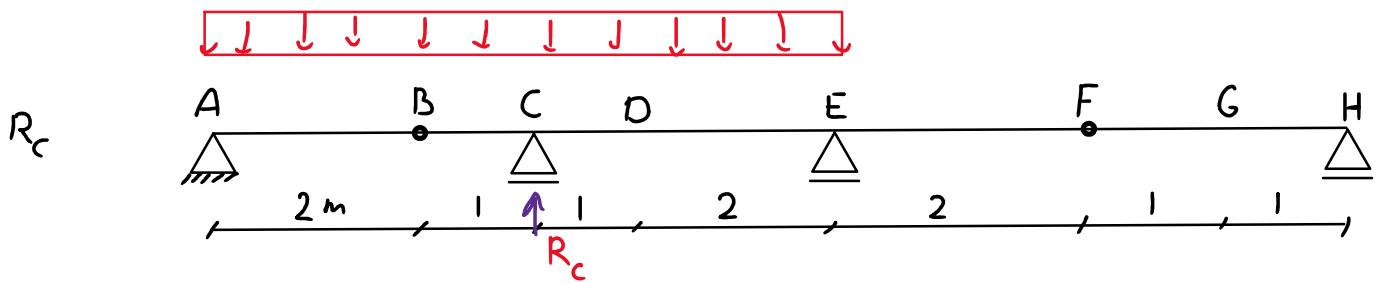


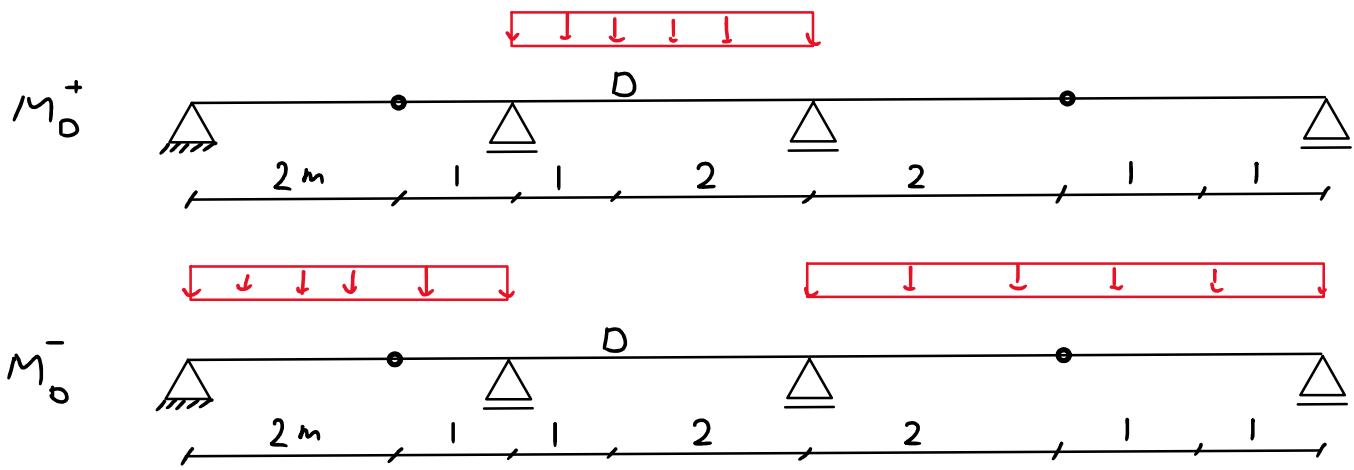


$$M_{max} = w A = 2 [1.08 \times 1] = \underline{\underline{2.16 \text{ t.m}}}$$

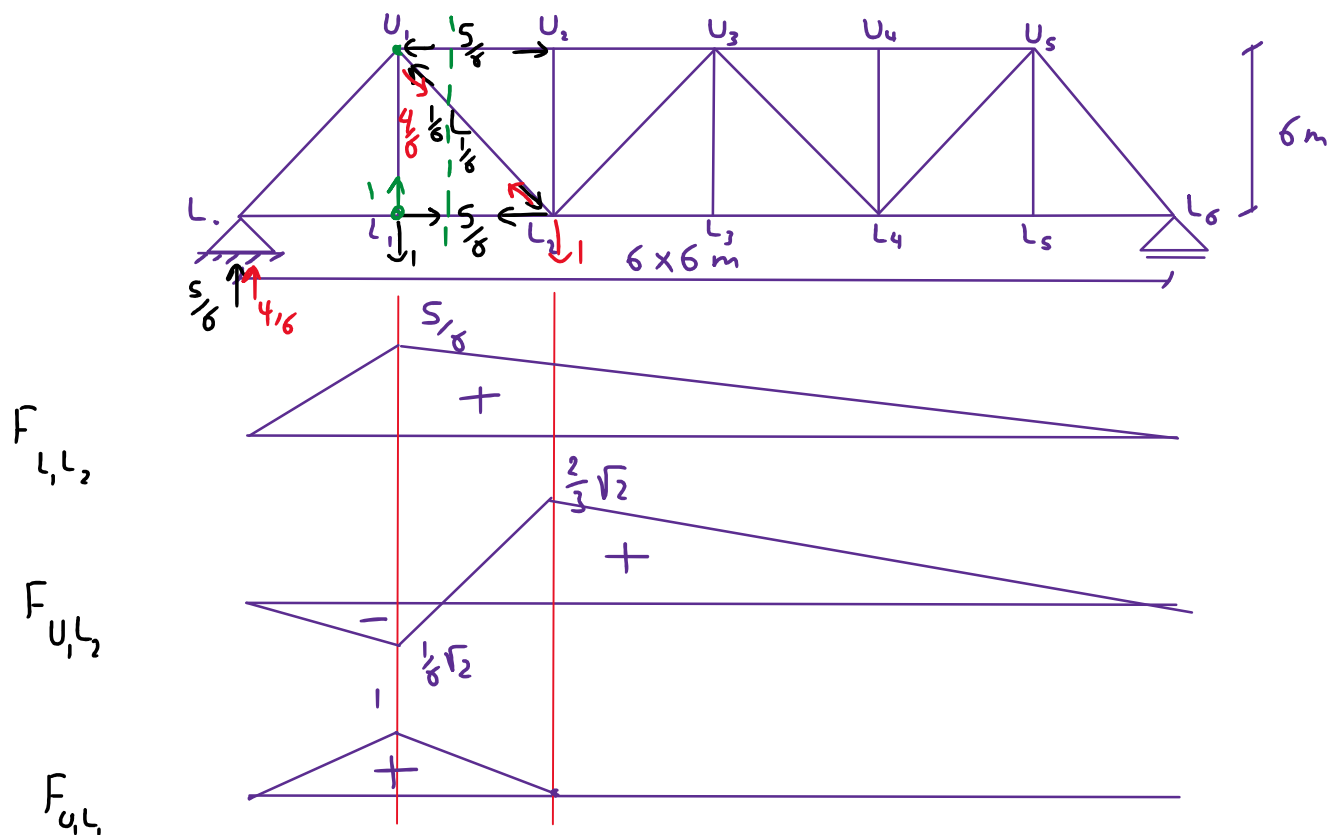


مثال: ماکزیمم R_c ، V_{cr} ، M_D و V_D در کدام حالت توزیع بار گسترده زنده اتفاق می افتد؟

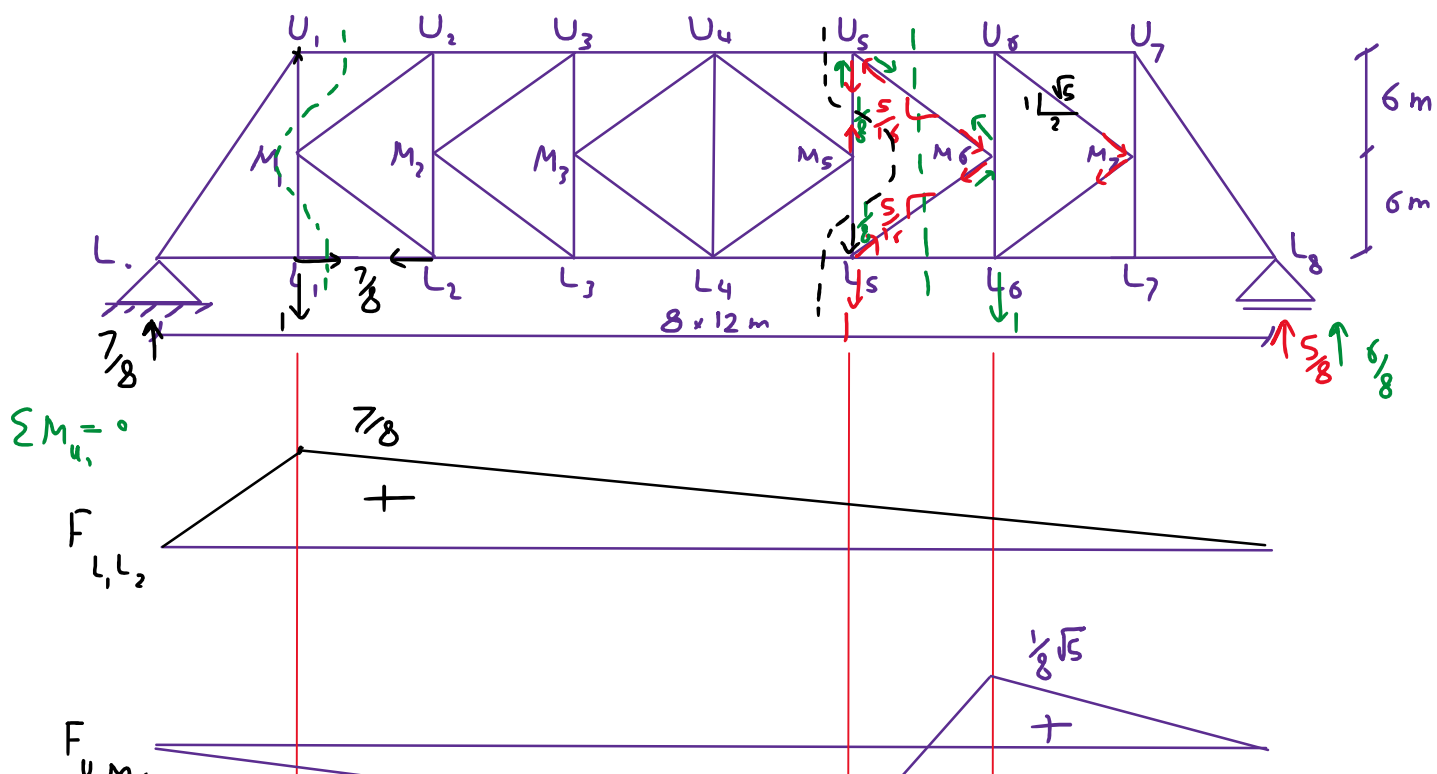


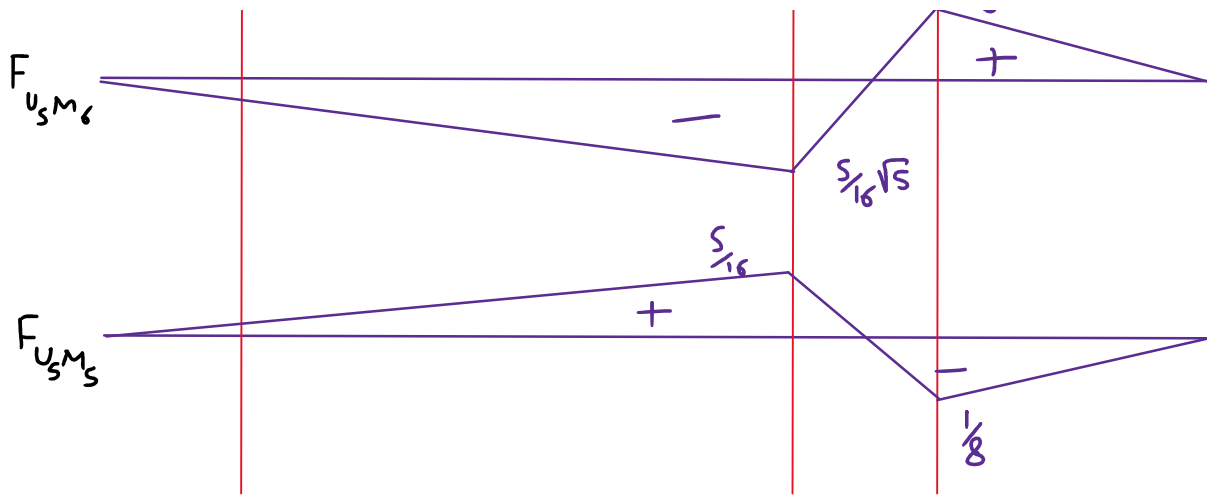


مثال: خط تأثیر بزرگ اعضای U_1L_2 و U_1L_1 را رسم نمایید.

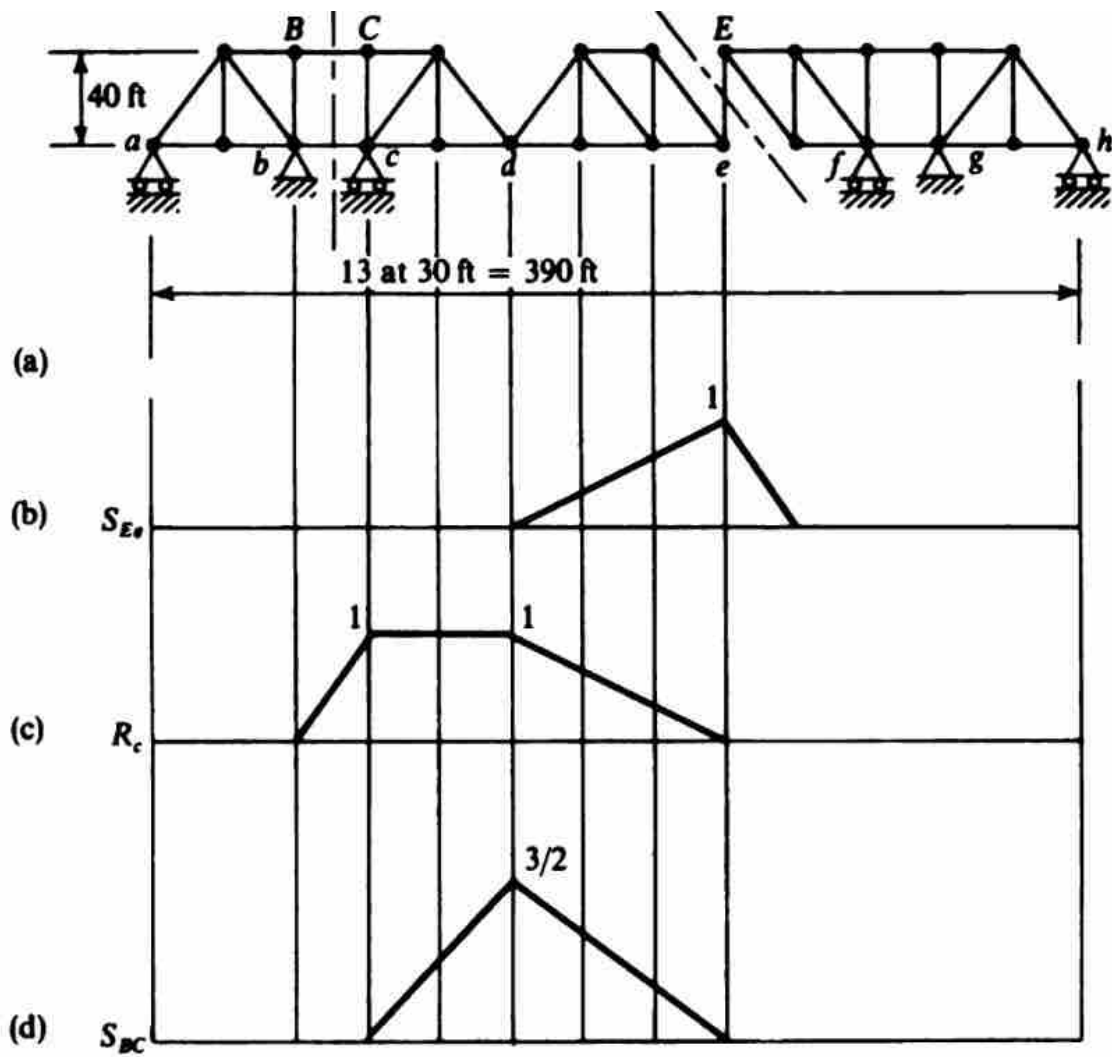


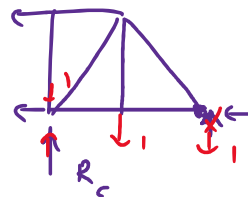
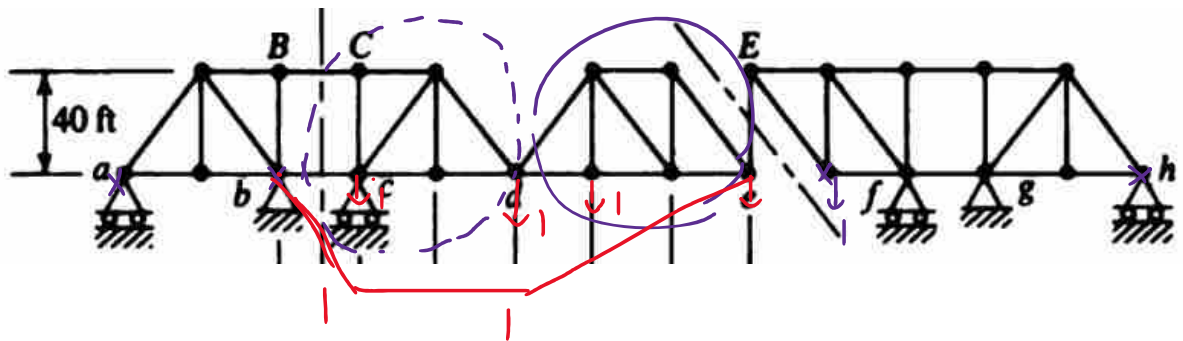
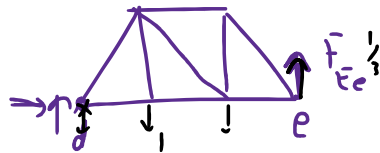
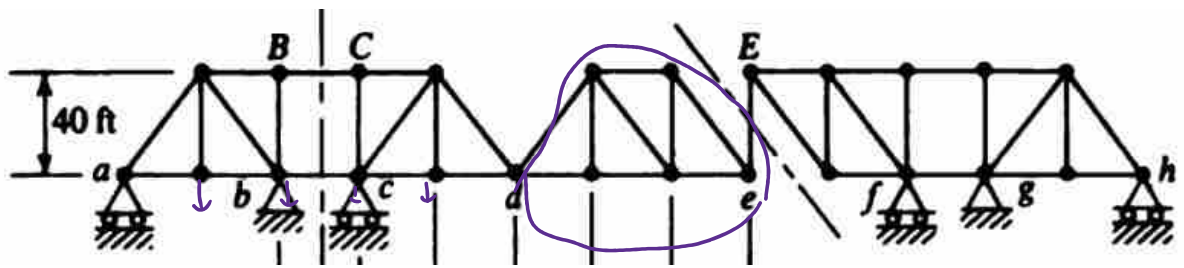
مثال: خط تأثیر بزرگ اعضای U_5M_6 و U_5M_5 را رسم نمایید.



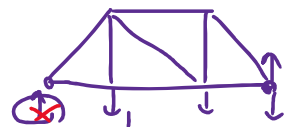
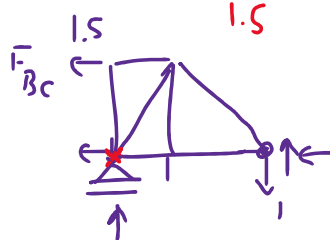
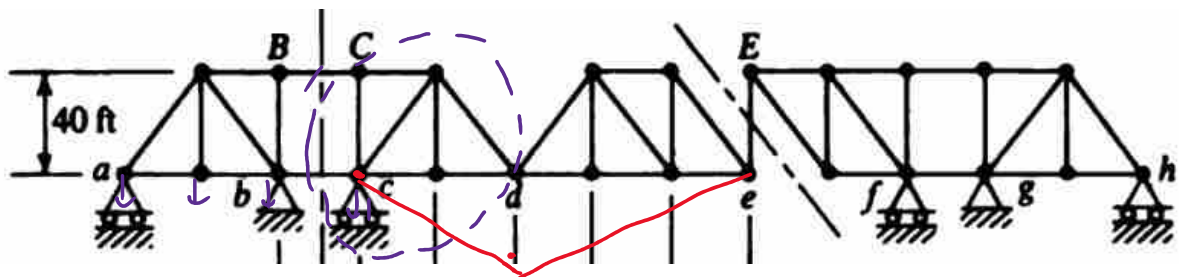
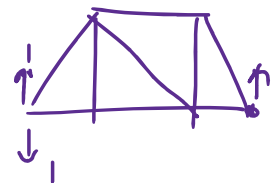
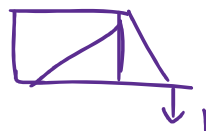
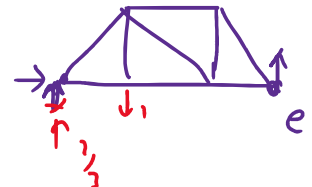


مثال: مطلوب است خط تأثیر اعف و Ee ، BC و عکس العمل تکیه گاه c .





$$\sum F_y = 0$$



مثال: خط تأثیر اعضاء EF, FD را رسم کنید.

$$1 \times 12 - \frac{4}{6} \times 4 - F_{FD} \times 12 = 0$$

$$1 - \frac{4}{6} = F_{FD} \rightarrow \frac{2}{3}$$

$$-\frac{1}{2} \times 4 = F_{FD} \times 12$$

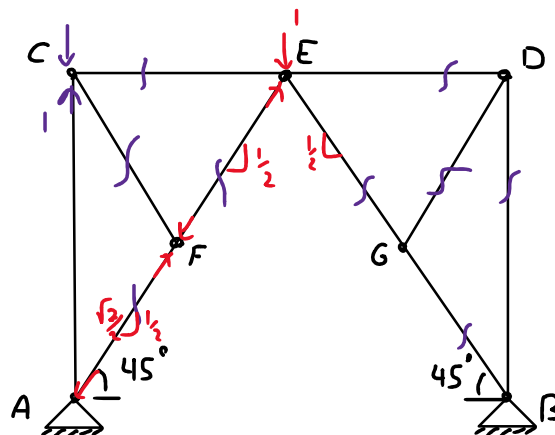
F_{FD}

$$\frac{4}{6} \times 8 + \frac{4}{6} F \times 2 + \frac{1}{6} F \times 4 = 0$$

F_{EF}

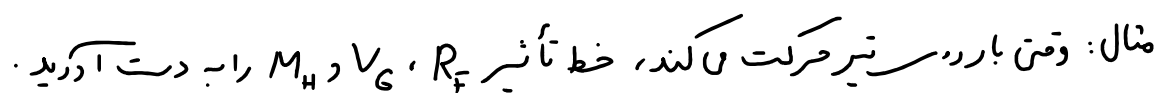
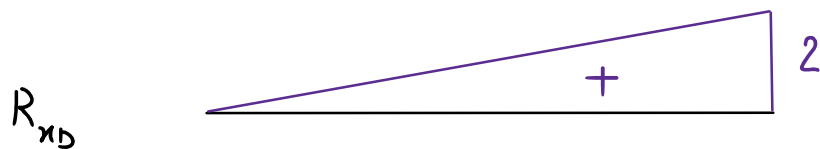
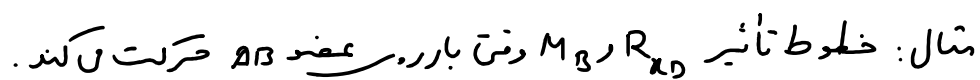
1.83

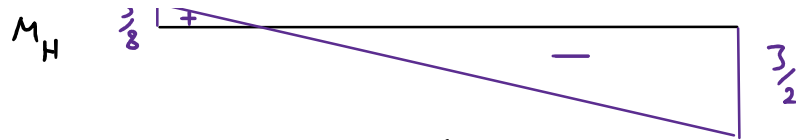
مثال: خط تأثیر اعضاء AC, AF را رسم نمایید؛ وقتی بار در CD حرکت می‌کند.



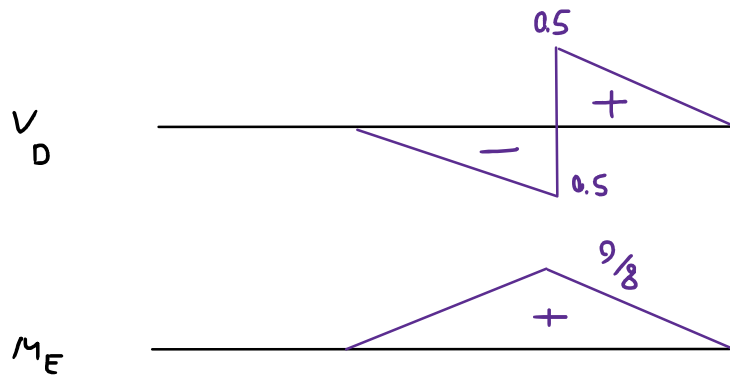
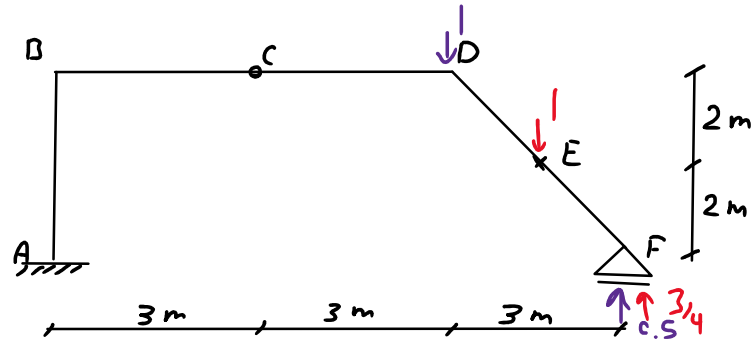
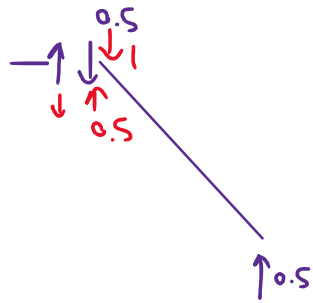
F_{AC}



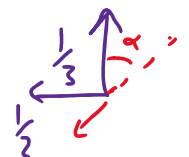
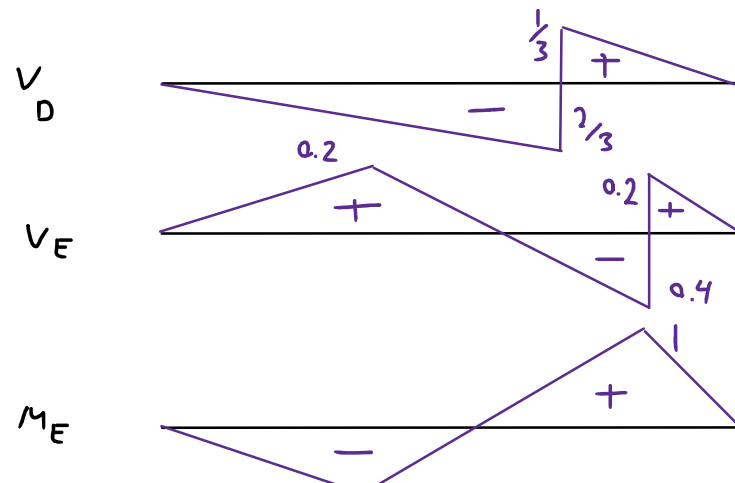
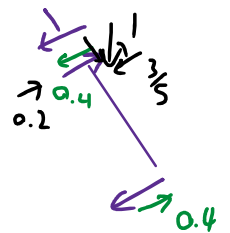
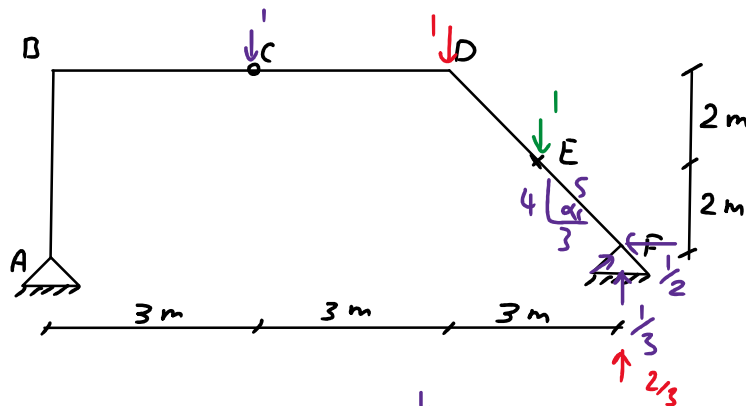
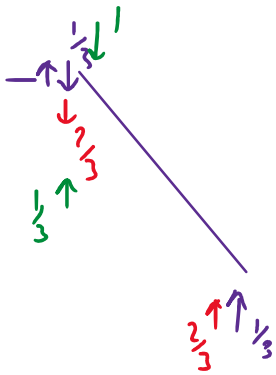




مثال: خط تأثیر V_D و M_E را وقتی بار روی B C D F حرکت می‌کند، رسم کنید.



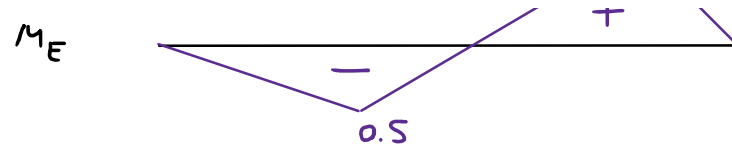
مثال: خط تأثیر V_D و V_E و M_E را وقتی بار روی B C D F حرکت می‌کند، رسم کنید.



$$\frac{1}{3} \cos \alpha - \frac{1}{2} \sin \alpha$$

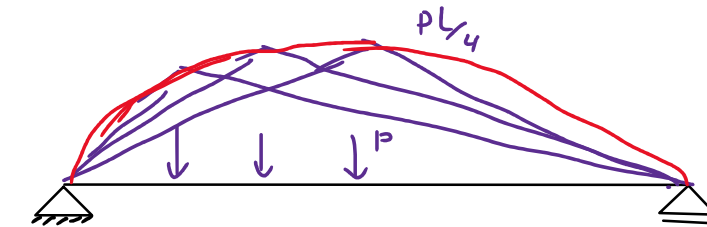
$$\frac{1}{3} \times \left(\frac{3}{5}\right) - \frac{1}{2} \left(\frac{4}{5}\right) = -\frac{1}{5}$$

$$\frac{8}{6} \left(\frac{3}{5}\right) - \frac{1}{8} \left(\frac{4}{5}\right) = 0.4$$



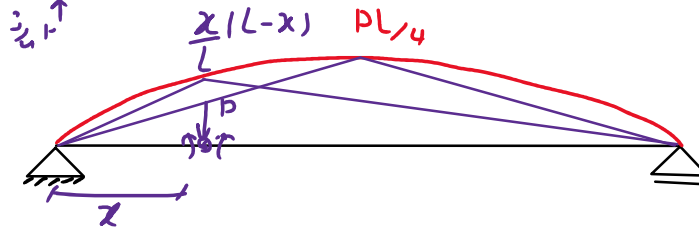
دیگرام پوشی (envelope)

① بار متحرک در حال حرکت



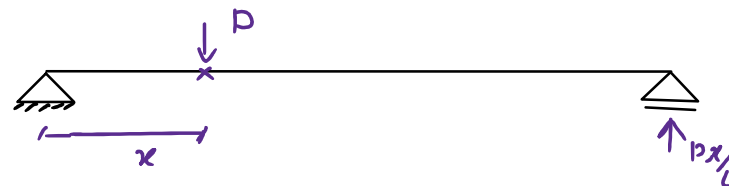
از خط تاثیر

تابع پوشی
 $M = P(x - \frac{x^2}{L})$

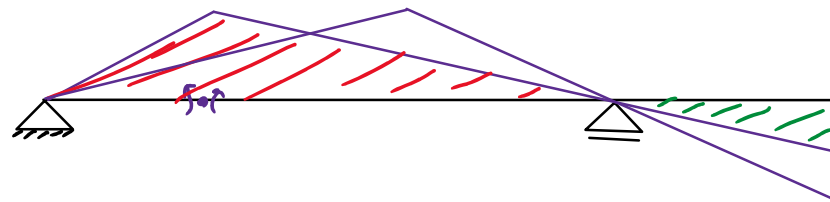
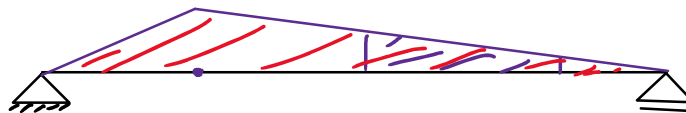


از تعیل

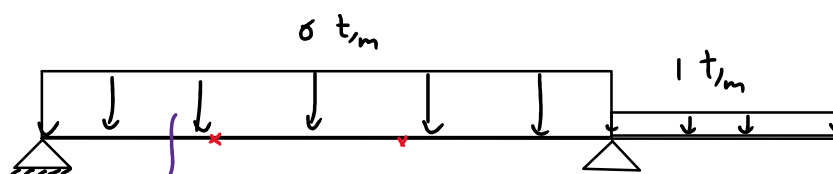
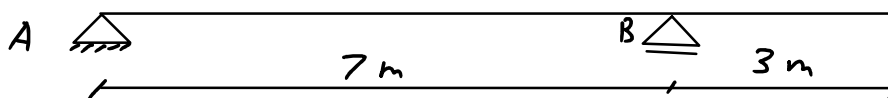
$M = \frac{Px}{L}(L-x)$
 تابع پوشی
 $M = P(x - \frac{x^2}{L})$



② بار متحرک با طول متغیر



مثال: دیگرام پوشی لنگر برای تشکیل زیر بار بدست آورید. بار متحرک ضربدار پیوسته ۱ و بار زنده ضربدار پیوسته ۵ می باشد.

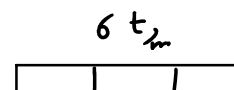


پوشی لنگر مثبت

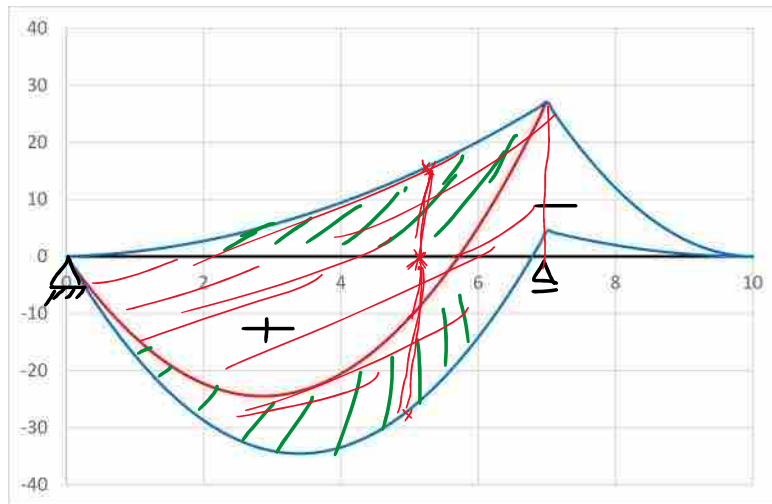
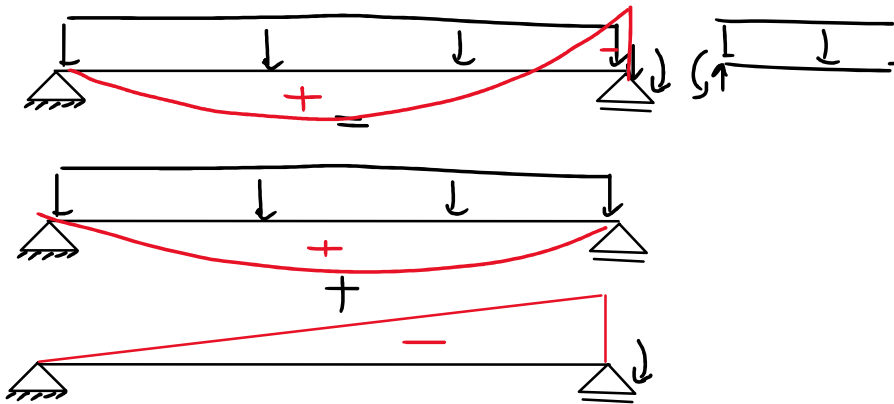
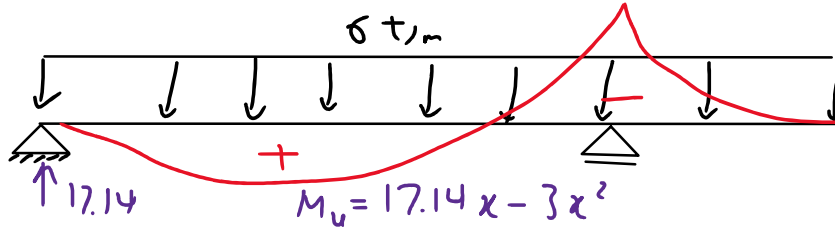
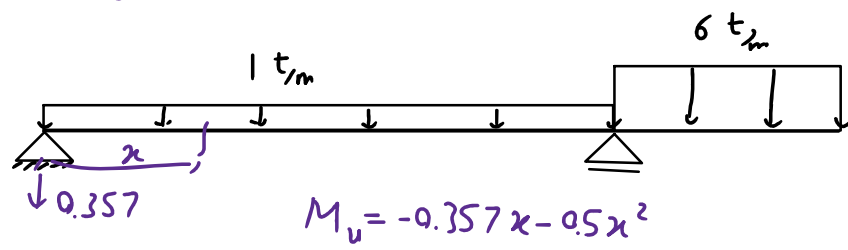


$M_u = 20.36x - 3x^2$

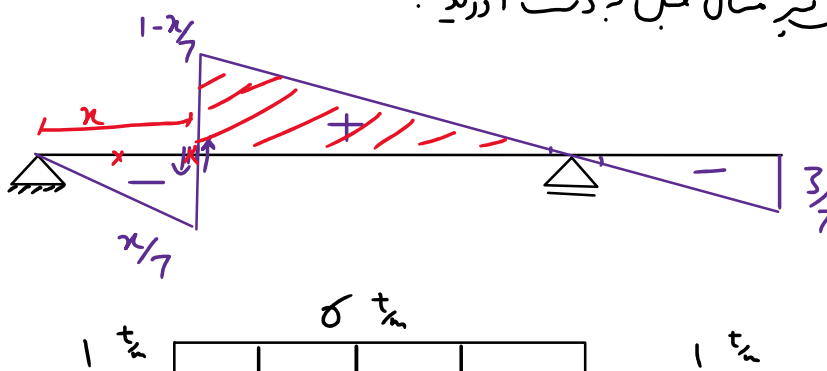
1 t/m

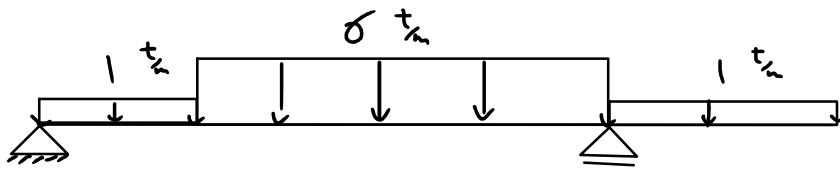


بوتی لگرمین



مثال : دیاگرام بدشی برش را برابر تیر مثال قبل بدست آورید .



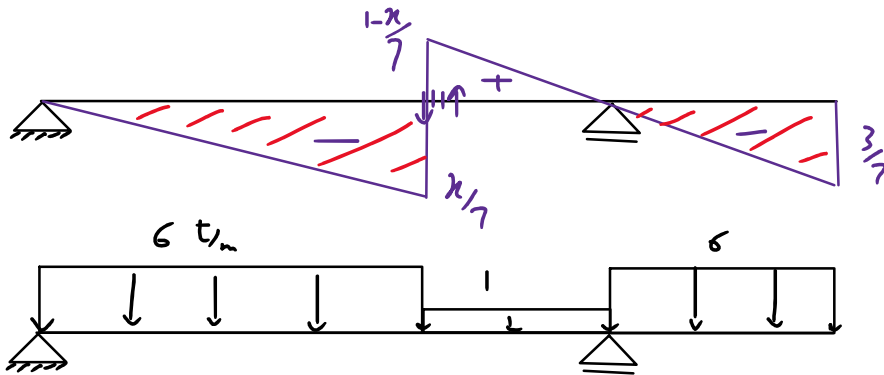


برش مثبت

$$V = -1\left(\frac{1}{2} \times \frac{x}{7} \times x\right) + 6\left(\frac{1}{2} \times \left(1 - \frac{x}{7}\right) \times (7 - x)\right) - 1\left(\frac{1}{2} \times \frac{3}{7} \times 3\right)$$

$$V = -\frac{1}{14}x^2 + \frac{3}{7}(49 + x^2 - 14x) - \frac{9}{14}$$

$$V = \frac{5}{14}x^2 - 6x + \frac{285}{14}$$

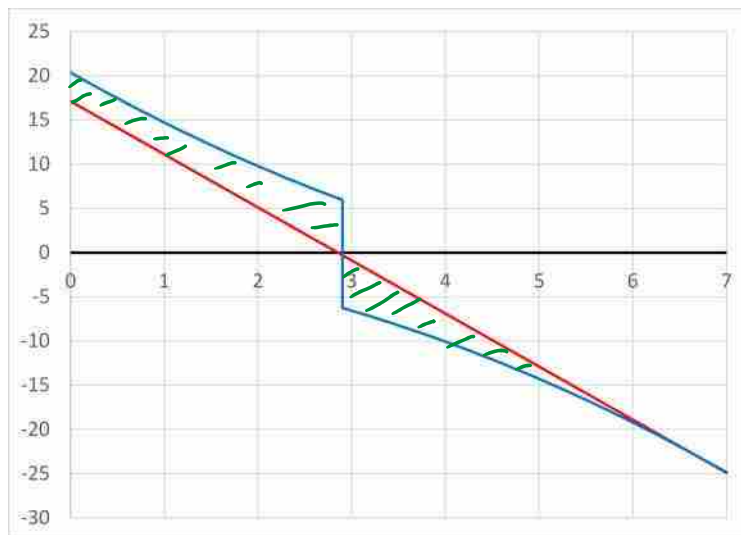


برش منفی

$$V = -6\left(\frac{1}{2} \times \frac{x}{7} \times x\right) + 1\left(\frac{1}{2} \times \left(1 - \frac{x}{7}\right) \times (7 - x)\right) - 6\left(\frac{1}{2} \times \frac{3}{7} \times 3\right)$$

$$V = -\frac{6}{14}x^2 + \frac{1}{14}(49 + x^2 - 14x) - \frac{54}{14}$$

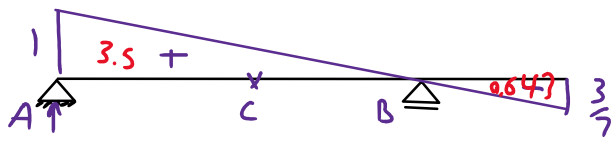
$$V = -\frac{5}{14}x^2 - x - \frac{5}{14}$$



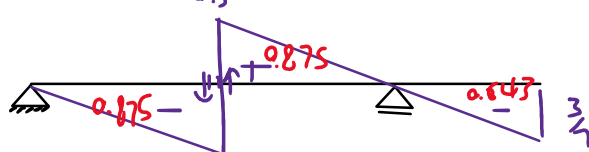
رسم بوش به صورت تقریبی

1

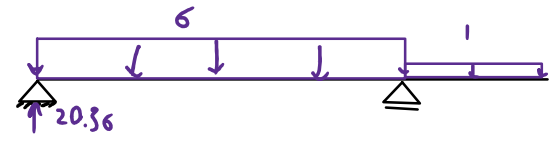
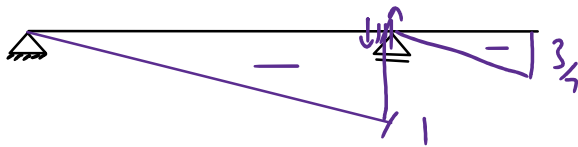
6



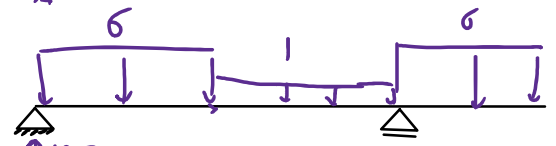
$$V_A = +6 \times 3.5 - 1 \times 0.643 = 20.36$$



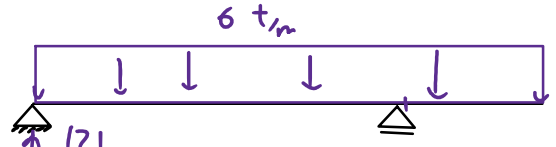
$$V_C^+ = -1 \times 0.875 + 6 \times 0.875 - 1 \times 0.643 = 3.73$$



$$V_A = 20.36$$



$$V_C^- = 12.76 - 6 \times 3.5 = -8.23$$



$$V_{BL} = 17.1 - 6 \times 7 = -24.9$$

$$V_{BR} = 6 \times 3 = 18$$

